PRIME WEAK HYPER BCK-IDEALS OF LOWER HYPER BCK-SEMI LATTICE

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ABSTRACT. In this manuscript, we introduce the concept of a prime weak hyper BCK-ideal of a lower hyper BCK-semi lattice and consider its property. Also, we introduce the concept of an irreducible weak hyper BCK-ideal of a lower hyper BCK-semi lattice and investigate its relation with the prime weak hyper BCK-ideal.

1. Introduction

The study of BCK-algebras was initiated by Y. Imai and K. Iséki[2] in 1966 as a generalization of the concept of set-theoretic difference and propositional calcului. Since then a great deal of literature has been produced on the theory of BCK-algebras. The hyper structure theory(called also multialgebras) was introduced in 1934 by F. Marty at the 8th congress of Scandinavian Mathematiciens. In [4], Y.B. Jun et al. applied the hyper structures to BCK-algebras, and introduced the notion of a hyper BCK-algebra which is a generalization of BCK-algebra, and investigated some related properties. In [1] R. A. Borzooei et al. introduced the concept of $\ll$-left ($\ll$-right) scalar element in a hyper BCK-algebra and considered the relation among them. Now, we follow [1] and [4] and introduce the concept of a prime and irreducible weak hyper BCK-ideals of a lower hyper BCK-semi lattice and investigate the relation between them.

2. Preliminaries

Let H be a non-empty set endowed with a hyper operation "$\circ$", that is, $\circ$ is a function from $H \times H$ to $\mathcal{P}^*(H) = \mathcal{P}(\mathcal{H})\backslash\{\phi\}$. For two subset A and B of H, denote by $A \circ B$ the set $\bigcup\limits_{a\in A, b\in B} a \circ b$. We shall use $x \circ y$ instead of $x \circ \{y\}$, $\{x\} \circ y$, or $\{x\} \circ \{y\}$.

Definition 2.1. *By a hyper BCK-algebra we mean a non-empty set H endowed with a hyper operation "$\circ$" and a constant 0 satisfying the following axioms:*

(HB1) $(x \circ z) \circ (y \circ z) \ll x \circ y$,

(HB2) $(x \circ y) \circ z = (x \circ z) \circ y$,

(HB3) $x \circ H \ll \{x\}$,

(HB4) $x \ll y$ *and* $y \ll x$ *imply* $x = y$

for all $x, y, z \in H$, where $x \ll y$ is defined by $0 \in x \circ y$ and for every $A, B \subseteq H$, $A \ll B$ is defined by $\forall a \in A$, $\exists b \in B$ such that $a \ll b$. In such case, we call "$\ll$" the hyper order in H.

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Theorem 2.2. *In any hyper* BCK*-algebra* H*, the following hold:*

(a1) $0 \circ 0 = \{0\}$,

(a2) $0 \ll x$,

(a3) $x \ll x$,

(a4) $A \ll A$,

(a5) $A \ll 0$ *implies* $A = \{0\}$,

(a6) $A \subseteq B$ *implies* $A \ll B$,

(a7) $0 \circ x = \{0\}$,

(a8) $x \circ y \ll x$,

(a9) $x \circ 0 = \{x\}$,

(a10) $y \ll z$ *implies* $x \circ z \ll x \circ y$,

(a11) $x \circ y = \{0\}$ *implies* $(x \circ z) \circ (y \circ z) = \{0\}$ *and* $x \circ z \ll y \circ z$,

(a12) $A \circ \{0\} = \{0\}$ *implies* $A = \{0\}$

for all $x, y, z \in H$ *and* $A, B \subseteq H$.

Definition 2.3. *Let* H *be a hyper* BCK*-algebra. Then*

(i) *A non-empty subset* S *of* H *is called a hyper subalgebra of* H *if* S *is a hyper* BCK*-algebra with respect to the hyper operation "* $\circ$ *" on* H,

(ii) *A non-empty subset* I *of* H *is called a weak hyper* BCK*-ideal of* H *if it satisfies:*

(w1) $0 \in I$,

(w2) $(\forall x, y \in H)(x \circ y \subseteq I$ *and* $y \in I \Longrightarrow x \in I)$.

