FUZZY INTERIOR IDEALS IN HYPERSEMIGROUPS

Niovi Kehayopulu

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Abstract

We introduce the concept of interior ideal and the concept of fuzzy interior ideal in hypersemigroups and we prove, among others, that in regular also in intra-regular hypersemigroups the interior ideals and the fuzzy interior ideals coincide. We also prove that an hypergroupoid H is simple if and only if every fuzzy ideal of H is a constant function; and that an hypersemigroup H is simple if and only if every fuzzy interior ideal of H is a constant function, equivalently if, for every element a of H, we have $H = H * \{a\} * H$.

1 Introduction

This paper is based on our paper [5] and partly on [6]. We first introduce the concept of an interior ideal and the concept of a fuzzy interior ideal of an hypersemigroup and we prove that if H is an hypersemigroup and A an interior ideal of H, then the characteristic mapping f_A is a fuzzy interior ideal of H. "Conversely", if A is a nonempty subset of H and f_A a fuzzy interior ideal of H, then the set A is an interior ideal of H. Then we prove that any fuzzy ideal of an hypersemigroup H is a fuzzy interior ideal of H and in regular, also in intra-regular hypersemigroups the concepts of interior ideals and fuzzy interior ideals coincide. We also prove that in a regular and in an intra-regular hypersemigroup H the interior ideals are subsemigroups of H. Following Kuroki, we call an hypergroupoid H fuzzy simple if every fuzzy ideal of H is a constant function. We prove that an hypergroupoid is simple if and only if it is fuzzy simple, and an hypersemigroup H is simple if and only $H = H*\{a\}*H$ for every $a \in H$, equivalently, if every fuzzy interior ideal of H is a constant function. As a consequence, for an hypersemigroup H, the following are equivalent: (1) H is simple. (2) $H = H * \{a\} * H$ for every $a \in H$. (3) H is fuzzy simple. (4) every fuzzy interior ideal of H is a constant function.

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2 Prerequisites

For the sake of completeness, we will give some definitions already given in [2]. An *hypergroupoid* is a nonempty set H with an hyperoperation

$\circ : H \times H \to \mathcal{P}^*(H) \mid (a,b) \to a \circ b$ on H and an operation

$* : \mathcal{P}^*(H) \times \mathcal{P}^*(H) \to \mathcal{P}^*(H) \mid (A,B) \to A * B$ on $\mathcal{P}^*(H)$ (induced by the operation of H) such that $A * B = \bigcup\limits_{(a,b)\in A\times B} (a \circ b)$ for every $A, B \in \mathcal{P}^*(H)$ ($\mathcal{P}^*(H)$ being the set of nonempty subsets of H). As the operation "$*$" depends on the hyperoperation "$\circ$", an hypergroupoid can be denoted by $(H,\circ)$ (instead of $(H,\circ,*)$). If $(H,\circ)$ is an hypergroupoid then, for every $x, y \in H$, we have $\{x\} * \{y\} = \bigcup\limits_{a\in\{x\},\, b\in\{y\}} (a \circ b) = x \circ y$. The following proposition, though clear, plays an essential role in the theory of hypergroupoids.

Proposition 2.1. *Let $(H,\circ)$ be an hypergroupoid, $x \in H$ and $A, B \in \mathcal{P}^*(H)$. Then we have the following:*
(1) *If $x \in A * B$, then $x \in a \circ b$ for some $a \in A$, $b \in B$ and*
(2) *If $a \in A$ and $b \in B$, then $a \circ b \subseteq A * B$.*

Proposition 2.2. *If $(H,\circ)$ is an hypergroupoid then, for every $A, B, C, D \in \mathcal{P}^*(H)$, we have*
(1) *$A \subseteq B \Rightarrow A * C \subseteq B * C$ and $C * A \subseteq C * B$, equivalently,*
$A \subseteq B$ and $C \subseteq D \Rightarrow A * C \subseteq B * D$.
(2) *$H * A \subseteq H$ and $A * H \subseteq H$.*

Definition 2.3. Let $(H,\circ)$ be an hypergroupoid. A nonempty subset A of H is called a *left* (resp. *right*) *ideal* of H if $H * A \subseteq A$ (resp. $A * H \subseteq A$). If A is both a left and a right ideal of H, then it is called an *ideal* of H. A nonempty subset A of H is called a *subgroupoid* of H if $A * A \subseteq A$.
Clearly, every left (resp. right) ideal of H is a subgroupoid of H.

