CLASSIFICATION OF QUASI UNION HYPER K-ALGEBRAS OF ORDER 6

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Abstract. In this manuscript, we classify non isomorphic quasi union hyper K-algebras of order 6 and show that conjecture 4.2[6] is not true generally, and finally modify it.

1 Introduction The study of BCK-algebra was initiated by Imai and Iséki[2] in 1966 as a generalization of the concept of set-theoretic difference and propositional calculi. The hyper structure theory (called also multi algebras) was introduced in 1934 by Marty[4] at the 8th congress of Scandinavian Mathematicians. Hyper structures have many applications to several sectors of both pure and applied sciences. Borzooei, et.al.[3] applied the hyper structure to BCK-algebras and introduced the concept of hyper BCK-algebra and hyper K-algebra in which, each of them is a generalization of BCK-algebra. Nasr-Azadani and Zahedi [5, 6] introduced quasi union hyper K-algebras and classified non isomorphic quasi union hyper K-algebras of order less than 6, and gave a conjecture for order n. Now we classify non isomorphic quasi union hyper K-algebras of order 6 and finally modify that conjecture.

2 Preliminaries Let H be a non-empty set, the set of all non-empty subset of H is denoted by $\mathcal{P}^*(H)$. A *hyperoperation* on H is a map $\circ : H \times H \to \mathcal{P}^*(H)$, where $(a,b) \longmapsto a \circ b, \forall a,b \in H$. A set H, endowed with a hyperoperation, " $\circ$ ", is called a *hyperstructure*. If $A,B \subseteq H$, then $A \circ B = \bigcup_{a\in A, b\in B} a \circ b$.

Definition 2.1. [1, 3] Let H be a non-empty set containing a constant " 0 " and " $\circ$ " be a hyperoperation on H. Then H is called a hyper K-algebra if it satisfies HK1-HK5.

HK1: $(x \circ z) \circ (y \circ z) < x \circ y$,

HK2: $(x \circ y) \circ z = (x \circ z) \circ y$,

HK3: $x < x$,

HK4: $x < y,\ y < x$, then $x = y$,

HK5: $0 < x$.

for all $x,y,z \in H$, where $x < y$ means $0 \in x \circ y$. Moreover for any $A,B \subseteq H$, $A < B$ if $\exists\, a \in A$, $\exists\, b \in B$ such that $a < b$.

For briefly the readers could see some definitions and results about hyper K-algebra in[1, 3].

Let H be a set containing "0", $\mathcal{P}_0(H) = \{A \subseteq H : 0 \in A\}$ and $\mathcal{S} = \{f | f : H \to \mathcal{P}_0(H)$ is a function$\}$. For convenience we use F^x instead of $f(x)$ for any $f \in \mathcal{S}$.

Theorem 2.2. *[5] Let X be a set and $H = X \cup \{0\}$. Then for any $f \in \mathcal{S}$, $\circ_f : H \times H \to \mathcal{P}^*(H)$ is defined by:*

$$x \circ_f y := \begin{cases} F^x & \text{if } x = y, \\ \{x\} & \text{otherwise.} \end{cases}$$

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is a hyperoperation. Moreover the following statements are equivalent.
(i) $(H, \circ_f, 0)$ *is a hyper K-algebra,*
(ii) $F^x \circ_f y = F^x$ *for all* $y \neq x, y \in H$,
(iii) if $x \neq y$ *and* $y \in F^x$, *then* $y \in F^y$ *and* $F^y \subseteq F^x$.
This hyper K-algebra is called quasi union hyper K-algebra with respect to f *and denoted by* H_f.

Corollary 2.3. *[5] Let* H_f *be a quasi union hyper K-algebra and* $x \neq y$. *If* $y \in F^x$ *and* $x \in F^y$, *then* $F^y = F^x$.

Definition 2.4. [1] Let H_1 and H_2 be two hyper K-algebras. A mapping $f : H_1 \to H_2$ is said to be a homomorphism if $f(0) = 0$ and $f(x \circ y) = f(x) \circ f(y)$, for all $x, y \in H_1$, $Kerf = \{x \in H_1 : f(x) = 0\}$. Moreover if f is bijection this homomorphism is called an isomorphism.

Definition 2.5. [6] Let H_f, H_g be two quasi union hyper K-algebras. Then we say that H_f is of type $(l_0^f, l_1^f, \cdots, l_{n-1}^f)$ and denote it by $t(H_f)$, if $l_i^f = |F^{x_i}|$, $0 \leq i \leq n-1$. Also H_f and H_g are co-type, if the type of H_f is a permutation of type H_g.

Theorem 2.6. *[6] Let* H_f *and* H_g *be quasi union hyper K-algebras and* v *be an isomorphism from* H_f *to* H_g. *Then* $|F^x| = |v(F^x)| = |G^{v(x)}|$.

Corollary 2.7. *[6] Let* H_f *and* H_g *be hyper K-algebras. Then* H_f *is not isomorphic to* H_g, *if one of the following statement holds.*
i) $|0 \circ_f 0| \neq |0 \circ_g 0|$,
ii) $F^{x_i} = G^{x_i}$ *for all* $1 \leq i \leq n$ *except for some* i, *(see Remark after Theorem 4.5),*
iii) $t^f(m) \neq t^g(m)$ *for some* m, $1 \leq m \leq n$, *where* $t^f(m) = |\{x_i \in H - \{0\} : |F^{x_i}| = m\}|$,
iv) $N_n(H_f) \neq N_n(H_g)$, *where* $N_i(H_f) = |\{F^{x_k} : x_k \in F^{x_k}, 0 \leq k \leq i \leq n\}|$,
v) H_f *and* H_g *are not co-type.*

Theorem 2.8. *[6] (Key Theorem) Let* H_f *be a quasi union hyper K-algebra of type* $(l_0^f, l_1^f, \cdots, l_n^f)$. *Then, there is a union hyper K-algebra* H_g *isomorphic to* H_f *such that* $t(H_g) = (l_0^g, l_1^g, \cdots, l_n^g)$ *where* $l_0^g \leq l_1^g \leq l_2^g, \cdots, \leq l_n^g$.

Theorem 2.9. *[6] Let* $H_i = \{0, 1, 2, \cdots, i-1\}, 1 \leq i \leq 5$. *Then there are*
(i) 1 non-isomorphic quasi union hyper K-algebra on H_1,
(ii) 3 non-isomorphic quasi union hyper K-algebras on H_2,
(iii) 9 non-isomorphic quasi union hyper K-algebras on H_3,
(iv) 30 non-isomorphic quasi union hyper K-algebras on H_4,
(v) 107 non-isomorphic quasi union hyper K-algebras on H_5, *(see Theorem 4.7).*

Conjecture 2.10. *[6] Let* $|\mathcal{H}_n|, n \in \mathbb{N}$, *be the number of non-isomorphic quasi union hyper K-algebras of order* n. *Then* $|\mathcal{H}_1| = 1, |\mathcal{H}_2| = 3$ *and for* $n > 2$

$$|\mathcal{H}_n| = 2|\mathcal{H}_{n-1}| - |\mathcal{H}_{n-2}| + \binom{2n-2}{n-2}.$$

3 review of the conjecture In this section we show that the number of non isomorphic quasi union hyper K-algebras of order n and type $(l_0, l_1, \ldots, l_{n-1})$ such that $l_0 = l_1$ is equal to $2|\mathcal{H}_{n-1}| - |\mathcal{H}_{n-2}|$. So for counting the number of non isomorphic quasi union hyper K-algebras of order n, it's sufficient to count the number of non isomorphic quasi union hyper K-algebras of type $(l_0, l_1, \ldots, l_{n-1})$ such that $l_0 = 1, l_1 \geq 2$.

Note: In this paper we denote the set of all non isomorphic quasi union hyper K-algebras of order n and type $(l_0, l_1, \ldots, l_{n-1})$ such that $l_i \leq l_{i+1}$ where $0 \leq i \leq n-1$, by $\mathcal{H}_n$, also $\mathcal{H}_n^{11} = \{H_n \in \mathcal{H}_n | l_0 = l_1 = 1\}$, $\mathcal{H}_n^{12} = \{H_n \in \mathcal{H}_n | l_0 = 1, l_1 \geq 2\}$ and $\mathcal{H}_n^{22} = \{H_n \in \mathcal{H}_n | l_0 \geq 2\}$.

Theorem 3.1. *(Going down)*
Let $H_n = \{x_0 = 0, x_1, x_2, \ldots, x_{n-1}\}$ and $(H_n, \circ, 0)$ be a quasi union hyper K-algebra. Then:
i) If $(H_n, \circ, 0) \in \mathcal{H}_n^{11}$, then there exists a quasi union hyper K-algebra, $D_1(H_n)$, of order $n-1$ and type $(l_0^d, l_1^d, \ldots, l_{n-2}^d)$ such that $l_i^d = l_{i+1}$, where $0 \leq i \leq n-2$.
ii) If $H_n \in \mathcal{H}_n^{22}$, then there exists a quasi union hyper K-algebra, $D_2(H_n)$, of order $n-1$ and type $(l_0^d, l_1^d, \ldots, l_{n-2}^d)$ such that $l_i^d = l_{i+1} - 1$, where $0 \leq i \leq n-2$.

Proof. i) Let $H_n \in \mathcal{H}_n^{11}$, $H_{n-1} = \{x_0 = 0, x_2, ..., x_{n-1}\}$ and " $\circ_{d_1}$ " be a restriction " $\circ$ " to H_{n-1}. Then it's clear that $(H_{n-1}, \circ_{d_1}, 0)$ is a quasi union hyper K-algebra and $l_i^d = l_{i+1}$ where $0 \leq i \leq n-2$.
ii) Let $H_n \in \mathcal{H}_n^{22}$, and $H_{n-1} = \{x_0 = 0, x_2, ..., x_{n-1}\}$. Then $(H_{n-1}, \circ_{d_2}, 0)$ is a quasi union hyper K-algebra where " $\circ_{d_2}$ " is defined as follows:

$$x_i \circ_{d_2} x_j = \begin{cases} x_i \circ x_i - \{x_1\} & \text{if } i = j, \\ \{x_i\} & i \neq j. \end{cases}$$

for all $x_i, x_j \in H_{n-1}$. By Theorem 2.2, it's sufficient to show that if $x_k \in x_i \circ_{d_2} x_i$, then $x_k \in x_k \circ_{d_2} x_k$ and $x_k \circ_{d_2} x_k \subseteq x_i \circ_{d_2} x_i$, for all $x_i, x_k \in H_{n-1}$. Let $x_k \in x_i \circ_{d_2} x_i$. Since $(H_n, \circ, 0)$ is a quasi union hyper K-algebra, then $x_k \in x_k \circ x_k - \{x_1\}$ and $x_k \circ x_k - \{x_1\} \subseteq x_i \circ x_i - \{x_1\}$. These imply that $x_k \in x_k \circ_{d_2} x_k$ and $x_k \circ_{d_2} x_k \subseteq x_i \circ_{d_2} x_i$. Moreover $|x_i \circ_{d_2} x_i| = |x_i \circ x_i| - 1$, i.e, $l_i^d = l_{i+1} - 1$, and these complete the proof. □

Theorem 3.2. *(Going up)*
Let $H_{n-1} = \{x_0 = 0, x_1, x_2, \ldots, x_{n-2}\}$ and $(H_{n-1}, \circ, 0)$ be a quasi union hyper K-algebra. Then:
i) If $(H_{n-1}, \circ, 0) \in \mathcal{H}_{n-1}^{11} \cup \mathcal{H}_{n-1}^{12}$, then there exists a quasi union hyper K-algebra, $U_1(H_{n-1}) \in H_n^{11}$, of order n and type $(l_0^u, l_1^u, \ldots, l_{n-1}^u)$ such that $l_0 = l_1 = 1$.
ii) If $(H_{n-1}, \circ, 0) \in \mathcal{H}_{n-1}$, then there exists a quasi union hyper K-algebra, $U_2(H_{n-1})$, of order n and type $(l_0^u, l_1^u, \ldots, l_{n-1}^u)$ such that $l_0 \geq 2$, i.e, $U(H_{n-1}) \in \mathcal{H}_n^{22}$.