Theorem 2.4. [4] *Let* S *be a non-empty subset of a hyper* BCK*-algebra* H*. Then* S *is a hyper subalgebra of* H *if and only if* $x \circ y \subseteq S$ *for all* $x, y \in S$.

Theorem 2.5. [4] *Let* H *be a hyper* BCK*-algebra. Then*

(i) *the set*

$$S(H) := \{x \in H | x \circ x = \{0\}\}$$

is a hyper subalgebra of H*, which is called* BCK*-part of* H.

Definition 2.6. [4] *Let* I *be a non-empty subset of a hyper* BCK*-algebra* H*. Then* I *is said to be a hyper* BCK*-ideal of* H *if*

(HI1) $0 \in I$,

(HI2) $x \circ y \ll I$ *and* $y \in I$ *imply* $x \in I$ *for all* $x, y \in H$.

Theorem 2.7. [3] *Let* A *be a subset of a hyper* BCK*-algebra* H*. If* I *is a hyper* BCK*-ideal of* H *such that* $A \ll I$*, then* A *is contained in* I.

Definition 2.8. [4] *Let* A *be a subset of hyper* BCK*-algebra* H*. The smallest weak hyper* BCK*-ideal containing* A *is called the weak hyper* BCK*-ideal generated by* A*, and is denoted by* $[A]_w$.

Theorem 2.9. [3] *Let A be a non-empty subset of hyper BCK-algebra H. If an element x of H satisfies the identity*

$$(...((x \circ a_1) \circ a_2) \circ ...) \circ a_n = \{0\}, \textit{for some } a_1, a_2, ..., a_n \in A,$$

then $x \in [A]_w$, i, e.,

$$[A]_w \supseteq \{x \in H | (...((x \circ a_1) \circ a_2) \circ ...) \circ a_n = \{0\} \textit{ for some } a_1, a_2, ..., a_n \in A\}$$

Moreover, if $A \subseteq S(H)$, then

$$[A]_w = \{x \in H | (...((x \circ a_1) \circ a_2) \circ ...) \circ a_n = \{0\} \textit{ for some } a_1, a_2, ..., a_n \in A\}.$$

Definition 2.10. [1] *Let H be a hyper BCK-algebra. Then,*

(i) *$a \in H$ is called a $\ll$-left scalar element, (resp. $\ll$-right scalar element) if whenever $a \ll x$ (respect. $x \ll a$) then $a \circ x = \{0\}$ (resp. $x \circ a = \{0\}$) for all $x \in H$.*
(ii) *$a \in H$ is called a $\ll$-scalar element if a is both $\ll$-left scalar and $\ll$-right scalar element.*
(iii) *$A \subseteq H$ is called a $\ll$-left scalar set (resp. $\ll$-right scalar set, $\ll$-scalar set) if every element of A is a $\ll$-left scalar (resp. $\ll$-right scalar, $\ll$-scalar) element.*

Theorem 2.11. [1] *Let H be a hyper BCK-algebra and $A \subseteq H$. Then $A \subseteq S(H)$ if and only if A is a $\ll$-left scalar set of H.*

Theorem 2.12. [1] *Let H be a hyper BCK-algebra and $A \subseteq H$. if A is a $\ll$-right scalar set of H, then $A \subseteq S(H)$*

Definition 2.13. [1] *Let H be a hyper BCK-algebra and $x, y, z \in H$. If $x \ll y$ and $y \ll z$ imply $x \ll z$, then we say that H satisfies the transitive condition.*

Corollary 2.14. *Let H be a hyper BCK-algebra which is satisfies the transitive condition. Then $(H, \ll)$ is a partially ordered set.*

Proof. By $(HB4)$, Theorem 2.1(a3) and Definition 2.8, the proof is clear. □

3. Prime weak hyper BCK-ideals of a lower hyper BCK-semi lattice

From now on, we let H be a hyper BCK-algebra.

Definition 3.1. *Let H satisfies the transitive condition, then we say H is lower semi lattice if $x \wedge y$ (greatest lower bound of x and y) exists and belongs to H for all $x, y \in H$.*

Example 3.2. (i) *Consider the hyper BCK-algebra $H = \{0, 1, 2\}$ with the following Cayley table:*

$\circ$	0	1	2
0	$\{0\}$	$\{0\}$	$\{0\}$
1	$\{1\}$	$\{0\}$	$\{0\}$
2	$\{2\}$	$\{1, 2\}$	$\{0, 1, 2\}$

The following table of $\wedge$ shows that H is a lower semi lattice.