Definition 2.4. An hypergroupoid $(H,\circ)$ is called *hypersemigroup* if

$$\{x\} * (y \circ z) = (x \circ y) * \{z\}$$

for every $x, y, z \in H$. Since $\{x\} * \{y\} = x \circ y$ for every $x, y \in H$, this is equivalent to saying that $\{x\} * \Big(\{y\} * \{z\}\Big) = \Big(\{x\} * \{y\}\Big) * \{z\}$ for every $x.y, z \in H$.

Proposition 2.5. ([1,2]; for its proof we refer to [4]) If $(H,\circ)$ be an hypersemigroup, then $(\mathcal{P}^*(H), *)$ is a semigroup.

As a result, for any $A, B, C \in \mathcal{P}^*(H)$, we write $A * (B * C) = (A * B) * C := A * B * C$; and in an expression of the form $A_1 * A_2 * ..... * A_n$, where the A_i $(i = 1, 2, ..., n)$ are elements of $\mathcal{P}^*(H)$ we can put parentheses in any place beginning with some A_i and ending in some A_j $(1 \le i, j \le n)$.

Following Zadeh, any mapping $f : H \to [0,1]$ of an hypergroupoid H into the closed interval $[0,1]$ of real numbers is called a *fuzzy subset* of H (or a *fuzzy* *set* in H) and, for any nonempty subset A of H, the characteristic function f_A of A, is the fuzzy subset of H defined by

$$f_A : H \to \{0,1\} \mid x \to f_A(x) = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{if } x \notin A. \end{cases}$$

The concepts of fuzzy left ideals and fuzzy right ideals of semigroups due to Kuroki [6], are the following: A fuzzy subset f of a semigroup S is called a fuzzy left (resp. fuzzy right) ideal of S if, for every $x, y \in S$, we have $f(xy) \geq f(y)$ (resp. $f(xy) \geq f(x)$). It is called a fuzzy ideal of S if it is both a fuzzy left and a fuzzy right ideal of S. These concepts can be transferred, in a natural way, to an hypergroupoid as follows:

Definition 2.6. [3] Let $(H, \circ)$ be an hypergroupoid. A fuzzy subset f of H is called a *fuzzy left ideal* of H if

$$f(x \circ y) \geq f(y) \text{ for all } x, y \in H,$$

in the sense that if $x, y \in H$ and $u \in x \circ y$, then $f(u) \geq f(y)$.
A fuzzy subset f of H is called a *fuzzy right ideal* of H if

$$f(x \circ y) \geq f(x) \text{ for all } x, y \in H,$$

meaning that if $x, y \in H$ and $u \in x \circ y$, then $f(u) \geq f(x)$.
A fuzzy subset f of H is called a *fuzzy ideal* of H it is both a fuzzy left ideal and a fuzzy right ideal of H. As one can easily see, a fuzzy subset f of H is a fuzzy ideal of H if and only $f(x \circ y) \geq \max\{f(x), f(y)\}$ for all $x, y \in H$, in the sense that $x, y \in H$ and $u \in x \circ y$ implies $f(u) \geq \max\{f(x), f(y)\}$.

3 Main results

Definition 3.1. Let H be an hypersemigroup. A nonempty subset A of H is called an *interior ideal* of H if

$$H * A * H \subseteq A.$$

By a *subidempotent interior ideal* of H we mean an interior ideal of H which is at the same time a subsemigroup of H.