Proof. i) Let $(H_{n-1}, \circ, 0) \in \mathcal{H}_{n-1}^{11} \cup \mathcal{H}_{n-1}^{12}$. Then we set $H_n = \{x_0 = 0, x_{n-1}, x_1, ..., x_{n-2}\}$ and we show that $(H_n, \circ_{u_1}, 0)$ is a quasi union hyper K-algebra, where

$$x_i \circ_{u_1} x_j = \begin{cases} x_i \circ x_i & \text{if} \quad i = j \neq n-1, \\ \{0\} & i = j = n-1, \\ \{x_i\} & i \neq j. \end{cases}$$

for all $x_i, x_j \in H_n$. It's clear that $(H_n, \circ_{u_1}, 0)$ is a quasi union hyper K-algebra of type $(l_0^u, l_1^u, \ldots, l_{n-1}^u)$ such that $l_0 = l_1 = 1$.
ii) Let $(H_{n-1}, \circ, 0) \in \mathcal{H}_{n-1}$ and $H_{n-1} = \{x_0 = 0, x_1, ..., x_{n-2}\}$. Then we set $H_n = \{x_0 = 0, x_{n-1}, x_1, ..., x_{n-2}\}$ and we show that $(H_n, \circ_{u_2}, 0)$ is a quasi union hyper K-algebra, where

$$x_i \circ_{u_2} x_j = \begin{cases} x_i \circ x_i \cup \{x_{n-1}\} & \text{if} \quad i = j \neq n-1, \\ x_0 \circ x_0 \cup \{x_{n-1}\} & i = j = n-1, \\ \{x_i\} & i \neq j. \end{cases}$$

for all $x_i, x_j \in H_n$. Since H_{n-1} is a quasi union hyper K-algebra and $x_0 \circ x_0 \subseteq x_i \circ x_i$ for all $0 \leq i \leq n-2$, if $x_k \in x_i \circ_{u_2} x_i$, then $x_k \in x_k \circ_{u_2} x_k$ and $x_k \circ_{u_2} x_k \subseteq x_i \circ_{u_2} x_i$, for all $x_i, x_k \in H_n$. Therefore $(H_n, \circ_{u_2}, 0)$ is a quasi union hyper K-algebra, of type $(l_0+1, l_0+1, l_1+1, \ldots, l_{n-2}+1)$ and we denote this quasi union hyper K-algebra by $U_2(H_{n-1})$ and it's type is equal to $(l_0^u, l_1^u, \ldots, l_{n-1}^u)$ where $l_i^u \leq l_{i+1}^u$ and $l_0^u \geq 2$. □

Note: By attention to the proof of these theorems, $U_i(H_{n-1}), i = 1, 2$, is obtained by inserting x_{n-1} between 0 and x_1 and $D_i(H_n), i = 1, 2$, is obtained by deleting x_1 from H_n.

Theorem 3.3. *$D_2U_2 = 1_{\mathcal{H}_{n-1}}$, $U_2D_2 = 1_{\mathcal{H}_n^{22}}$, $D_1U_1 = 1_{\mathcal{H}_{n-1}^{11} \cup \mathcal{H}_{n-1}^{12}}$ and $U_1D_1 = 1_{\mathcal{H}_n^{11}}$.*

Proof. Let $H_{n-1} = \{x_0, x_1, \ldots, x_{n-2}\}$. Then $U_2(H_{n-1}) = H_n = H_{n-1} \cup \{x_{n-1}\}$ and

$$x_i \circ_{u_2} x_j = \begin{cases} x_i \circ x_i \cup \{x_{n-1}\} & \text{if} \quad i = j \neq n-1, \\ x_0 \circ x_0 \cup \{x_{n-1}\} & i = j = n-1, \\ \{x_i\} & i \neq j. \end{cases}$$

By definition of D_2 we have $D_2(U_2(H_{n-1})) = H_{n-1}$, also we have:

$$x_i \circ_{d_2} x_j = \begin{cases} x_i \circ_{u_2} x_i - \{x_{n-1}\} & \text{if } i = j, \\ \{x_i\} & i \neq j, \end{cases}$$

$$= \begin{cases} x_i \circ x_i \cup \{x_{n-1}\} - \{x_{n-1}\} & \text{if } i = j, \\ \{x_i\} & i \neq j, \end{cases}$$

$$= \begin{cases} x_i \circ x_i & \text{if } i = j, \\ \{x_i\} & i \neq j. \end{cases}$$

So $D_2U_2 = 1_{\mathcal{H}_{n-1}}$. Similarly we can show that $U_2D_2 = 1_{\mathcal{H}_n^{22}}$, $D_1U_1 = 1_{\mathcal{H}_{n-1}^{11} \cup \mathcal{H}_{n-1}^{12}}$ and $U_1D_1 = 1_{\mathcal{H}_n^{11}}$. □

Lemma 3.4. *i) $|\mathcal{H}_n^{22}| = |\mathcal{H}_{n-1}|$, ii)$|\mathcal{H}_n^{11}| = |\mathcal{H}_{n-1}^{11}| + |\mathcal{H}_{n-1}^{12}|$.*

Proof. i) We define functions ψ and ϕ as follows:

$$\psi : \mathcal{H}_{n-1} \longrightarrow \mathcal{H}_n^{22} \qquad\qquad \phi : \mathcal{H}_n^{11} \longrightarrow \mathcal{H}_{n-1}^{11} \cup \mathcal{H}_{n-1}^{12}$$
$$\psi(H_{n-1}) = U_2(H_{n-1}), \qquad\qquad \phi(H_n) = D_1(H_n).$$

By Theorem 3.3, it's clear that ψ and ϕ are bijection functions. So $|\mathcal{H}_n^{22}| = |\mathcal{H}_{n-1}|$ and since H_{n-1}^{11} and H_{n-1}^{12} are distinct sets we have $|\mathcal{H}_n^{11}| = |\mathcal{H}_{n-1}^{11}| + |\mathcal{H}_{n-1}^{12}|$. □

By attention to the last results, the Conjecture 2.10 is modified as follows:

Theorem 3.5. *(modified conjecture) $|\mathcal{H}_n| = 2|\mathcal{H}_{n-1}| - |\mathcal{H}_{n-2}| + |\mathcal{H}_n^{12}|$.*

Proof. Since $\mathcal{H}_n = \mathcal{H}_n^{11} \cup \mathcal{H}_n^{12} \cup \mathcal{H}_n^{22}$ and $\mathcal{H}_n^{11}$, $\mathcal{H}_n^{12}$ and $\mathcal{H}_n^{22}$ are distinct sets, then $|\mathcal{H}_n| = |\mathcal{H}_n^{11}| + |\mathcal{H}_n^{12}| + |\mathcal{H}_n^{22}|$. By Lemma 3.4, we have $|\mathcal{H}_n^{22}| = |\mathcal{H}_{n-1}|$ and $|\mathcal{H}_n^{11}| = |\mathcal{H}_{n-1}^{11}| + |\mathcal{H}_{n-1}^{12}|$. So

$$\begin{aligned}
|\mathcal{H}_n| &= |\mathcal{H}_n^{11}| + |\mathcal{H}_n^{12}| + |\mathcal{H}_n^{22}| \\
&= (|\mathcal{H}_{n-1}^{11}| + |\mathcal{H}_{n-1}^{12}|) + |\mathcal{H}_n^{12}| + |\mathcal{H}_{n-1}| \\
&= (|\mathcal{H}_{n-1}| - |\mathcal{H}_{n-1}^{22}|) + |\mathcal{H}_n^{12}| + |\mathcal{H}_{n-1}| \\
&= 2|\mathcal{H}_{n-1}| + |\mathcal{H}_n^{12}| - |\mathcal{H}_{n-1}^{22}| \\
&= 2|\mathcal{H}_{n-1}| + |\mathcal{H}_n^{12}| - |\mathcal{H}_{n-2}| \\
&= 2|\mathcal{H}_{n-1}| - |\mathcal{H}_{n-2}| + |\mathcal{H}_n^{12}|.
\end{aligned}$$

□

Open problem: What's the number of $|\mathcal{H}_n^{12}|$ respect to n?

4 non isomorphic quasi union hyper K-algebras of order 6

In this section at first we prove some theorems that help us to distinguish which quasi union hyper K-algebras are non isomorphic. Then we introduce the elements of $\mathcal{H}_6^{12}$, and show that $|\mathcal{H}_6^{12}| = 346$ which is not equal to $\begin{pmatrix} 2\times 6-2 \\ 6-2 \end{pmatrix}$. So the Conjecture 2.10 is not true in general.

Note. Let $H_n = \{x_0 = 0, x_1, x_2, \dots, x_{n-1}\}$ and $(H_n, \circ, 0)$ is a quasi union hyper K-algebra. For simplicity, we show $(H_n, \circ, 0)$ by ordered set $\{F^{x_i} : 0 \le i \le n-1\}$, where $F^{x_i} = x_i \circ x_i$.

Theorem 4.1. *Let H_n be a quasi union hyper K-algebra and $x \sim y$ if and only if $|F^x| = |F^y|$. Then $\sim$ is an equivalence relation on H_n.*

Proof. Straightforward. □

Definition 4.2. Let H_n be a quasi union hyper K-algebra and $x, y \in H_n$. We say that x and y are co-class if and only if $x \sim y$. We denote the equivalence classes of $\sim$ by $E_0, E_1, ..., E_k$, $1 \le k \le n$.