$\wedge$	0	1	2
0	0	0	0
1	0	1	1
2	0	1	2

(ii) Let $H = N \cup \{0, \alpha, \beta\}$, where $(0 \neq)\alpha \notin N$, $(0 \neq)\beta \notin N$ and $\alpha \neq \beta$. Define a hyper operation "$\circ$" on H as follows:

$$x \circ y = \begin{cases} \{0, x\} & , \ \text{if } (x \le y,\ x, y \in N \cup \{0\}) \text{ or } (x \in N \cup \{0\},\ y = \alpha, \beta) \\ \{x\} & , \ \text{if } x > y \text{ and } x, y \in N \cup \{0\} \\ \{\alpha\} & , \ \text{if } x = \alpha,\ y \neq \alpha, \\ \{\beta\} & , \ \text{if } x = \beta,\ y \neq \beta, \\ \{0\} & , \ \text{if } x = y = \alpha \text{ or } x = y = \beta, \end{cases}$$

for all $x, y \in H$. Then $(H, \circ, 0)$ is a hyper BCK-algebra. But, it is not a lower semi lattice, since $x \ll \alpha, \beta$ for all $x \in N \cup \{0\}$ and so $\alpha \wedge \beta$ does not exist.

Notation 3.3. *For any elements a, x of H and $m \in N$, we denote*

$a \circ^m x = (...((a \circ x) \circ x) \circ ...) \circ x$, in which x occurs m times.

Proposition 3.4. [1] *Let H be a lower semi lattice and $|x \circ y| < \infty$ for all $x, y \in H$. If $a \circ^m x = \{0\}$ and $a \circ^n y = \{0\}$ for some $m, n \in N$ and $a, x, y \in H$, and $x \wedge y$ is a $\ll$-right scalar element of H, then there exists a natural number p such that $a \circ^p (x \wedge y) = \{0\}$.*

Proposition 3.5. [1] *Let A and B be non-empty subsets of H. If an element x of H satisfies the identity*

$$(...((x \circ c_1) \circ c_2) \circ ...) \circ c_n = \{0\}$$

for some $c_1, c_2, ..., c_n \in A \cup B$, then there exist $a_1, a_2, ..., a_k \in A$ and $b_1, b_2, ..., b_l \in B$ such that

$$((...((x \circ a_1) \circ a_2) \circ ...) \circ a_k) \circ b_1) \circ b_2)...) \circ b_l = \{0\}.$$

Theorem 3.6. *Let H be a lower semi lattice such that $|x \circ y| < \infty$ for all $x, y \in H$, and let A be a subset of H and $x, y \in H$ such that $A \cup \{x, y\}$ is contained in $S(H)$. If $x \wedge y \in A$ is a $\ll$-right scalar element, then*

$$[A \cup \{x\}]_w \cap [A \cup \{y\}]_w = [A]_w$$

Proof. Obviously, it is sufficient to show that $[A \cup \{x\}]_w \cap [A \cup \{y\}]_w \subseteq [A]_w$. Let $z \in [A \cup \{x\}]_w \cap [A \cup \{y\}]_w$. Then by Theorem 2.9 and Proposition 3.5, there exist natural numbers l, k such that

$$((...((z \circ a_1) \circ a_2) \circ ...) \circ a_m) \circ^l x = \{0\} \text{ and } ((...((z \circ b_1) \circ b_2) \circ ...) \circ b_n) \circ^k y = \{0\}$$

for some $a_1, a_2, ..., a_m, b_1, b_2, ..., b_n \in A$. Hence by using (HB2) of H and Theorem 2.2(a7), we get

$$((...((((...((z \circ a_1) \circ a_2) \circ ...) \circ a_m) \circ b_1) \circ b_2)...) \circ b_n) \circ^l x = \{0\} \text{ and}$$

$$((...((((...((z \circ a_1) \circ a_2) \circ ...) \circ a_m) \circ b_1) \circ b_2)...) \circ b_n) \circ^k y = \{0\}$$

