A GOAL PROGRAMMING MODEL FOR MULTIOBJECTIVE FACILITY LOCATION PROBLEMS IN A COMPETITIVE ENVIRONMENT

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Abstract. An objective of the competitive facility location problems is mainly to maximize customers using her/his facilities. This paper proposes a new multiobjective competitive facility location problem by adding another new objective about the convenience for customers. A solution algorithm based upon the goal programming is proposed for the formulated multiobjective problem. It is shown the result of applying the solution algorithm to numerical examples for the multiobjective competitive facility location problem.

1 Introduction A competitive facility location problem (CFLP) is one of the optimal location problems for competitive facilities, e.g. shops and supermarkets, and an objective for CFLPs is mainly to capture as many customers as possible. Mathematical studies on CFLPs are originated by Hotelling [9]. He considered a CFLP that each of decision makers (DMs) can locate on a line segment and move her/his facility at any times under the conditions that customers are uniformly distributed on the line segment and only use the nearest facility. CFLPs on a plain were studied by Eaton and Lipsey [5], Okabe and Suzuki [15], etc. As extension of Hotelling's CFLP, Wendell and McKelvey [20] assumed that there exist customers on a finite number of points, called "demand points" (DPs), and they considered a CFLP on a network whose nodes are DPs. Hakimi [8] considered a CFLP on the network under the conditions that the DM locates her/his facilities on a network that competitive facilities were already located. Drezner [4] extended Hakimi's CFLP to a CFLP on a plane that there are DPs and competitive facilities. In the studies of CFLPs including the above CFLPs, it is assumed that customers only use the nearest facility from them. Karkazis [13] extended Hakimi's CFLP by separately considering both the distance between customers and facilities and the quality level of facilities, e.g., scale of facilities and quality of service provided by facilities. Uno and Sakawa [18] extended Drezner's CFLP by simultaneously considering both the distance and the quality by introducing the Huff's attractive function [10, 11].

The objective of the above CFLPs is only to maximize the number of customers using her/his facilities. However, their optimal locations are often inconvenient for customers; e.g. for Hotelling's CFLP with two facilities, both of their optimal sites are at the middle point of the line segment, and such a location is inconvenient for many customers, especially customers on the edge of the line segment. In this paper, we propose a new multiobjective CFLP by considering another objective function about convenience for customers. Weber [19] proposed an optimal location problem without competitiveness, called "Weber problem", whose objective is to optimize the convenience for customers. References to the Weber problem were written by many researchers up to the present [2, 6]. Then, by introducing the objective function of the Weber problem to the CFLPs, the CFLPs are extended

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to multiobjective CFLPs with the two objective functions. From the point of view of the DM, the former objective generally takes precedence over the latter. The goal programming is one of the methods to deal with multiobjective programming problems under the assumption that the DM specifies goals or aspiration levels for the objective functions; for details of the goal programming, the reader can refer to the studies of Charnes and Cooper [3], Ignizio [12], and Sakawa [17]. We propose a solution algorithm for the multiobjective CFLP based upon the goal programming.

The remainder of this paper is organized as follows. In Section 2, we formulate multiobjective CFLP by introducing the Huff's attractive function and extending Drezner's CFLP. In order to solve the multiobjective CFLP, we propose the solution algorithm based upon the goal programming in Section 3. In Section 4, we illustrate the solution algorithm by numerical examples for the multiobjective CFLP. Finally, in Section 5, concluding comments and future studies are summarized.

2 Formulation of multiobjective CFLP In the following CFLPs, we assume that all customers only exist on demand points (DPs) in plain R^2. For convenience sake, by aggregating all customers on the same DP, we regard one DP as one customer.

First, we formulate the CFLP suggested by Uno and Sakawa [18]. There are n DPs on the plane R^2, and let $I = \{1, \dots, n\}$ be a set of indices of the DPs. In R^2, there exists a competitive facility, which is indicated by A, and the DM locates a new facility, which is indicated by B.

In this CFLP, the Huff's attractive function [10, 11] plays an important role. We first introduce the Huff's attractive function, which depends upon the distance between the customers and the facilities and the quality of the facilities. Let $\boldsymbol{v}_i \in R^2$ be the site of DP $i \in I$, and let $\boldsymbol{x}_A$ and $\boldsymbol{x}_B \in R^2$ be the sites of the competitive and the new facility, respectively. Then, the distances between DP i and the competitive and the new facility are denoted by $||\boldsymbol{x}_A - \boldsymbol{v}_i||$ and $||\boldsymbol{u}_B - \boldsymbol{v}_i||$, respectively, where $||\cdot||$ is the Euclid norm in R^2. On the other hand, let l_A and $l_B \in \{1, \dots, L\}$ be the level of qualities of the competitive and the new facility, respectively, where, L is the maximal quality level of these facilities. Then, we represent the quality of the facility whose level is l by using the function $q(l)$ which is satisfied that $1 = q(1) < \cdots < q(L) < \infty$. Then, for representing the attractive power of the facility whose site and level is $\boldsymbol{x}$ and l for DP i, the Huff's attractive function is represented as follows:

$$a_i(\boldsymbol{x}, l) := \begin{cases} \dfrac{q(l)}{||\boldsymbol{v}_i - \boldsymbol{x}||^2}, & \text{if } ||\boldsymbol{v}_i - \boldsymbol{x}|| > \varepsilon, \\ \dfrac{q(l)}{\varepsilon^2}, & \text{if } ||\boldsymbol{v}_i - \boldsymbol{x}|| \le \varepsilon, \end{cases} \tag{1}$$

where $\varepsilon > 0$ is an upper limit of the distance that customers can move without any trouble.