The concept of fuzzy interior ideal of semigroups is also due to Kuroki [6], and it is the following: A fuzzy subset f of a semigroup S is called a fuzzy interior ideal of S if, for any $x, a, y \in S$, we have $f(xay) \geq f(a)$. This concept can be naturally transferred to an hypersemigroup as follows:

Definition 3.2. Let H be an hypersemigroup. A fuzzy subset f of H is called a *fuzzy interior ideal* of H if

$$f\Big((x \circ a) * \{y\}\Big) \geq f(a) \text{ for every } x, a, y \in H,$$

in the sense that if $x, a, y \in H$ and $u \in (x \circ a) * \{y\}$, then $f(u) \geq f(a)$.
For an hypersemigroup, we clearly have

$$(x \circ a) * \{y\} = \{x\} * (a \circ y) = \{x\} * \{a\} * \{y\}.$$

Proposition 3.3. *Let H be an hypersemigroup. If A is an interior ideal of H, then the characteristic function f_A is a fuzzy interior ideal of H. "Conversely", if A is a nonempty subset of H such that f_A is a fuzzy interior ideal of H, then A is an interior ideal of H.*

Proof. $\Longrightarrow$. Let $x, a, y \in H$. Then $f_A\Big((x \circ a) * \{y\}\Big) \geq f_A(a)$. In fact: Let $u \in (x \circ a) * \{y\}$. If $a \in A$, then $f_A(a) = 1$. Since A is an interior ideal of H, we have $H * A * H \subseteq A$. So we have $u \in \{x\} * \{a\} * \{y\} \subseteq H * A * H \subseteq A$. Then $u \in A$, and $f_A(u) = 1$. Thus we get $f_A(u) \geq f_A(a)$. Let now $a \notin A$. Then $f_A(a) = 0$. Since f_A is a fuzzy subset of H and $u \in H$, we have $f_A(u) \geq 0$. Thus we have $f_A(u) \geq f_A(a)$.
$\Longleftarrow$. Let A be a nonempty subset of H and f_A a fuzzy interior ideal of H. Then $H * A * H \subseteq A$. Indeed: Let $u \in H * A * H$. Then $u \in v \circ y$ for some $v \in H * A$, $y \in H$ and $v \in x \circ a$ for some $x \in H$, $a \in A$. Since $v \circ y \subseteq (x \circ a) * \{y\}$, we have $u \in (x \circ a) * \{y\}$, where $x, y \in H$ and $a \in A$. Since f_A a fuzzy interior ideal of H, we have $f_A(u) \geq f_A(a) = 1$. Since f_A is a fuzzy subset of H and $u \in H$, we have $f_A(u) \leq 1$. So we have $f_A(u) = 1$, and $u \in A$. □

Proposition 3.4. *Let H be an hypersemigroup. If f is a fuzzy ideal of H, then f is a fuzzy interior ideal of H.*

Proof. Let $x, a, y \in H$. Then $f\Big((x \circ a) * \{y\}\Big) \geq f(a)$. In fact:
Let $u \in (x \circ a) * \{y\}$. By Proposition 2.1, there exists $v \in x \circ a$ such that $u \in v \circ y$. Since $v \in x \circ a$ and f is a fuzzy left ideal of H, we have $f(v) \geq f(a)$. Since $u \in v \circ y$ and f is a fuzzy right ideal of H, we have $f(u) \geq f(v)$. Then we have $f(u) \geq f(a)$, and the proof is complete. □

Definition 3.5. (cf. also [3]) An hypersemigroup H is called *regular* if for every $a \in H$ there exists $x \in H$ such that $a \in \{a\} * (x \circ a)$.

Lemma 3.6. [3; Lemma 1.2] *Let H be an hypersemigroup. The following are equivalent:*
(1) *H is regular.*
(2) *$a \in \{a\} * \{x\} * \{a\}$ for every $a \in H$.*
(3) *$A \subseteq A * H * A$ for every nonempty subset A of H.*

Proposition 3.7. *Let H be a regular hypersemigroup and A an interior ideal of H. Then A is a subsemigroup of H.*

Proof. Since A is an interior ideal of H, we have $H * A * H \subseteq A$. Since H is regular, we have $A \subseteq A * H * A$. Then we have

$$A * A \quad \subseteq \quad (A * H * A) * A = (A * H) * A * A \subseteq H * A * H \subseteq A,$$

so A is a subsemigroup of H. □

Proposition 3.8. *Let H be a regular hypersemigroup and f a fuzzy interior ideal of H. Then f is a fuzzy ideal of H.*