For simplicity, we will denote by T the set $(...((((...((z \circ a_1) \circ a_2) \circ ...) \circ a_m) \circ b_1) \circ b_2)...) \circ b_n$. Thus by Proposition 3.4 , for any $t \in T$ there exists a natural number p_t such that

$$t \circ^{p_t} (x \wedge y) = \{0\} \tag{1}$$

Since $| T | < \infty$ by hypothesis, there exists a natural number p such that $p = \max \{p_t : t \in T\}$. We note that for all natural numbers l and k with $l \le k$ and $\forall\ b, c \in H$ if $b \circ^l c = \{0\}$, then $b \circ^k c = \{0\}$ by using Theorem 2.2(a7) repeatedly. Hence, by using (HB2) of H and Theorem 2.2(a7) it follow from (1) and $p_t \le p$ that $t \circ^p (x \wedge y) = \{0\}$ for all $t \in T$, and so we have

$$T \circ^p (x \wedge y) = \bigcup \{t \circ^p (x \wedge y) \mid t \in T\} = \{0\},$$

that is

$$((...((((...((z \circ a_1) \circ a_2) \circ ...) \circ a_m) \circ b_1) \circ b_2)...) \circ b_n) \circ^p (x \wedge y) = \{0\},$$

and so $z \in [A]_w$ by Theorem 2.9. Therefore $[A \cup \{x\}]_w \cap [A \cup \{y\}]_w \subseteq [A]_w$, which completes the proof. □

Definition 3.7. *A non-empty subset S of H is said to be closed set related to the hyper order $\ll$ (briefly, $\ll$-closed set) if*

$$(\forall x \in H)\ (\forall s \in S)(x \ll s \Longrightarrow x \in S).$$

Note that every hyper BCK-ideal is a $\ll$-closed set by Theorem 2.7. But the following example shows that the converse may not be true in general.

Example 3.8. [5, *Page* 260] *Consider the hyper BCK-algebra $H = \{0, 1, 2, 3, 4\}$ with the following Cayley table:*

$\circ$	0	1	2	3	4
0	$\{0\}$	$\{0\}$	$\{0\}$	$\{0\}$	$\{0\}$
1	$\{1\}$	$\{0\}$	$\{0\}$	$\{1\}$	$\{0\}$
2	$\{2\}$	$\{1\}$	$\{0\}$	$\{2\}$	$\{0\}$
3	$\{3\}$	$\{3\}$	$\{3\}$	$\{0\}$	$\{3\}$
4	$\{4\}$	$\{4\}$	$\{4\}$	$\{4\}$	$\{0\}$

It is easy to check that $S := \{0, 1\}$ is a $\ll$-closed set but is not a hyper BCK-ideal of H, since $2 \circ 1 = \{1\} \ll S$ and $1 \in S$ but $2 \notin S$.

Lemma 3.9. *Every $\ll$-closed set of a hyper BCK-algebra is a hyper subalgebra.*

Proof. Let S be a $\ll$-closed set of the hyper BCK-algebra H, and let $x, y \in S$. Since $x \circ y \ll x$ by Theorem 2.2(a8), it follows that $t \ll x$ for all $t \in x \circ y$. Hence $t \in S$ because $x \in S$ and S is a $\ll$-closed set. Thus $x \circ y \subseteq S$. This implies that S is a hyper subalgebra of H. □

The following example shows that the converse of Lemma 3.9 may not be true in general.

Example 3.10. *Consider the hyper BCK-algebra $H = \{0, 1, 2\}$ with the following Cayley table:*

$\circ$	0	1	2
0	$\{0\}$	$\{0\}$	$\{0\}$
1	$\{1\}$	$\{0\}$	$\{0\}$
2	$\{2\}$	$\{2\}$	$\{0, 2\}$

It is easy to check that $S := \{0, 2\}$ is a hyper subalgebra of H but it is not a $\ll$-closed set since $1 \ll 2$ and $1 \notin S$.

Definition 3.11. *A proper weak hyper BCK-ideal P of a lower semi lattice H is said to be prime if $x \wedge y \in P$ implies $x \in P$ or $y \in P$ for all $x, y \in H$.*

Definition 3.12. *A lower semi lattice H is said to be integer if whenever $x \wedge y = 0$, then $x = 0$ or $y = 0$ for all $x, y \in H$.*

One can easily observe that a non-trivial lower semi lattice H is integer if and only if the weak hyper BCK-ideal $\{0\}$ of H is prime.