Definition 4.3. Let $H_n = \{x_0 = 0, x_1, \dots, x_{n-1}\}$ be a quasi union hyper K-algebra. We relate matrix $A(H_n) = (a_{ij})$ to H_n such that $a_{ij} = \chi_{F^{x_j}}(x_i), i = 0, \dots, n-1$, where $\chi_{F^{x_j}}$ is a characteristic function. The column that consists of sum of row's elements is called last column and so the row that consists of sum of column's elements is called last row. If H_f and H_g be two quasi union hyper K-algebras of order n, then two row's of $A(H_f)$ and $A(H_g)$ are called co-row if one is a permutation of the other and If each row of $A(H_f)$ is co-row with a row of $A(H_g)$, then we say $A(H_f)$ is co-matrix with $A(H_g)$.

Example 4.4. Let $H_f = \{\{0\}, \{0, 1\}, \{0, 2\}, \{0, 1, 3, 4\}, \{0, 1, 3, 4\}\}$ and $H_g = \{\{0\}, \{0, 1\}, \{0, 2\}, \{0, 2, 3, 4\}, \{0, 2, 3, 4\}\}$ be two isomorphic quasi union hyper K-algebras on $H = \{0, 1, 2, 3, 4\}$. Then $A(H_f)$ and $A(H_g)$ are as follows:

$A(H_f)$	0	1	2	3	4	last column
$0 \circ 0$	1	0	0	0	0	1
$1 \circ 1$	1	1	0	0	0	2
$2 \circ 2$	1	0	1	0	0	2
$3 \circ 3$	1	1	0	1	1	4
$4 \circ 4$	1	1	0	1	1	4
last row	5	3	1	2	2	

$A(H_g)$	0	1	2	3	4	last column
$0 \circ 0$	1	0	0	0	0	1
$1 \circ 1$	1	1	0	0	0	2
$2 \circ 2$	1	0	1	0	0	2
$3 \circ 3$	1	0	1	1	1	4
$4 \circ 4$	1	0	1	1	1	4
last row	5	1	3	2	2	

Theorem 4.5. *Let H_f and H_g be two isomorphic quasi union hyper K-algebras of type $(l_0, l_1, \ldots, l_{n-1})$, Then:*
i) The last column of $A(H_f)$ and $A(H_g)$ are the same, and they are types of H_f and H_g.
ii) The last row of $A(H_f)$ and $A(H_g)$ are co-row.
iii) $A(H_f)$ is co-matrix with $A(H_g)$.
iv) If σ is isomorphism between H_f and H_g, then $\sigma(E_i) = E_i$.

Proof. By Theorem 2.6, the proof is clear. □

Remark. The corollary 2.7(ii) is not true generally. See next example.

Example 4.6. Consider two quasi union hyper K-algebras H_f and H_g on $H = \{0, 1, 2, 3, 4, 5\}$ as follows:

$$H_f = \{\{0\}, \{0,1\}, \{0,2\}, \{0,1\}, \{0,2\}, \{0,1\}\},$$
$$H_g = \{\{0\}, \{0,1\}, \{0,2\}, \{0,1\}, \{0,2\}, \{0,2\}\}.$$

$F^{x_i} = G^{x_i}$ for $i = 0, 1, 2, 3, 4$ and $F^5 \neq G^5$, but H_f and H_g are isomorphic to each other by permutation (12)(34).
In the next, we classify non isomorphic quasi union hyper K-algebras of order 6, and also we show that the number of non isomorphic quasi union hyper K-algebras of order 5 gave in [6] is not true. Then we give quasi union hyper K-algebras of order 5 which are not counted in [6].

Theorem 4.7. *Let $H_i = \{0, 1, 2, \cdots, i-1\}, 1 \leq i \leq 6$. Then there are*
(i) 1 non-isomorphic quasi union hyper K-algebra on H_1,
(ii) 3 non-isomorphic quasi union hyper K-algebras on H_2,
(iii) 9 non-isomorphic quasi union hyper K-algebras on H_3,
(iv) 30 non-isomorphic quasi union hyper K-algebras on H_4,
(v) 118 non-isomorphic quasi union hyper K-algebras on H_5,
(vi) 552 non-isomorphic quasi union hyper K-algebras on H_6.

Proof. To prove (i)-(iv) see Theorem 4.1[6]. To prove (v) we refer to Theorem 4.1(v)[6], but there are non isomorphic quasi union hyper K-algebras of order 5 which were not counted in [6], and they are as follows:

Type (1,2,2,2,3):

$1 - \{\{0\}, \{0,1\}, \{0,1\}, \{0,3\}, \{0,3,4\}\}$.

Type (1,2,2,3,3):

$2 - \{\{0\}, \{0,1\}, \{0,1\}, \{0,3,4\}, \{0,3,4\}\}$,
$3 - \{\{0\}, \{0,1\}, \{0,2\}, \{0,1,2\}, \{0,1,4\}\}$,
$4 - \{\{0\}, \{0,1\}, \{0,2\}, \{0,1,3\}, \{0,2,4\}\}$,
$5 - \{\{0\}, \{0,1\}, \{0,2\}, \{0,3,4\}, \{0,3,4\}\}$.

Type (1,2,2,3,4):

$6 - \{\{0\},\{0,1\},\{0,2\},\{0,1,2\},\{0,1,2,4\}\}$,
$7 - \{\{0\},\{0,1\},\{0,2\},\{0,1,3\},\{0,1,3,4\}\}$.

Type (1,2,2,4,4):

$8 - \{\{0\},\{0,1\},\{0,2\},\{0,1,2,3\},\{0,1,2,4\}\}$.

Type (1,2,3,3,3):

$9 - \{\{0\},\{0,1\},\{0,1,2\},\{0,3,4\},\{0,3,4\}\}$.

Type (1,2,3,3,4):

$10 - \{\{0\},\{0,1\},\{0,1,2\},\{0,1,2\},\{0,1,2,4\}\}$,
$11 - \{\{0\},\{0,1\},\{0,2,3\},\{0,2,3\},\{0,1,2,3\}\}$.

Type (1,2,3,4,4):

$12 - \{\{0\},\{0,1\},\{0,1,2\},\{0,1,3,4\},\{0,1,3,4\}\}$.
Also $\{\{0\},\{0,1,2,3\},\{0,1,2,3\},\{0,1,2,3\},\{0,1,2,4\}\}$ of type $(1,4,4,4,4)$ is not a quasi union hyper K-algebra, and was more counted in [6]. So $|\mathcal{H}_5^{12}| = 56+11 = 67$ and $|\mathcal{H}_5| = 118$.
Proof of vi) To obtain all non isomorphic quasi union hyper K-algebras of order 6 and type $(l_0, l_1, \dots, l_5)$ such that $l_0 = 1, l_1 \geq 2$, we wrote a computer program and by using it we obtained all quasi union hyper K-algebras of order 6. Then by Theorem 4.5, we classified them. Finally we obtained 346 non isomorphic quasi union hyper K-algebras of order 6 and type $(l_0, l_1, \dots, l_5)$ where $l_0 = 1$ and $l_1 \geq 2$. By Theorem 3.2(Going up), we can obtain all elements of $\mathcal{H}_6^{11}$ and $\mathcal{H}_6^{22}$ which we don't bring them.
Elements of $\mathcal{H}_6^{12}$ are as follows:

Type (1,2,2,2,2,2):

$1 - \{\{0\},\{0,1\},\{0,1\},\{0,1\},\{0,1\},\{0,1\}\}$,
$2 - \{\{0\},\{0,1\},\{0,1\},\{0,1\},\{0,4\},\{0,1\}\}$,
$3 - \{\{0\},\{0,1\},\{0,1\},\{0,1\},\{0,4\},\{0,4\}\}$,
$4 - \{\{0\},\{0,1\},\{0,1\},\{0,1\},\{0,4\},\{0,5\}\}$,
$5 - \{\{0\},\{0,1\},\{0,1\},\{0,3\},\{0,3\},\{0,5\}\}$,
$6 - \{\{0\},\{0,1\},\{0,1\},\{0,3\},\{0,4\},\{0,5\}\}$,
$7 - \{\{0\},\{0,1\},\{0,2\},\{0,3\},\{0,4\},\{0,5\}\}$.

Type (1,2,2,2,2,3):

$8 - \{\{0\},\{0,1\},\{0,1\},\{0,1\},\{0,1\},\{0,1,5\}\}$,
$9 - \{\{0\},\{0,1\},\{0,1\},\{0,1\},\{0,4\},\{0,1,4\}\}$,
$10 - \{\{0\},\{0,1\},\{0,1\},\{0,3\},\{0,3\},\{0,1,3\}\}$,
$11 - \{\{0\},\{0,1\},\{0,1\},\{0,1\},\{0,4\},\{0,1,5\}\}$,
$12 - \{\{0\},\{0,1\},\{0,2\},\{0,3\},\{0,2\},\{0,1,2\}\}$,
$13 - \{\{0\},\{0,1\},\{0,1\},\{0,1\},\{0,4\},\{0,4,5\}\}$,
$14 - \{\{0\},\{0,1\},\{0,2\},\{0,3\},\{0,2\},\{0,1,3\}\}$,
$15 - \{\{0\},\{0,1\},\{0,1\},\{0,3\},\{0,3\},\{0,1,5\}\}$,
$16 - \{\{0\},\{0,1\},\{0,1\},\{0,3\},\{0,4\},\{0,1,5\}\}$,
$17 - \{\{0\},\{0,1\},\{0,1\},\{0,3\},\{0,4\},\{0,3,5\}\}$,
$18 - \{\{0\},\{0,1\},\{0,2\},\{0,3\},\{0,4\},\{0,1,2\}\}$,
$19 - \{\{0\},\{0,1\},\{0,2\},\{0,3\},\{0,4\},\{0,1,5\}\}$.

Type (1,2,2,2,2,4):

$20-\{\{0\},\{0,1\},\{0,1\},\{0,1\},\{0,4\},\{0,1,4,5\}\}$,
$21-\{\{0\},\{0,1\},\{0,1\},\{0,3\},\{0,4\},\{0,1,3,4\}\}$,
$22-\{\{0\},\{0,1\},\{0,1\},\{0,3\},\{0,4\},\{0,1,3,5\}\}$,
$23-\{\{0\},\{0,1\},\{0,1\},\{0,3\},\{0,4\},\{0,3,4,5\}\}$,
$24-\{\{0\},\{0,1\},\{0,2\},\{0,3\},\{0,4\},\{0,1,2,3\}\}$,
$25-\{\{0\},\{0,1\},\{0,2\},\{0,3\},\{0,4\},\{0,1,2,5\}\}$.