Proof. Let $a, b \in H$. Then $f(a \circ b) \geq f(a)$ and $f(a \circ b) \geq f(b)$. In fact:
Let $u \in a \circ b$. Then $f(u) \geq f(a)$. Indeed: Since $a \in H$ and H is regular, there exists $x \in H$ such that $a \in \{a\} * \{x\} * \{a\}$. Then

$$a \circ b \subseteq \{a\} * \{x\} * \{a\} * \{b\} = (a \circ x) * (a \circ b),$$

from which $u \in v \circ w$ for some $v \in a \circ x$, $w \in a \circ b$. We have $u \in v \circ w \subseteq \{v\} * (a \circ b)$ and $f\Big(\{v\} * (a \circ b)\Big) \geq f(a)$, thus we have $f(u) \geq f(a)$, and f is a fuzzy right ideal of H. We also have $f(u) \geq f(b)$. Indeed: Since $b \in H$ and H is regular, there exists $y \in H$ such that $b \in \{b\} * \{y\} * \{b\}$. Then we have

$$u \in a \circ b \subseteq \{a\} * \{b\} * \{y\} * \{b\} = (a \circ b) * (y \circ b).$$

Then $u \in s \circ t$ for some $s \in a \circ b$, $t \in y \circ b$. Then we have

$$u \in s \circ t \subseteq (a \circ b) * \{t\} = \{a\} * (b \circ t).$$

Since $f\Big(\{a\} * (b \circ t)\Big) \geq f(b)$, we obtain $f(u) \geq f(b)$, and f is a fuzzy left ideal of H. Therefore f is a fuzzy ideal of H. □

From Propositions 3.4 and 3.8 we have the following

Theorem 3.9. *In regular hypersemigroups the concepts of fuzzy ideals and fuzzy interior ideals coincide.*

Definition 3.10. (cf. also [3]) An hypersemigroup H is called *intra-regular* if for every $a \in H$ there exist $x, y \in H$ such that $a \in (x \circ a) * (a \circ y)$.

Lemma 3.11. *Let H be an hypersemigroup. The following are equivalent:*

(1) *H is intra-regular.*
(2) *$a \in H * \{a\} * \{a\} * H$ for every $a \in H$.*
(3) *$A \subseteq H * A * A * H$ for every nonempty subset of H.*

Proof. The implication (1) $\Rightarrow$ (2) and the equivalence (2) $\Leftrightarrow$ (3) are obvious. Let us prove the implication (2) $\Rightarrow$ (1). Let $a \in H$. By (2), we have $a \in \Big(H * \{a\}\Big) * \Big(\{a\} * H\Big)$. By Proposition 2.1, $a \in u \circ v$ for some $u \in H * \{a\}$, $v \in \{a\} * H$, $u \in x \circ a$ and $v \in a \circ y$ for some $x, y \in H$. Then we have $a \in u \circ v \subseteq (x \circ a) * (a \circ y)$, then $a \in (x \circ a) * (a \circ y)$, where $x, y \in H$ and so H is intra-regular. □

Proposition 3.12. *Let H be an intra-regular hypersemigroup and A an interior ideal of H. Then A is a subsemigroup of H.*

Proof. Since A is an interior ideal of H, we have $H * A * H \subseteq A$. Since H is intra-regular, we have $A \subseteq H * A * A * H$. Then we have

$$\begin{aligned} A * A &\subseteq (H * A * A * H) * A = (H * A) * A * (H * A) \\ &\subseteq H * A * H \subseteq A, \end{aligned}$$

so A is a subsemigroup of H. □

By Propositions 3.7 and 3.12, we have the following

Corollary 3.13. *In regular and in intra-regular hypersemigroups the interior ideals and the subidempotent interior ideals coincide.*

Proposition 3.14. *Let H be an intra-regular hypersemigroup and f is a fuzzy interior ideal of H. Then f is a fuzzy ideal of H.*