Example 3.13. *Consider the lower semi lattice $H = \{0, 1, 2\}$ with the following Cayley table:*

$\circ$	0	1	2
0	$\{0\}$	$\{0\}$	$\{0\}$
1	$\{1\}$	$\{0, 1\}$	$\{0, 1\}$
2	$\{2\}$	$\{1, 2\}$	$\{0, 1, 2\}$

It easy to check that H is integer and the weak hyper BCK-ideal $I := \{0, 1\}$ of H is prime.

Example 3.14. *Consider the lower semi lattice $H = \{0, 1, 2, 3\}$ with the following Cayley table:*

$\circ$	0	1	2	3
0	$\{0\}$	$\{0\}$	$\{0\}$	$\{0\}$
1	$\{1\}$	$\{0, 1\}$	$\{0\}$	$\{1\}$
2	$\{2\}$	$\{2\}$	$\{0, 1\}$	$\{2\}$
3	$\{3\}$	$\{3\}$	$\{3\}$	$\{0, 3\}$

It easy to check that H is not integer, and the weak hyper BCK-ideal $I := \{0, 1\}$ of H is not prime because $2 \wedge 3 = 0 \in I$ but $2 \notin I$ and $3 \notin I$.

Notation 3.15. *For any hyper BCK-algebra H,*
(i) denote by $\mathcal{I}_\omega(H)$ the set of all weak hyper BCK-ideals of H.
(ii) denote by $\mathcal{I}^r_\omega(H)$ the set of all weak hyper BCK-ideals of H which are $\ll$-right scalar.

Theorem 3.16. *Let H be an integer lower semi lattice with properties:*
(i) $|x \circ y| < \infty$ for all $x, y \in H$,
(ii) for all $\ll$-right scalar set $A \subseteq H$, $[A]_w \in \mathcal{I}^r_w(H)$,
and let S be a $\ll$-closed set of H such that $H - S$ is a $\ll$-right scalar set. Then the set

$$\sum := \{I \in \mathcal{I}^r_w(H) : I \cap T = \phi, \textit{where } T = S - \{0\}\}$$

has a maximal weak hyper BCK-ideal P of H such that $P \cap T = \phi$. Moreover, P is prime .

Proof. Clearly, $\{0\} \in \sum$ and so $\sum \neq \phi$. The existence of maximal weak hyper BCK-ideal P of H such that $P \cap T = \phi$ easily follows from Zorn's lemma. We will prove that P is prime. If not, then there exist $x, y \in H$ such that $x \wedge y \in P$, $x \notin P$ and $y \notin P$. Hence P is properly contained in both $[P \cup \{x\}]_w = P_1$ and $[P \cup \{y\}]_w = P_2$. Since H is integer and $x, y \neq 0$, we get $x \wedge y \neq 0$. If $x \in T \subseteq S$, then, since S is a $\ll$-closed set and $x \wedge y \ll x$, we have $x \wedge y \in T$ and so $x \wedge y \in P \cap T$, which is impossible. Hence $x \notin T$, and so $x \in H - S$ is a $\ll$-right scalar. Moreover, P is a $\ll$-right scalar set because $P \in \mathcal{I}^r_w(H)$. Hence, $P_1 \in \mathcal{I}^r_w(H)$ by hypothesis (ii). Similarly, we have $y \notin T$ and so $P_2 \in \mathcal{I}^r_w(H)$. Hence, by the maximality of P there exist $s_i \in P_i \cap T$ for $i = 1, 2$. Since $0 \notin T$ and $s_1 \in T$, it follows that $s_1 \neq 0$. Similarly, we have $s_2 \neq 0$. Since H is integer, we get $s_1 \wedge s_2 \neq 0$. Since $s_1 \in T \subseteq S$ and S is $\ll$-closed, it follows from $s_1 \wedge s_2 \ll s_1$ that $s_1 \wedge s_2 \in S$. Hence $s_1 \wedge s_2 \in T$. Now, we show that $s_1 \wedge s_2 \in P_1$. Since $s_1 \wedge s_2 \ll s_1$ and P_1 is a $\ll$-right scaler, it follows that $(s_1 \wedge s_2) \circ s_1 = \{0\}$ and so $s_1 \wedge s_2 \in [P_1]_w$ by Theorem 2.9. Hence, since P_1 is a weak hyper BCK-ideal of H, we have $[P_1]_w = P_1$ and so $s_1 \wedge s_2 \in P_1$. Similarly, we can show that $s_1 \wedge s_2 \in P_2$. Since $P \cup \{x, y\} \subseteq H - S$, it follow from the hypothesis and Theorem 2.12 that $P \cup \{x, y\} \subseteq S(H)$. Hence by Theorem 3.6, we have