Type (1,2,2,2,2,5):

$26-\{\{0\},\{0,1\},\{0,1\},\{0,3\},\{0,4\},\{0,1,3,4,5\}\}$,
$27-\{\{0\},\{0,1\},\{0,2\},\{0,3\},\{0,4\},\{0,1,2,3,4\}\}$,
$28-\{\{0\},\{0,1\},\{0,2\},\{0,3\},\{0,4\},\{0,1,2,3,5\}\}$.

Type (1,2,2,2,2,6):

$29-\{\{0\},\{0,1\},\{0,2\},\{0,3\},\{0,4\},\{0,1,2,3,4,5\}\}$,

Type (1,2,2,2,3,3):

$30-\{\{0\},\{0,1\},\{0,1\},\{0,1\},\{0,1,4\},\{0,1,4\}\}$,
$31-\{\{0\},\{0,1\},\{0,1\},\{0,3\},\{0,1,3\},\{0,1,3\}\}$,
$32-\{\{0\},\{0,1\},\{0,1\},\{0,1\},\{0,1,4\},\{0,1,5\}\}$,
$33-\{\{0\},\{0,1\},\{0,2\},\{0,3\},\{0,2,3\},\{0,1,2\}\}$,
$34-\{\{0\},\{0,1\},\{0,1\},\{0,1\},\{0,4,5\},\{0,4,5\}\}$,
$35-\{\{0\},\{0,1\},\{0,1\},\{0,3\},\{0,1,3\},\{0,1,5\}\}$,
$36-\{\{0\},\{0,1\},\{0,1\},\{0,3\},\{0,1,3\},\{0,3,5\}\}$,
$37-\{\{0\},\{0,1\},\{0,1\},\{0,3\},\{0,1,4\},\{0,1,4\}\}$,
$38-\{\{0\},\{0,1\},\{0,1\},\{0,3\},\{0,3,4\},\{0,3,4\}\}$,
$39-\{\{0\},\{0,1\},\{0,2\},\{0,3\},\{0,2,3\},\{0,2,3\}\}$,
$40-\{\{0\},\{0,1\},\{0,1\},\{0,3\},\{0,1,4\},\{0,1,5\}\}$,
$41-\{\{0\},\{0,1\},\{0,2\},\{0,3\},\{0,2,3\},\{0,2,5\}\}$,
$42-\{\{0\},\{0,1\},\{0,1\},\{0,3\},\{0,1,4\},\{0,3,5\}\}$,
$43-\{\{0\},\{0,1\},\{0,2\},\{0,3\},\{0,3,4\},\{0,1,2\}\}$,
$44-\{\{0\},\{0,1\},\{0,1\},\{0,3\},\{0,3,4\},\{0,3,5\}\}$,
$45-\{\{0\},\{0,1\},\{0,2\},\{0,3\},\{0,3,4\},\{0,3,4\}\}$,
$46-\{\{0\},\{0,1\},\{0,1\},\{0,3\},\{0,4,5\},\{0,4,5\}\}$,
$47-\{\{0\},\{0,1\},\{0,2\},\{0,3\},\{0,1,4\},\{0,1,5\}\}$,
$48-\{\{0\},\{0,1\},\{0,2\},\{0,3\},\{0,4,5\},\{0,4,5\}\}$.

Type (1,2,2,2,3,4):

$49-\{\{0\},\{0,1\},\{0,1\},\{0,1\},\{0,1,4\},\{0,1,4,5\}\}$,
$50-\{\{0\},\{0,1\},\{0,1\},\{0,3\},\{0,1,3\},\{0,1,3,5\}\}$,
$51-\{\{0\},\{0,1\},\{0,1\},\{0,3\},\{0,1,4\},\{0,1,3,4\}\}$,
$52-\{\{0\},\{0,1\},\{0,1\},\{0,3\},\{0,3,4\},\{0,1,3,4\}\}$,
$53-\{\{0\},\{0,1\},\{0,2\},\{0,3\},\{0,1,2\},\{0,1,2,3\}\}$,
$54-\{\{0\},\{0,1\},\{0,1\},\{0,3\},\{0,1,4\},\{0,1,3,5\}\}$,
$55-\{\{0\},\{0,1\},\{0,2\},\{0,3\},\{0,2,4\},\{0,2,3,4\}\}$,
$56-\{\{0\},\{0,1\},\{0,1\},\{0,3\},\{0,1,4\},\{0,1,4,5\}\}$,
$57-\{\{0\},\{0,1\},\{0,2\},\{0,3\},\{0,3,4\},\{0,1,2,3\}\}$,
$58-\{\{0\},\{0,1\},\{0,1\},\{0,3\},\{0,3,4\},\{0,1,3,5\}\}$,
$59-\{\{0\},\{0,1\},\{0,1\},\{0,3\},\{0,3,4\},\{0,3,4,5\}\}$,

$60 - \{\{0\}, \{0,1\}, \{0,2\}, \{0,3\}, \{0,1,2\}, \{0,1,2,5\}\}$,
$61 - \{\{0\}, \{0,1\}, \{0,2\}, \{0,3\}, \{0,1,2\}, \{0,1,3,5\}\}$,
$62 - \{\{0\}, \{0,1\}, \{0,2\}, \{0,3\}, \{0,3,4\}, \{0,1,3,4\}\}$,
$63 - \{\{0\}, \{0,1\}, \{0,2\}, \{0,3\}, \{0,1,4\}, \{0,1,2,5\}\}$,
$64 - \{\{0\}, \{0,1\}, \{0,2\}, \{0,3\}, \{0,1,4\}, \{0,1,4,5\}\}$,
$65 - \{\{0\}, \{0,1\}, \{0,2\}, \{0,3\}, \{0,1,4\}, \{0,2,3,5\}\}$.

Type (1,2,2,2,3,5):

$66 - \{\{0\}, \{0,1\}, \{0,1\}, \{0,3\}, \{0,1,4\}, \{0,1,3,4,5\}\}$,
$67 - \{\{0\}, \{0,1\}, \{0,1\}, \{0,3\}, \{0,3,4\}, \{0,1,3,4,5\}\}$,
$68 - \{\{0\}, \{0,1\}, \{0,2\}, \{0,3\}, \{0,1,2\}, \{0,1,2,3,5\}\}$,
$69 - \{\{0\}, \{0,1\}, \{0,2\}, \{0,3\}, \{0,1,4\}, \{0,1,2,3,4\}\}$,
$70 - \{\{0\}, \{0,1\}, \{0,2\}, \{0,3\}, \{0,1,4\}, \{0,1,2,3,5\}\}$,
$71 - \{\{0\}, \{0,1\}, \{0,2\}, \{0,3\}, \{0,1,4\}, \{0,1,2,4,5\}\}$.

Type (1,2,2,2,3,6):

$72 - \{\{0\}, \{0,1\}, \{0,2\}, \{0,3\}, \{0,1,4\}, \{0,1,2,3,4,5\}\}$.

Type (1,2,2,2,4,4):

$73 - \{\{0\}, \{0,1\}, \{0,1\}, \{0,1\}, \{0,1,4,5\}, \{0,1,4,5\}\}$,
$74 - \{\{0\}, \{0,1\}, \{0,1\}, \{0,3\}, \{0,1,3,4\}, \{0,1,3,4\}\}$,
$75 - \{\{0\}, \{0,1\}, \{0,2\}, \{0,3\}, \{0,1,2,3\}, \{0,1,2,3\}\}$,
$76 - \{\{0\}, \{0,1\}, \{0,1\}, \{0,3\}, \{0,1,3,4\}, \{0,1,3,5\}\}$,
$77 - \{\{0\}, \{0,1\}, \{0,1\}, \{0,3\}, \{0,1,4,5\}, \{0,1,4,5\}\}$,
$78 - \{\{0\}, \{0,1\}, \{0,1\}, \{0,3\}, \{0,3,4,5\}, \{0,3,4,5\}\}$,
$79 - \{\{0\}, \{0,1\}, \{0,2\}, \{0,3\}, \{0,1,2,3\}, \{0,1,2,5\}\}$,
$80 - \{\{0\}, \{0,1\}, \{0,2\}, \{0,3\}, \{0,1,2,4\}, \{0,1,2,4\}\}$,
$81 - \{\{0\}, \{0,1\}, \{0,2\}, \{0,3\}, \{0,1,2,4\}, \{0,1,2,5\}\}$,
$82 - \{\{0\}, \{0,1\}, \{0,2\}, \{0,3\}, \{0,1,2,4\}, \{0,1,3,5\}\}$,
$83 - \{\{0\}, \{0,1\}, \{0,2\}, \{0,3\}, \{0,1,4,5\}, \{0,1,4,5\}\}$.

Type (1,2,2,2,4,5):

$84 - \{\{0\}, \{0,1\}, \{0,1\}, \{0,3\}, \{0,1,3,4\}, \{0,1,3,4,5\}\}$,
$85 - \{\{0\}, \{0,1\}, \{0,2\}, \{0,3\}, \{0,1,2,3\}, \{0,1,2,3,5\}\}$,
$86 - \{\{0\}, \{0,1\}, \{0,2\}, \{0,3\}, \{0,1,2,4\}, \{0,1,2,3,4\}\}$,
$87 - \{\{0\}, \{0,1\}, \{0,2\}, \{0,3\}, \{0,1,2,4\}, \{0,1,2,3,5\}\}$,
$88 - \{\{0\}, \{0,1\}, \{0,2\}, \{0,3\}, \{0,1,2,4\}, \{0,1,2,4,5\}\}$.

Type (1,2,2,2,4,6):

$89 - \{\{0\}, \{0,1\}, \{0,2\}, \{0,3\}, \{0,1,2,4\}, \{0,1,2,3,4,5\}\}$.

Type (1,2,2,2,5,5):

$90 - \{\{0\}, \{0,1\}, \{0,1\}, \{0,3\}, \{0,1,3,4,5\}, \{0,1,3,4,5\}\}$,
$91 - \{\{0\}, \{0,1\}, \{0,2\}, \{0,3\}, \{0,1,2,3,4\}, \{0,1,2,3,4\}\}$,
$92 - \{\{0\}, \{0,1\}, \{0,2\}, \{0,3\}, \{0,1,2,3,4\}, \{0,1,2,3,5\}\}$,
$93 - \{\{0\}, \{0,1\}, \{0,2\}, \{0,3\}, \{0,1,2,4,5\}, \{0,1,2,4,5\}\}$.

Type (1,2,2,2,5,6):

$94 - \{\{0\}, \{0,1\}, \{0,2\}, \{0,3\}, \{0,1,2,3,4\}, \{0,1,2,3,4,5\}\}$.

Type (1,2,2,2,6,6):

95 – $\{\{0\},\{0,1\},\{0,2\},\{0,3\},\{0,1,2,3,4,5\},\{0,1,2,3,4,5\}\}$.