Proof. Let $a, b \in H$ and $u \in a \circ b$. Since $a \in H$ and H is intra-regular, there exist $x, y \in H$ such that $a \in \{x\} * \{a\} * \{a\} * \{y\}$. Then

$$a \circ b \subseteq \{x\} * \{a\} * \{a\} * \{y\} * \{b\} = (x \circ a) * \Big((a \circ y) * \{b\}\Big).$$

Then $u \in v \circ w$ for some $v \in x \circ a$, $w \in (a \circ y) * \{b\}$. We have

$$u \in v \circ w \subseteq (x \circ a) * \{w\}$$

and, since f is a fuzzy interior ideal of H, $f\Big((x \circ a) * \{w\}\Big) \geq f(a)$. Thus we get $f(u) \geq f(a)$, and f is a fuzzy right ideal of H. Since $b \in H$ and H is intra-regular, there exist $z, t \in H$ such that $b \in \{z\} * \{b\} * \{b\} * \{t\}$, then we have

$$a \circ b \subseteq \{a\} * \{z\} * \{b\} * \{b\} * \{t\} = \Big((a \circ z) * \{b\}\Big) * (b \circ t).$$

Then $u \in c \circ d$ for some $c \in (a \circ z) * \{b\}$, $d \in b \circ t$. Since $u \in c \circ d \subseteq \{c\} * (b \circ t)$ and $f\Big(\{c\} * (b \circ t)\Big) \geq f(b)$, we have $f(u) \geq f(b)$, and f is a fuzzy left ideal of H. Hence f is a fuzzy ideal of H. □

By Propositions 3.4 and 3.14, we have the following theorem

Theorem 3.15. *In intra-regular hypersemigroups the concepts of fuzzy ideals and fuzzy interior ideals coincide.*

An ideal A of an hypergroupoid H is called *proper* if $A \neq H$.

Definition 3.16. An hypergroupoid H is called *simple* if does not contain proper ideals, that is, for every ideal A of H, we have $A = H$.

The concept of fuzzy simple semigroups due to Kuroki [6] can be naturally transferred to hypergroupoids as follows:

Definition 3.17. An hypergroupoid H is called *fuzzy simple* if every fuzzy ideal of H is a constant function, that is, for every fuzzy ideal f of H and every $a, b \in H$, we have $f(a) = f(b)$.

Notation 3.18. Let H be an hypergroupoid and $a \in H$. We denote by I_a the subset of H defined as follows:

$$I_a = \{b \in H \mid f(b) \geq f(a)\}.$$

Lemma 3.19. *Let H be an hypergroupoid and f a fuzzy right (resp. fuzzy left) ideal of H. Then the set I_a is a right (resp. left) ideal of H for every $a \in H$.*

Proof. Let $a \in H$ and f a fuzzy right ideal of H. The set I_a is a right ideal of H. Indeed: Since $a \in I_a$, the set I_a is a nonempty subset of H. Moreover, $I_a * H \subseteq I_a$. Indeed: Let $x \in I_a * H$. Then $x \in u \circ v$ for some $u \in I_a$, $v \in H$. Since $x \in u \circ v$ and f is a fuzzy right ideal of H, we have $f(x) \geq f(u)$. Since $u \in I_a$, we have $f(u) \geq f(a)$, thus we have $f(x) \geq f(a)$. Since $u \in I_a$, we have $u \in H$. Since $u, v \in H$, we have $u \circ v \subseteq H * H \subseteq H$, so $x \in H$. Since $x \in H$ and $f(x) \geq f(a)$, we have $x \in I_a$. Thus I_a is a right ideal of H. Similarly, if f is a fuzzy left ideal of H, then the set I_a is a left ideal of H for every $a \in H$. □

Corollary 3.20. *If H is an hypergroupoid and f a fuzzy ideal of H, then the set I_a is an ideal of H for every $a \in H$.*

Lemma 3.21. *Let H be an hypergroupoid. If A a left (resp. right) ideal or an ideal of H, then the characteristic function f_A is a fuzzy left (resp. fuzzy right) ideal or a fuzzy ideal of H. "Conversely", if A is a nonempty subset of H and f_A a fuzzy left (resp. fuzzy right) ideal or a fuzzy ideal of H, then A is a left (resp. right) ideal or an ideal of H.*