$$P_1 \cap P_2 = [P \cup \{x\}]_w \cap [P \cup \{y\}]_w = [P]_w = P.$$

This implies that $s_1 \wedge s_2 \in P \cap T$, which is a contradiction. Therefore P is prime. □

Definition 3.17. *Let H be a lower semi lattice, and let $I \in \mathcal{I}_\omega^r(H)$. Then I is said to be irreducible if $I = A \cap B$ implies $I = A$ or $I = B$ for all $A, B \in \mathcal{I}_\omega^r(H)$.*

Example 3.18. *Consider the lower semi lattice $H = \{0, a, b, c\}$ with the following Cayley table:*

$\circ$	0	a	b	c
0	$\{0\}$	$\{0\}$	$\{0\}$	$\{0\}$
a	$\{a\}$	$\{0, a\}$	$\{0\}$	$\{a\}$
b	$\{b\}$	$\{b\}$	$\{0, b\}$	$\{b\}$
c	$\{c\}$	$\{c\}$	$\{c\}$	$\{0\}$

Then it is easy to check that

$$\mathcal{I}_\omega^r = \{\{0\}, \{0, c\}\}.$$

and so $I := \{0, c\}$ is irreducible.

Proposition 3.19. *Let H be a lower semi lattice and $|x \circ y| < \infty$ for all $x, y \in H$. If $I \in \mathcal{I}_\omega^r(H)$ is prime, then the following implication holds.*

$$(\forall A, B \in \mathcal{I}_\omega^r(H)(A \cap B \subseteq I \Longrightarrow A \subseteq I \text{ or } B \subseteq I.)$$

Proof. Let $I \in \mathcal{I}_\omega^r(H)$ be prime. Suppose $A, B \in \mathcal{I}_\omega^r(H)$ such that $A \cap B \subseteq I$, but $A \not\subseteq I$ and $B \not\subseteq I$, by contrary. Then there exist a and b such that $a \in A - I$ and $b \in B - I$. Since A is a $\ll$-right scalar set and $a \wedge b \ll a$, we get $(a \wedge b) \circ a = \{0\}$ which implies $a \wedge b \in [A]_w$ by Theorem 2.9. Since A is a weak hyper BCK- ideal of H, $[A]_w = A$. Hence $a \wedge b \in A$. Similarly, we have $a \wedge b \in B$. Thus $a \wedge b \in A \cap B \subseteq I$. Hence, since I is prime, we have $a \in I$ or $b \in I$, which is impossible. This completes the proof. □

Proposition 3.20. *Let H be a lower semi lattice and $|x \circ y| < \infty$ for all $x, y \in H$. If $I \in \mathcal{I}_\omega^r(H)$ satisfies the implication*

$$(\forall A, B \in \mathcal{I}_\omega^r(H)(A \cap B \subseteq I \Longrightarrow A \subseteq I \text{ or } B \subseteq I),$$

then I is irreducible.

Proof. Suppose that $I = A \cap B$ for some weak hyper BCK-ideals A and B belong to $\mathcal{I}_\omega^r(H)$. Then $I \subseteq A$ and $I \subseteq B$. On the other hand, $A \subseteq I$ or $B \subseteq I$ by hypothesis. Therefore $I = A$ or $I = B$, which completes the proof. □

Corollary 3.21. *Let H be a lower semi lattice and $|x \circ y| < \infty$ for all $x, y \in H$. If $I \in \mathcal{I}_\omega^r(H)$ is prime, then I is irreducible.*

Proof. By Propositions 3.19 and 3.20 the result holds. □

Open Problem 3.22. *Does the converse of Corollary 3.21 holds? Prove it or find a contra example.*

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