Type (1,2,2,3,3,3):

96 – $\{\{0\},\{0,1\},\{0,1\},\{0,1,3\},\{0,1,3\},\{0,1,3\}\}$,
97 – $\{\{0\},\{0,1\},\{0,2\},\{0,1,2\},\{0,1,2\},\{0,1,2\}\}$,
98 – $\{\{0\},\{0,1\},\{0,1\},\{0,1,3\},\{0,1,3\},\{0,1,5\}\}$,
99 – $\{\{0\},\{0,1\},\{0,1\},\{0,3,4\},\{0,3,4\},\{0,3,4\}\}$,
100 – $\{\{0\},\{0,1\},\{0,2\},\{0,1,2\},\{0,1,2\},\{0,1,5\}\}$,
101 – $\{\{0\},\{0,1\},\{0,2\},\{0,1,2\},\{0,1,4\},\{0,1,4\}\}$,
102 – $\{\{0\},\{0,1\},\{0,2\},\{0,1,3\},\{0,1,3\},\{0,1,3\}\}$,
103 – $\{\{0\},\{0,1\},\{0,1\},\{0,1,3\},\{0,1,4\},\{0,1,5\}\}$,
104 – $\{\{0\},\{0,1\},\{0,2\},\{0,2,3\},\{0,2,4\},\{0,1,2\}\}$,
105 – $\{\{0\},\{0,1\},\{0,1\},\{0,1,3\},\{0,4,5\},\{0,4,5\}\}$,
106 – $\{\{0\},\{0,1\},\{0,2\},\{0,2,3\},\{0,2,4\},\{0,2,3\}\}$,
107 – $\{\{0\},\{0,1\},\{0,2\},\{0,1,2\},\{0,1,4\},\{0,2,5\}\}$,
108 – $\{\{0\},\{0,1\},\{0,2\},\{0,1,2\},\{0,4,5\},\{0,4,5\}\}$,
109 – $\{\{0\},\{0,1\},\{0,2\},\{0,1,3\},\{0,1,3\},\{0,2,5\}\}$,
110 – $\{\{0\},\{0,1\},\{0,2\},\{0,3,4\},\{0,3,4\},\{0,3,4\}\}$,
111 – $\{\{0\},\{0,1\},\{0,2\},\{0,1,3\},\{0,1,4\},\{0,1,5\}\}$,
112 – $\{\{0\},\{0,1\},\{0,2\},\{0,1,3\},\{0,1,4\},\{0,2,5\}\}$,
113 – $\{\{0\},\{0,1\},\{0,2\},\{0,1,3\},\{0,4,5\},\{0,4,5\}\}$.

Type (1,2,2,3,3,4):

114 – $\{\{0\},\{0,1\},\{0,1\},\{0,1,3\},\{0,1,3\},\{0,1,3,5\}\}$,
115 – $\{\{0\},\{0,1\},\{0,1\},\{0,1,3\},\{0,1,4\},\{0,1,3,4\}\}$,
116 – $\{\{0\},\{0,1\},\{0,1\},\{0,3,4\},\{0,3,4\},\{0,1,3,4\}\}$,
117 – $\{\{0\},\{0,1\},\{0,2\},\{0,1,2\},\{0,1,2\},\{0,1,2,5\}\}$,
118 – $\{\{0\},\{0,1\},\{0,2\},\{0,1,2\},\{0,1,4\},\{0,1,2,4\}\}$,
119 – $\{\{0\},\{0,1\},\{0,2\},\{0,1,3\},\{0,1,3\},\{0,1,2,3\}\}$,
120 – $\{\{0\},\{0,1\},\{0,1\},\{0,1,3\},\{0,1,4\},\{0,1,3,5\}\}$,
121 – $\{\{0\},\{0,1\},\{0,2\},\{0,3,4\},\{0,3,4\},\{0,1,3,4\}\}$,
122 – $\{\{0\},\{0,1\},\{0,1\},\{0,3,4\},\{0,3,4\},\{0,3,4,5\}\}$,
123 – $\{\{0\},\{0,1\},\{0,2\},\{0,1,2\},\{0,1,4\},\{0,1,2,5\}\}$,
124 – $\{\{0\},\{0,1\},\{0,2\},\{0,1,2\},\{0,1,4\},\{0,1,4,5\}\}$,
125 – $\{\{0\},\{0,1\},\{0,2\},\{0,1,3\},\{0,1,3\},\{0,1,2,5\}\}$,
126 – $\{\{0\},\{0,1\},\{0,2\},\{0,1,3\},\{0,1,3\},\{0,1,3,5\}\}$,
127 – $\{\{0\},\{0,1\},\{0,2\},\{0,1,3\},\{0,1,4\},\{0,1,2,3\}\}$,
128 – $\{\{0\},\{0,1\},\{0,2\},\{0,1,3\},\{0,1,4\},\{0,1,3,4\}\}$,
129 – $\{\{0\},\{0,1\},\{0,2\},\{0,1,3\},\{0,2,4\},\{0,1,2,3\}\}$,
130 – $\{\{0\},\{0,1\},\{0,2\},\{0,1,3\},\{0,1,4\},\{0,1,2,5\}\}$,
131 – $\{\{0\},\{0,1\},\{0,2\},\{0,1,3\},\{0,1,4\},\{0,1,3,5\}\}$,
132 – $\{\{0\},\{0,1\},\{0,2\},\{0,1,3\},\{0,2,4\},\{0,1,2,5\}\}$,
133 – $\{\{0\},\{0,1\},\{0,2\},\{0,1,3\},\{0,2,4\},\{0,1,3,5\}\}$,
134 – $\{\{0\},\{0,1\},\{0,2\},\{0,3,4\},\{0,3,4\},\{0,1,2,5\}\}$,
135 – $\{\{0\},\{0,1\},\{0,2\},\{0,3,4\},\{0,3,4\},\{0,3,4,5\}\}$.

Type (1,2,2,3,3,5):

136 – $\{\{0\},\{0,1\},\{0,1\},\{0,1,3\},\{0,1,4\},\{0,1,3,4,5\}\}$,
137 – $\{\{0\},\{0,1\},\{0,1\},\{0,3,4\},\{0,3,4\},\{0,1,3,4,5\}\}$,
138 – $\{\{0\},\{0,1\},\{0,2\},\{0,1,2\},\{0,1,4\},\{0,1,2,4,5\}\}$,
139 – $\{\{0\},\{0,1\},\{0,2\},\{0,1,3\},\{0,1,3\},\{0,1,2,3,5\}\}$,
140 – $\{\{0\},\{0,1\},\{0,2\},\{0,1,3\},\{0,1,4\},\{0,1,2,3,4\}\}$,
141 – $\{\{0\},\{0,1\},\{0,2\},\{0,1,3\},\{0,2,4\},\{0,1,2,3,4\}\}$,
142 – $\{\{0\},\{0,1\},\{0,2\},\{0,3,4\},\{0,3,4\},\{0,1,2,3,4\}\}$,
143 – $\{\{0\},\{0,1\},\{0,2\},\{0,1,3\},\{0,1,4\},\{0,1,2,3,5\}\}$,
144 – $\{\{0\},\{0,1\},\{0,2\},\{0,1,3\},\{0,1,4\},\{0,1,3,4,5\}\}$,
145 – $\{\{0\},\{0,1\},\{0,2\},\{0,1,3\},\{0,2,4\},\{0,1,2,3,5\}\}$,
146 – $\{\{0\},\{0,1\},\{0,2\},\{0,3,4\},\{0,3,4\},\{0,1,3,4,5\}\}$.

Type (1,2,2,3,3,6):

147 – $\{\{0\},\{0,1\},\{0,2\},\{0,1,3\},\{0,1,4\},\{0,1,2,3,4,5\}\}$,
148 – $\{\{0\},\{0,1\},\{0,2\},\{0,1,3\},\{0,2,4\},\{0,1,2,3,4,5\}\}$,
149 – $\{\{0\},\{0,1\},\{0,2\},\{0,3,4\},\{0,3,4\},\{0,1,2,3,4,5\}\}$.

Type (1,2,2,3,4,4):

150 – $\{\{0\},\{0,1\},\{0,1\},\{0,1,3\},\{0,1,3,4\},\{0,1,3,4\}\}$,
151 – $\{\{0\},\{0,1\},\{0,2\},\{0,1,2\},\{0,1,2,4\},\{0,1,2,4\}\}$,
152 – $\{\{0\},\{0,1\},\{0,2\},\{0,1,3\},\{0,1,2,3\},\{0,1,2,3\}\}$,
153 – $\{\{0\},\{0,1\},\{0,1\},\{0,1,3\},\{0,1,3,4\},\{0,1,3,5\}\}$,
154 – $\{\{0\},\{0,1\},\{0,1\},\{0,1,3\},\{0,1,4,5\},\{0,1,4,5\}\}$,
155 – $\{\{0\},\{0,1\},\{0,2\},\{0,1,2\},\{0,1,2,4\},\{0,1,2,5\}\}$,
156 – $\{\{0\},\{0,1\},\{0,2\},\{0,1,2\},\{0,1,4,5\},\{0,1,4,5\}\}$,
157 – $\{\{0\},\{0,1\},\{0,2\},\{0,1,3\},\{0,1,2,3\},\{0,1,2,5\}\}$,
158 – $\{\{0\},\{0,1\},\{0,2\},\{0,1,3\},\{0,1,2,3\},\{0,1,3,5\}\}$,
159 – $\{\{0\},\{0,1\},\{0,2\},\{0,1,3\},\{0,1,2,4\},\{0,1,2,3\}\}$,
160 – $\{\{0\},\{0,1\},\{0,2\},\{0,1,3\},\{0,1,2,4\},\{0,1,2,4\}\}$,
161 – $\{\{0\},\{0,1\},\{0,2\},\{0,1,3\},\{0,1,3,4\},\{0,1,2,3\}\}$,
162 – $\{\{0\},\{0,1\},\{0,2\},\{0,1,3\},\{0,1,3,4\},\{0,1,3,4\}\}$,
163 – $\{\{0\},\{0,1\},\{0,2\},\{0,1,3\},\{0,1,2,4\},\{0,1,2,5\}\}$,
164 – $\{\{0\},\{0,1\},\{0,2\},\{0,1,3\},\{0,1,2,4\},\{0,1,3,5\}\}$,
165 – $\{\{0\},\{0,1\},\{0,2\},\{0,1,3\},\{0,1,3,4\},\{0,1,3,5\}\}$,
166 – $\{\{0\},\{0,1\},\{0,2\},\{0,1,3\},\{0,1,4,5\},\{0,1,4,5\}\}$,
167 – $\{\{0\},\{0,1\},\{0,2\},\{0,1,3\},\{0,2,4,5\},\{0,2,4,5\}\}$.