Proof. Let A be a left ideal of H, $x, y \in H$ and $u \in x \circ y$. Then $f_A(u) \geq f_A(y)$. Indeed: If $y \in A$, then $x \circ y \subseteq H * A \subseteq A$, then $u \in A$ and $f_A(u) = 1 \geq f_A(y)$. If $y \notin A$, then $f_A(y) = 0 \leq f_A(u)$, so f_A is a fuzzy left ideal of H. Let now f_A be a fuzzy left ideal of H. Then $H * A \subseteq A$. Indeed: Let $u \in H * A$. Then $u \in x \circ y$ for some $x \in H$, $y \in A$. Since $u \in x \circ y$, we have $f_A(u) \geq f_A(y) = 1$. Then $f_A(u) = 1$, and $u \in A$. The "dual" (for right-fuzzy right ideals) can be proved in a similar way, this completes the proof. □

Theorem 3.22. *An hypergroupoid H is simple if and only if it is fuzzy simple.*

Proof. $\Longrightarrow$. Let f be a fuzzy ideal of H and $a, b \in H$. Since f is a fuzzy ideal of H and $a \in H$, by Corollary 3.20, the set I_a is an ideal of H. Since H is simple, we have $I_a = H$. Then $b \in I_a$, so $f(b) \geq f(a)$. By symmetry, we get $f(a) \geq f(b)$. Thus we have $f(a) = f(b)$, and H is fuzzy simple.
$\Longleftarrow$. Let H be fuzzy simple and I an ideal of H. Then $I = H$. Indeed: Let $x \in H$. Since I is an ideal of H, by Lemma 3.21, the characteristic function f_I is a fuzzy ideal of H. Since H is fuzzy simple, f_I is a constant function, that is, $f_I(y) = f_I(z)$ for every $y, z \in H$. Take an element $a \in I$ ($I \neq \emptyset$). Then we have $f_I(x) = f_I(a) = 1$, so $x \in I$. Thus H is simple. □

Theorem 3.23. *If H is an hypersemigroup, then the following are equivalent:*
(1) *H is simple.*
(2) *$H = H * \{a\} * H$ for every $a \in H$.*
(3) *Every fuzzy interior ideal of H is a constant function.*

Proof. (1) $\Longrightarrow$ (2). Let $a \in H$. The set $H * \{a\} * H$ is an ideal of H. Indeed, it is a nonempty subset of H, and we have
$H * (H * \{a\} * H) = (H * H) * \{a\} * H \subseteq H * \{a\} * H$ and
$(H * \{a\} * H) * H = H * \{a\} * (H * H) \subseteq H * \{a\} * H$.
Since H is simple, we have $H * \{a\} * H = H$.
(2) $\Longrightarrow$ (3). Let f be a fuzzy interior ideal of H and $a, b \in H$. Then $f(a) = f(b)$.

Indeed: Since $b \in H$, by hypothesis, we have $b \in (x \circ a) * \{y\}$ for some $x, y \in H$. Since f is a fuzzy interior ideal of H, we have $f(b) \geq f(a)$. By symmetry, we get $f(a) \geq f(b)$, so $f(a) = f(b)$.
(3) $\Longrightarrow$ (1). Let f is a fuzzy ideal of H. By Proposition 3.4, f is a fuzzy interior ideal of H. By hypothesis, f is a constant function. Thus H is fuzzy simple. Then, by Theorem 3.22, H is simple. □

Summarizing, in case of an hypersemigroup the following are equivalent: (1) H is simple; (2) $H = H * \{a\} * H$ for every $a \in H$; (3) $H = H * A * H$ for every $A \in \mathcal{P}^*(H)$; (4) H is fuzzy simple; (5) every fuzzy interior ideal of H is a constant function. Clearly $H = H * \{a\} * H$ for every $a \in H$ is equivalent to $H = H * A * H$ for every nonempty subset A of H.

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Communicated by *Klaus Denecke*

University of Athens
Department of Mathematics
15784 Panepistimiopolis, Greece
email: nkehayop@math.uoa.gr