Type (1,2,2,3,4,5):

168 – $\{\{0\},\{0,1\},\{0,1\},\{0,1,3\},\{0,1,3,4\},\{0,1,3,4,5\}\}$,
169 – $\{\{0\},\{0,1\},\{0,2\},\{0,1,2\},\{0,1,2,4\},\{0,1,2,4,5\}\}$,
170 – $\{\{0\},\{0,1\},\{0,2\},\{0,1,3\},\{0,1,2,3\},\{0,1,2,3,5\}\}$,
171 – $\{\{0\},\{0,1\},\{0,2\},\{0,1,3\},\{0,1,2,4\},\{0,1,2,3,4\}\}$,
172 – $\{\{0\},\{0,1\},\{0,2\},\{0,1,3\},\{0,1,3,4\},\{0,1,2,3,4\}\}$,
173 – $\{\{0\},\{0,1\},\{0,2\},\{0,1,3\},\{0,1,2,4\},\{0,1,2,3,5\}\}$,
174 – $\{\{0\},\{0,1\},\{0,2\},\{0,1,3\},\{0,1,2,4\},\{0,1,2,4,5\}\}$,
175 – $\{\{0\},\{0,1\},\{0,2\},\{0,1,3\},\{0,1,3,4\},\{0,1,2,3,5\}\}$,
176 – $\{\{0\},\{0,1\},\{0,2\},\{0,1,3\},\{0,1,3,4\},\{0,1,3,4,5\}\}$.

Type (1,2,2,3,4,6):

$177-\{\{0\},\{0,1\},\{0,2\},\{0,1,3\},\{0,1,2,4\},\{0,1,2,3,4,5\}\}$,
$178-\{\{0\},\{0,1\},\{0,2\},\{0,1,3\},\{0,1,3,4\},\{0,1,2,3,4,5\}\}$.

Type (1,2,2,3,5,5):

$179-\{\{0\},\{0,1\},\{0,1\},\{0,1,3\},\{0,1,3,4,5\},\{0,1,3,4,5\}\}$,
$180-\{\{0\},\{0,1\},\{0,2\},\{0,1,2\},\{0,1,2,4,5\},\{0,1,2,4,5\}\}$,
$181-\{\{0\},\{0,1\},\{0,2\},\{0,1,3\},\{0,1,2,3,4\},\{0,1,2,3,4\}\}$,
$182-\{\{0\},\{0,1\},\{0,2\},\{0,1,3\},\{0,1,2,3,4\},\{0,1,2,3,5\}\}$,
$183-\{\{0\},\{0,1\},\{0,2\},\{0,1,3\},\{0,1,2,4,5\},\{0,1,2,4,5\}\}$,
$184-\{\{0\},\{0,1\},\{0,2\},\{0,1,3\},\{0,1,3,4,5\},\{0,1,3,4,5\}\}$.

Type (1,2,2,3,5,6):

$185-\{\{0\},\{0,1\},\{0,2\},\{0,1,3\},\{0,1,2,3,4\},\{0,1,2,3,4,5\}\}$.

Type (1,2,2,3,6,6):

$186-\{\{0\},\{0,1\},\{0,2\},\{0,1,3\},\{0,1,2,3,4,5\},\{0,1,2,3,4,5\}\}$.

Type (1,2,2,4,4,4):

$187-\{\{0\},\{0,1\},\{0,1\},\{0,1,3,4\},\{0,1,3,4\},\{0,1,3,4\}\}$,
$188-\{\{0\},\{0,1\},\{0,2\},\{0,1,2,3\},\{0,1,2,3\},\{0,1,2,3\}\}$,
$189-\{\{0\},\{0,1\},\{0,1\},\{0,3,4,5\},\{0,3,4,5\},\{0,3,4,5\}\}$,
$190-\{\{0\},\{0,1\},\{0,2\},\{0,1,2,3\},\{0,1,2,3\},\{0,1,2,5\}\}$,
$191-\{\{0\},\{0,1\},\{0,2\},\{0,1,3,4\},\{0,1,3,4\},\{0,1,3,4\}\}$,
$192-\{\{0\},\{0,1\},\{0,2\},\{0,1,2,3\},\{0,1,2,4\},\{0,1,2,5\}\}$,
$193-\{\{0\},\{0,1\},\{0,2\},\{0,1,2,3\},\{0,1,4,5\},\{0,1,4,5\}\}$,
$194-\{\{0\},\{0,1\},\{0,2\},\{0,3,4,5\},\{0,3,4,5\},\{0,3,4,5\}\}$.

Type(1,2,2,4,4,5):

$195-\{\{0\},\{0,1\},\{0,1\},\{0,1,3,4\},\{0,1,3,4\},\{0,1,3,4,5\}\}$,
$196-\{\{0\},\{0,1\},\{0,2\},\{0,1,2,3\},\{0,1,2,3\},\{0,1,2,3,5\}\}$,
$197-\{\{0\},\{0,1\},\{0,2\},\{0,1,2,3\},\{0,1,2,4\},\{0,1,2,3,4\}\}$,
$198-\{\{0\},\{0,1\},\{0,2\},\{0,1,3,4\},\{0,1,3,4\},\{0,1,2,3,4\}\}$,
$199-\{\{0\},\{0,1\},\{0,2\},\{0,1,2,3\},\{0,1,2,4\},\{0,1,2,3,5\}\}$,
$200-\{\{0\},\{0,1\},\{0,2\},\{0,1,3,4\},\{0,1,3,4\},\{0,1,3,4,5\}\}$.

Type (1,2,2,4,4,6):

$201-\{\{0\},\{0,1\},\{0,2\},\{0,1,2,3\},\{0,1,2,4\},\{0,1,2,3,4,5\}\}$,
$202-\{\{0\},\{0,1\},\{0,2\},\{0,1,3,4\},\{0,1,3,4\},\{0,1,2,3,4,5\}\}$.

Type (1,2,2,4,5,5):

$203-\{\{0\},\{0,1\},\{0,2\},\{0,1,2,3\},\{0,1,2,3,4\},\{0,1,2,3,4\}\}$,
$204-\{\{0\},\{0,1\},\{0,2\},\{0,1,2,3\},\{0,1,2,3,4\},\{0,1,2,3,5\}\}$,
$205-\{\{0\},\{0,1\},\{0,2\},\{0,1,2,3\},\{0,1,2,4,5\},\{0,1,2,4,5\}\}$.

Type (1,2,2,4,5,6):

$206-\{\{0\},\{0,1\},\{0,2\},\{0,1,2,3\},\{0,1,2,3,4\},\{0,1,2,3,4,5\}\}$.

Type (1,2,2,4,6,6):

$207-\{\{0\},\{0,1\},\{0,2\},\{0,1,2,3\},\{0,1,2,3,4,5\},\{0,1,2,3,4,5\}\}$.

Type(1,2,2,5,5,5):

208 − $\{\{0\},\{0,1\},\{0,1\},\{0,1,3,4,5\},\{0,1,3,4,5\},\{0,1,3,4,5\}\}$,
209 − $\{\{0\},\{0,1\},\{0,2\},\{0,1,2,3,4\},\{0,1,2,3,4\},\{0,1,2,3,4\}\}$,
210 − $\{\{0\},\{0,1\},\{0,2\},\{0,1,3,4,5\},\{0,1,3,4,5\},\{0,1,3,4,5\}\}$.

Type (1,2,2,5,5,6):

211 − $\{\{0\},\{0,1\},\{0,2\},\{0,1,2,3,4\},\{0,1,2,3,4\},\{0,1,2,3,4,5\}\}$.

Type (1,2,2,6,6,6):

212 − $\{\{0\},\{0,1\},\{0,2\},\{0,1,2,3,4,5\},\{0,1,2,3,4,5\},\{0,1,2,3,4,5\}\}$.

Type(1,2,3,3,3,3):

213 − $\{\{0\},\{0,1\},\{0,1,2\},\{0,1,2\},\{0,1,2\},\{0,1,2\}\}$,
214 − $\{\{0\},\{0,1\},\{0,1,2\},\{0,1,2\},\{0,1,2\},\{0,1,5\}\}$,
215 − $\{\{0\},\{0,1\},\{0,2,3\},\{0,2,3\},\{0,2,3\},\{0,2,3\}\}$,
216 − $\{\{0\},\{0,1\},\{0,1,2\},\{0,1,2\},\{0,1,4\},\{0,1,5\}\}$,
217 − $\{\{0\},\{0,1\},\{0,1,2\},\{0,1,2\},\{0,4,5\},\{0,4,5\}\}$,
218 − $\{\{0\},\{0,1\},\{0,1,2\},\{0,3,4\},\{0,3,4\},\{0,3,4\}\}$,
219 − $\{\{0\},\{0,1\},\{0,1,2\},\{0,1,3\},\{0,1,4\},\{0,1,5\}\}$,
220 − $\{\{0\},\{0,1\},\{0,1,2\},\{0,1,3\},\{0,4,5\},\{0,4,5\}\}$,
221 − $\{\{0\},\{0,1\},\{0,2,3\},\{0,2,3\},\{0,4,5\},\{0,4,5\}\}$.

Type (1,2,3,3,3,4):

222 − $\{\{0\},\{0,1\},\{0,1,2\},\{0,1,2\},\{0,1,2\},\{0,1,2,5\}\}$,
223 − $\{\{0\},\{0,1\},\{0,1,2\},\{0,1,2\},\{0,1,4\},\{0,1,2,4\}\}$,
224 − $\{\{0\},\{0,1\},\{0,2,3\},\{0,2,3\},\{0,2,3\},\{0,1,2,3\}\}$,
225 − $\{\{0\},\{0,1\},\{0,1,2\},\{0,1,2\},\{0,1,4\},\{0,1,2,5\}\}$,
226 − $\{\{0\},\{0,1\},\{0,1,2\},\{0,1,3\},\{0,1,4\},\{0,1,2,3\}\}$,
227 − $\{\{0\},\{0,1\},\{0,1,2\},\{0,3,4\},\{0,3,4\},\{0,1,3,4\}\}$,
228 − $\{\{0\},\{0,1\},\{0,2,3\},\{0,2,3\},\{0,2,3\},\{0,2,3,5\}\}$,
229 − $\{\{0\},\{0,1\},\{0,1,2\},\{0,1,3\},\{0,1,4\},\{0,1,2,5\}\}$,
230 − $\{\{0\},\{0,1\},\{0,1,2\},\{0,3,4\},\{0,3,4\},\{0,1,2,5\}\}$,
231 − $\{\{0\},\{0,1\},\{0,1,2\},\{0,3,4\},\{0,3,4\},\{0,3,4,5\}\}$.

Type (1,2,3,3,3,5):

232 − $\{\{0\},\{0,1\},\{0,1,2\},\{0,1,2\},\{0,1,4\},\{0,1,2,4,5\}\}$,
233 − $\{\{0\},\{0,1\},\{0,1,2\},\{0,1,3\},\{0,1,4\},\{0,1,2,3,4\}\}$,
234 − $\{\{0\},\{0,1\},\{0,1,2\},\{0,3,4\},\{0,3,4\},\{0,1,2,3,4\}\}$,
235 − $\{\{0\},\{0,1\},\{0,2,3\},\{0,2,3\},\{0,2,3\},\{0,1,2,3,5\}\}$,
236 − $\{\{0\},\{0,1\},\{0,1,2\},\{0,1,3\},\{0,1,4\},\{0,1,2,3,5\}\}$,
237 − $\{\{0\},\{0,1\},\{0,1,2\},\{0,3,4\},\{0,3,4\},\{0,1,3,4,5\}\}$.

Type (1,2,3,3,3,6):

238 − $\{\{0\},\{0,1\},\{0,1,2\},\{0,1,3\},\{0,1,4\},\{0,1,2,3,4,5\}\}$,
239 − $\{\{0\},\{0,1\},\{0,1,2\},\{0,3,4\},\{0,3,4\},\{0,1,2,3,4,5\}\}$.

Type (1,2,3,3,4,4):

$240 - \{\{0\},\{0,1\},\{0,1,2\},\{0,1,2\},\{0,1,2,4\},\{0,1,2,4\}\}$,
$241 - \{\{0\},\{0,1\},\{0,1,2\},\{0,1,3\},\{0,1,2,3\},\{0,1,2,3\}\}$,
$242 - \{\{0\},\{0,1\},\{0,2,3\},\{0,2,3\},\{0,1,2,3\},\{0,1,2,3\}\}$,
$243 - \{\{0\},\{0,1\},\{0,1,2\},\{0,1,2\},\{0,1,2,4\},\{0,1,2,5\}\}$,
$244 - \{\{0\},\{0,1\},\{0,1,2\},\{0,1,2\},\{0,1,4,5\},\{0,1,4,5\}\}$,
$245 - \{\{0\},\{0,1\},\{0,1,2\},\{0,1,3\},\{0,1,2,3\},\{0,1,2,5\}\}$,
$246 - \{\{0\},\{0,1\},\{0,1,2\},\{0,1,3\},\{0,1,2,4\},\{0,1,2,4\}\}$,
$247 - \{\{0\},\{0,1\},\{0,2,3\},\{0,2,3\},\{0,1,2,3\},\{0,2,3,5\}\}$,
$248 - \{\{0\},\{0,1\},\{0,2,3\},\{0,2,3\},\{0,2,3,4\},\{0,2,3,4\}\}$,
$249 - \{\{0\},\{0,1\},\{0,1,2\},\{0,1,3\},\{0,1,2,4\},\{0,1,2,5\}\}$,
$250 - \{\{0\},\{0,1\},\{0,1,2\},\{0,1,3\},\{0,1,4,5\},\{0,1,4,5\}\}$,
$251 - \{\{0\},\{0,1\},\{0,2,3\},\{0,2,3\},\{0,1,4,5\},\{0,1,4,5\}\}$,
$252 - \{\{0\},\{0,1\},\{0,2,3\},\{0,2,3\},\{0,2,3,4\},\{0,2,3,5\}\}$.

Type (1,2,3,3,4,5):

$253 - \{\{0\},\{0,1\},\{0,1,2\},\{0,1,2\},\{0,1,2,4\},\{0,1,2,4,5\}\}$,
$254 - \{\{0\},\{0,1\},\{0,1,2\},\{0,1,3\},\{0,1,2,3\},\{0,1,2,3,5\}\}$,
$255 - \{\{0\},\{0,1\},\{0,1,2\},\{0,1,3\},\{0,1,2,4\},\{0,1,2,3,4\}\}$,
$256 - \{\{0\},\{0,1\},\{0,2,3\},\{0,2,3\},\{0,1,2,3\},\{0,1,2,3,5\}\}$,
$257 - \{\{0\},\{0,1\},\{0,2,3\},\{0,2,3\},\{0,2,3,4\},\{0,1,2,3,4\}\}$,
$258 - \{\{0\},\{0,1\},\{0,1,2\},\{0,1,3\},\{0,1,2,4\},\{0,1,2,3,5\}\}$,
$259 - \{\{0\},\{0,1\},\{0,1,2\},\{0,1,3\},\{0,1,2,4\},\{0,1,2,4,5\}\}$,
$260 - \{\{0\},\{0,1\},\{0,2,3\},\{0,2,3\},\{0,2,3,4\},\{0,1,2,3,5\}\}$,
$261 - \{\{0\},\{0,1\},\{0,2,3\},\{0,2,3\},\{0,2,3,4\},\{0,2,3,4,5\}\}$.

Type (1,2,3,3,4,6):

$262 - \{\{0\},\{0,1\},\{0,1,2\},\{0,1,3\},\{0,1,2,4\},\{0,1,2,3,4,5\}\}$,
$263 - \{\{0\},\{0,1\},\{0,2,3\},\{0,2,3\},\{0,2,3,4\},\{0,1,2,3,4,5\}\}$.

Type (1,2,3,3,5,5):

$264 - \{\{0\},\{0,1\},\{0,1,2\},\{0,1,2\},\{0,1,2,4,5\},\{0,1,2,4,5\}\}$,
$265 - \{\{0\},\{0,1\},\{0,1,2\},\{0,1,3\},\{0,1,2,3,4\},\{0,1,2,3,4\}\}$,
$266 - \{\{0\},\{0,1\},\{0,2,3\},\{0,2,3\},\{0,1,2,3,4\},\{0,1,2,3,4\}\}$,
$267 - \{\{0\},\{0,1\},\{0,1,2\},\{0,1,3\},\{0,1,2,3,4\},\{0,1,2,3,5\}\}$,
$268 - \{\{0\},\{0,1\},\{0,1,2\},\{0,1,3\},\{0,1,2,4,5\},\{0,1,2,4,5\}\}$,
$269 - \{\{0\},\{0,1\},\{0,2,3\},\{0,2,3\},\{0,1,2,3,4\},\{0,1,2,3,5\}\}$,
$270 - \{\{0\},\{0,1\},\{0,2,3\},\{0,2,3\},\{0,2,3,4,5\},\{0,2,3,4,5\}\}$.

Type (1,2,3,3,5,6):

$271 - \{\{0\},\{0,1\},\{0,1,2\},\{0,1,3\},\{0,1,2,3,4\},\{0,1,2,3,4,5\}\}$,
$272 - \{\{0\},\{0,1\},\{0,2,3\},\{0,2,3\},\{0,1,2,3,4\},\{0,1,2,3,4,5\}\}$.

Type (1,2,3,3,6,6):

$273 - \{\{0\},\{0,1\},\{0,1,2\},\{0,1,3\},\{0,1,2,3,4,5\},\{0,1,2,3,4,5\}\}$,
$274 - \{\{0\},\{0,1\},\{0,2,3\},\{0,2,3\},\{0,1,2,3,4,5\},\{0,1,2,3,4,5\}\}$.

Type (1,2,3,4,4,4):

$275 - \{\{0\}, \{0,1\}, \{0,1,2\}, \{0,1,2,3\}, \{0,1,2,3\}, \{0,1,2,3\}\}$,
$276 - \{\{0\}, \{0,1\}, \{0,1,2\}, \{0,1,2,3\}, \{0,1,2,3\}, \{0,1,2,5\}\}$,
$277 - \{\{0\}, \{0,1\}, \{0,1,2\}, \{0,1,3,4\}, \{0,1,3,4\}, \{0,1,3,4\}\}$,
$278 - \{\{0\}, \{0,1\}, \{0,1,2\}, \{0,1,2,3\}, \{0,1,2,4\}, \{0,1,2,5\}\}$,
$279 - \{\{0\}, \{0,1\}, \{0,1,2\}, \{0,1,2,3\}, \{0,1,4,5\}, \{0,1,4,5\}\}$,
$280 - \{\{0\}, \{0,1\}, \{0,1,2\}, \{0,3,4,5\}, \{0,3,4,5\}, \{0,3,4,5\}\}$.

Type (1,2,3,4,4,5):

$281 - \{\{0\}, \{0,1\}, \{0,1,2\}, \{0,1,2,3\}, \{0,1,2,3\}, \{0,1,2,3,5\}\}$,
$282 - \{\{0\}, \{0,1\}, \{0,1,2\}, \{0,1,2,3\}, \{0,1,2,4\}, \{0,1,2,3,4\}\}$,
$283 - \{\{0\}, \{0,1\}, \{0,1,2\}, \{0,1,3,4\}, \{0,1,3,4\}, \{0,1,2,3,4\}\}$,
$284 - \{\{0\}, \{0,1\}, \{0,1,2\}, \{0,1,2,3\}, \{0,1,2,4\}, \{0,1,2,3,5\}\}$,
$285 - \{\{0\}, \{0,1\}, \{0,1,2\}, \{0,1,3,4\}, \{0,1,3,4\}, \{0,1,3,4,5\}\}$.

Type (1,2,3,4,4,6):

$286 - \{\{0\}, \{0,1\}, \{0,1,2\}, \{0,1,2,3\}, \{0,1,2,4\}, \{0,1,2,3,4,5\}\}$,
$287 - \{\{0\}, \{0,1\}, \{0,1,2\}, \{0,1,3,4\}, \{0,1,3,4\}, \{0,1,2,3,4,5\}\}$.

Type (1,2,3,4,5,5):

$288 - \{\{0\}, \{0,1\}, \{0,1,2\}, \{0,1,2,3\}, \{0,1,2,3,4\}, \{0,1,2,3,4\}\}$,
$289 - \{\{0\}, \{0,1\}, \{0,1,2\}, \{0,1,2,3\}, \{0,1,2,3,4\}, \{0,1,2,3,5\}\}$,
$290 - \{\{0\}, \{0,1\}, \{0,1,2\}, \{0,1,2,3\}, \{0,1,2,4,5\}, \{0,1,2,4,5\}\}$.

Type (1,2,3,4,5,6):

$291 - \{\{0\}, \{0,1\}, \{0,1,2\}, \{0,1,2,3\}, \{0,1,2,3,4\}, \{0,1,2,3,4,5\}\}$.

Type (1,2,3,4,6,6):

$292 - \{\{0\}, \{0,1\}, \{0,1,2\}, \{0,1,2,3\}, \{0,1,2,3,4,5\}, \{0,1,2,3,4,5\}\}$.

Type (1,2,3,5,5,5):

$293 - \{\{0\}, \{0,1\}, \{0,1,2\}, \{0,1,2,3,4\}, \{0,1,2,3,4\}, \{0,1,2,3,4\}\}$,
$294 - \{\{0\}, \{0,1\}, \{0,1,2\}, \{0,1,3,4,5\}, \{0,1,3,4,5\}, \{0,1,3,4,5\}\}$.

Type (1,2,3,5,5,6):

$295 - \{\{0\}, \{0,1\}, \{0,1,2\}, \{0,1,2,3,4\}, \{0,1,2,3,4\}, \{0,1,2,3,4,5\}\}$.

Type (1,2,3,6,6,6):

$296 - \{\{0\}, \{0,1\}, \{0,1,2\}, \{0,1,2,3,4,5\}, \{0,1,2,3,4,5\}, \{0,1,2,3,4,5\}\}$.

Type (1,2,4,4,4,4):

$297 - \{\{0\}, \{0,1\}, \{0,1,2,3\}, \{0,1,2,3\}, \{0,1,2,3\}, \{0,1,2,3\}\}$,
$298 - \{\{0\}, \{0,1\}, \{0,2,3,4\}, \{0,2,3,4\}, \{0,2,3,4\}, \{0,2,3,4\}\}$,
$299 - \{\{0\}, \{0,1\}, \{0,1,2,3\}, \{0,1,2,3\}, \{0,1,4,5\}, \{0,1,4,5\}\}$.

Type (1,2,4,4,4,5):

$300 - \{\{0\}, \{0,1\}, \{0,1,2,3\}, \{0,1,2,3\}, \{0,1,2,3\}, \{0,1,2,3,5\}\}$,
$301 - \{\{0\}, \{0,1\}, \{0,2,3,4\}, \{0,2,3,4\}, \{0,2,3,4\}, \{0,1,2,3,4\}\}$,
$302 - \{\{0\}, \{0,1\}, \{0,2,3,4\}, \{0,2,3,4\}, \{0,2,3,4\}, \{0,2,3,4,5\}\}$.

Type (1,2,4,4,4,6):

303 – $\{\{0\},\{0,1\},\{0,2,3,4\},\{0,2,3,4\},\{0,2,3,4\},\{0,1,2,3,4,5\}\}$.

Type (1,2,4,4,5,5):

304 – $\{\{0\},\{0,1\},\{0,1,2,3\},\{0,1,2,3\},\{0,1,2,3,4\},\{0,1,2,3,4\}\}$,
305 – $\{\{0\},\{0,1\},\{0,1,2,3\},\{0,1,2,3\},\{0,1,2,3,4\},\{0,1,2,3,5\}\}$.

Type (1,2,4,4,5,6):

306 – $\{\{0\},\{0,1\},\{0,1,2,3\},\{0,1,2,3\},\{0,1,2,3,4\},\{0,1,2,3,4,5\}\}$.

Type (1,2,4,4,6,6):

307 – $\{\{0\},\{0,1\},\{0,1,2,3\},\{0,1,2,3\},\{0,1,2,3,4,5\},\{0,1,2,3,4,5\}\}$.

Type (1,2,5,5,5,5):

308 – $\{\{0\},\{0,1\},\{0,1,2,3,4\},\{0,1,2,3,4\},\{0,1,2,3,4\},\{0,1,2,3,4\}\}$,
309 – $\{\{0\},\{0,1\},\{0,2,3,4,5\},\{0,2,3,4,5\},\{0,2,3,4,5\},\{0,2,3,4,5\}\}$.

Type (1,2,5,5,5,6):

310 – $\{\{0\},\{0,1\},\{0,1,2,3,4\},\{0,1,2,3,4\},\{0,1,2,3,4\},\{0,1,2,3,4,5\}\}$.

Type (1,2,6,6,6,6):

311 – $\{\{0\},\{0,1\},\{0,1,2,3,4,5\},\{0,1,2,3,4,5\},\{0,1,2,3,4,5\},\{0,1,2,3,4,5\}\}$.

Type (1,3,3,3,3,3):

312 – $\{\{0\},\{0,1,2\},\{0,1,2\},\{0,1,2\},\{0,1,2\},\{0,1,2\}\}$,
313 – $\{\{0\},\{0,1,2\},\{0,1,2\},\{0,1,2\},\{0,4,5\},\{0,4,5\}\}$.

Type (1,3,3,3,3,4):

314 – $\{\{0\},\{0,1,2\},\{0,1,2\},\{0,1,2\},\{0,1,2\},\{0,1,2,5\}\}$,
315 – $\{\{0\},\{0,1,2\},\{0,1,2\},\{0,3,4\},\{0,3,4\},\{0,1,2,5\}\}$.

Type (1,3,3,3,3,5):

316 – $\{\{0\},\{0,1,2\},\{0,1,2\},\{0,3,4\},\{0,3,4\},\{0,1,2,3,4\}\}$.

Type (1,3,3,3,3,6):

317 – $\{\{0\},\{0,1,2\},\{0,1,2\},\{0,3,4\},\{0,3,4\},\{0,1,2,3,4,5\}\}$.

Type (1,3,3,3,4,4):

318 – $\{\{0\},\{0,1,2\},\{0,1,2\},\{0,1,2\},\{0,1,2,4\},\{0,1,2,4\}\}$,
319 – $\{\{0\},\{0,1,2\},\{0,1,2\},\{0,1,2\},\{0,1,2,4\},\{0,1,2,5\}\}$.

Type (1,3,3,3,4,5):

320 – $\{\{0\},\{0,1,2\},\{0,1,2\},\{0,1,2\},\{0,1,2,4\},\{0,1,2,4,5\}\}$.

Type (1,3,3,3,5,5):

321 – $\{\{0\},\{0,1,2\},\{0,1,2\},\{0,1,2\},\{0,1,2,4,5\},\{0,1,2,4,5\}\}$.

Type (1,3,3,4,4,4):

322 – $\{\{0\},\{0,1,2\},\{0,1,2\},\{0,1,2,3\},\{0,1,2,3\},\{0,1,2,3\}\}$,
323 – $\{\{0\},\{0,1,2\},\{0,1,2\},\{0,1,2,3\},\{0,1,2,3\},\{0,1,2,5\}\}$,
324 – $\{\{0\},\{0,1,2\},\{0,1,2\},\{0,1,2,3\},\{0,1,2,4\},\{0,1,2,5\}\}$,
325 – $\{\{0\},\{0,1,2\},\{0,1,2\},\{0,3,4,5\},\{0,3,4,5\},\{0,3,4,5\}\}$.

Type (1,3,3,4,4,5):

326 – $\{\{0\},\{0,1,2\},\{0,1,2\},\{0,1,2,3\},\{0,1,2,3\},\{0,1,2,3,5\}\}$,
327 – $\{\{0\},\{0,1,2\},\{0,1,2\},\{0,1,2,3\},\{0,1,2,4\},\{0,1,2,3,4\}\}$,
328 – $\{\{0\},\{0,1,2\},\{0,1,2\},\{0,1,2,3\},\{0,1,2,4\},\{0,1,2,3,5\}\}$.

Type (1,3,3,4,4,6):

329 – $\{\{0\},\{0,1,2\},\{0,1,2\},\{0,1,2,3\},\{0,1,2,4\},\{0,1,2,3,4,5\}\}$.

Type (1,3,3,4,5,5):

330 – $\{\{0\},\{0,1,2\},\{0,1,2\},\{0,1,2,3\},\{0,1,2,3,4\},\{0,1,2,3,4\}\}$,
331 – $\{\{0\},\{0,1,2\},\{0,1,2\},\{0,1,2,3\},\{0,1,2,3,4\},\{0,1,2,3,5\}\}$,
332 – $\{\{0\},\{0,1,2\},\{0,1,2\},\{0,1,2,3\},\{0,1,2,4,5\},\{0,1,2,4,5\}\}$.

Type (1,3,3,4,5,6):

333 – $\{\{0\},\{0,1,2\},\{0,1,2\},\{0,1,2,3\},\{0,1,2,3,4\},\{0,1,2,3,4,5\}\}$.

Type (1,3,3,4,6,6):

334 – $\{\{0\},\{0,1,2\},\{0,1,2\},\{0,1,2,3\},\{0,1,2,3,4,5\},\{0,1,2,3,4,5\}\}$.

Type (1,3,3,5,5,5):

335 – $\{\{0\},\{0,1,2\},\{0,1,2\},\{0,1,2,3,4\},\{0,1,2,3,4\},\{0,1,2,3,4\}\}$.

Type (1,3,3,5,5,6):

336 – $\{\{0\},\{0,1,2\},\{0,1,2\},\{0,1,2,3,4\},\{0,1,2,3,4\},\{0,1,2,3,4,5\}\}$.

Type (1,3,3,6,6,6):

337 – $\{\{0\},\{0,1,2\},\{0,1,2\},\{0,1,2,3,4,5\},\{0,1,2,3,4,5\},\{0,1,2,3,4,5\}\}$.

Type (1,4,4,4,4,4):

338 – $\{\{0\},\{0,1,2,3\},\{0,1,2,3\},\{0,1,2,3\},\{0,1,2,3\},\{0,1,2,3\}\}$.

Type (1,4,4,4,4,5):

339 – $\{\{0\},\{0,1,2,3\},\{0,1,2,3\},\{0,1,2,3\},\{0,1,2,3\},\{0,1,2,3,5\}\}$.

Type (1,4,4,4,5,5):

340 – $\{\{0\},\{0,1,2,3\},\{0,1,2,3\},\{0,1,2,3\},\{0,1,2,3,4\},\{0,1,2,3,4\}\}$,
341 – $\{\{0\},\{0,1,2,3\},\{0,1,2,3\},\{0,1,2,3\},\{0,1,2,3,4\},\{0,1,2,3,5\}\}$.

Type (1,4,4,4,5,6):

342 – $\{\{0\},\{0,1,2,3\},\{0,1,2,3\},\{0,1,2,3\},\{0,1,2,3,4\},\{0,1,2,3,4,5\}\}$.

Type (1,4,4,4,6,6):

343 – $\{\{0\},\{0,1,2,3\},\{0,1,2,3\},\{0,1,2,3\},\{0,1,2,3,4,5\},\{0,1,2,3,4,5\}\}$.

Type (1,5,5,5,5,5):

$344-\{\{0\},\{0,1,2,3,4\},\{0,1,2,3,4\},\{0,1,2,3,4\},\{0,1,2,3,4\},\{0,1,2,3,4\}\}$.

Type (1,5,5,5,5,6):

$345-\{\{0\},\{0,1,2,3,4\},\{0,1,2,3,4\},\{0,1,2,3,4\},\{0,1,2,3,4\},\{0,1,2,3,4,5\}\}$.

Type (1,6,6,6,6,6):

$346-\{\{0\},\{0,1,2,3,4,5\},\{0,1,2,3,4,5\},\{0,1,2,3,4,5\},\{0,1,2,3,4,5\},\{0,1,2,3,4,5\}\}$.
Moreover by Theorem 3.5, we have $|\mathcal{H}_6^{11}|+|\mathcal{H}_6^{22}|=2|\mathcal{H}_5|-|\mathcal{H}_4|=2\times 118-30=206$, so $|\mathcal{H}_6|=206+|\mathcal{H}_6^{12}|=206+346=552$, and these complete the proof of (vi). □

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