It is assumed that DP i uses the new facility if $a_i(\boldsymbol{x}_B, l_B) \ge a_i(\boldsymbol{x}_A, l_A) + \epsilon$, where ϵ is a sufficiently small positive number. Let $a_i^A := a_i(\boldsymbol{x}_A, l_A) + \epsilon$. Then, the set of the DPs using the new facility is represented as follows:

$$N_B(\boldsymbol{x}_B, l_B) := \{i \mid a_i(\boldsymbol{x}_B, l_B) \ge a_i^A\}. \tag{2}$$

Let $w_i > 0$ be the number of customers on DP i. It is assumed that sales of the new facility increase in proportion to the number of customers who use facilities. Let $\alpha > 0$ be the coefficient to represent the sales per a customer. An objective of the DM is maximizing the sales of the new facility without increasing the building cost of the new facility, denoted by

$c(l_B)$, which is satisfied that $0 < c(1) < \cdots < c(L) < \infty$. Then, we represent an objective function of the CFLP by using the following profit function:

$$f_1(\boldsymbol{x}_B, l_B) := \alpha \sum_{i \in N_B(\boldsymbol{x}_B, l_B)} w_i - c(l_B). \tag{3}$$

Then, the CFLP is formulated as the following profit maximizing problem:

$$\begin{array}{rl} \text{maximize} & f_1(\boldsymbol{x}_B, l_B) \\ \text{subject to} & \boldsymbol{x}_B \in R^2, \ l_B \in \{1, \ldots, L\}. \end{array} \tag{4}$$

Next, we introduce another objective function about the convenience for the customers, and then extend the CFLP (3) to a multiobjective CFLP. The objective of the original Weber problem [19] is represented as minimizing the sum of the distances between the customer and the facilities. On the other hand, it is desirable for the customers that the quality of the facilities is higher. Then, we represent the other objective function of the CFLP by using the following convenience function:

$$f_2(\boldsymbol{x}_B, l_B) := \frac{1}{q(l_B)} \sum_{i \in N_B(\boldsymbol{x}_B, l_B)} w_i ||\boldsymbol{v}_i - \boldsymbol{x}_B||. \tag{5}$$

Note that the DM only needs to consider the customers who use the new facility. Therefore, our proposing multiobjective CFLP is formulated as follows:

$$\begin{array}{rl} \text{maximize} & f_1(\boldsymbol{x}_B, l_B) \\ \text{minimize} & f_2(\boldsymbol{x}_B, l_B) \\ \text{subject to} & \boldsymbol{x}_B \in R^2, \ l_B \in \{1, \ldots, L\}. \end{array} \tag{6}$$

3 Solution algorithm for multiobjective CFLP

3.1 Goal Programming The term goal programming first appeared in a text by Charnes et al [3] to deal with multiobjective linear programming problems under the assumption that the DM specifies goals or aspiration levels for the objective functions. The key idea behind goal programming is to minimize the deviations from goals or aspiration levels set by the DM. For details of the goal programming, the reader can refer to the studies of Charnes and Cooper [3], Ignizio [12], and Sakawa [17]. In this section, we propose a solution algorithm for the multiobjective CFLP (6) based upon the goal programming.

At the goal programming, the DM first sets a goal for each objective, and next finds the solution to minimize the deviations from their goals. Let $\hat{f}_1$ and $\hat{f}_2$ be the goals of the first and second objective functions of (6), respectively. Then, we represent the deviations about the first and the second objective functions as follows, respectively:

$$d_1(\boldsymbol{x}_B, l_B) := \begin{cases} \hat{f}_1 - f_1(\boldsymbol{x}_B, l_B), & \text{if } f_1(\boldsymbol{x}_B, l_B) < \hat{f}_1, \\ 0, & \text{otherwise,} \end{cases} \tag{7}$$

$$d_2(\boldsymbol{x}_B, l_B) := \begin{cases} f_2(\boldsymbol{x}_B, l_B) - \hat{f}_2, & \text{if } f_2(\boldsymbol{x}_B) > \hat{f}_2, \\ 0, & \text{otherwise.} \end{cases} \tag{8}$$

Let λ_1 and λ_2 be the nonnegative weights to the first and the second objective function, respectively. Then, the goal programming problem for (6) is formulated as follows:

$$\begin{array}{rl} \text{minimize} & \lambda_1 d_1(\boldsymbol{x}_B, l_B) + \lambda_2 d_2(\boldsymbol{x}_B, l_B) \\ \text{subject to} & \boldsymbol{x}_B \in R^2, \ l_B \in \{1, \ldots, L\}. \end{array} \tag{9}$$

For the most multiobjective CFLPs, the DM regards the first objective value as more important than that of the second objective. The DM thus sets λ_1 and λ_2 such that $\lambda_1 >> \lambda_2$.

The process to solve the goal programming problem is divided into the following two stages. At the first stage, we try to achieve the goal about the first objective function. At the second stage, we try to satisfy the goal about the second objective function, keeping the attained goal about the first objective function. In the next two subsections, we propose the solution algorithm at each of the two stages.

3.2 Solution algorithm at the first stage At the first stage of the goal programming, we need to solve the following problem:

$$\begin{array}{rl} \text{minimize} & d_1(\boldsymbol{x}_B, l_B) \\ \text{subject to} & \boldsymbol{x}_B \in R^2,\ l_B \in \{1, \ldots, L\}. \end{array} \tag{10}$$

Since (10) is a nonlinear and nonconvex programming problem, it is difficult to find its optimal solution directly. Then we reformulate (10) to a combinational optimization problem.

For a subset of DPs $D \subseteq I$, let $(\boldsymbol{x}_B, l_B)$ be satisfied the following inequation:

$$a_i(\boldsymbol{x}_B, l_B) \geq a_i^A, \quad \forall i \in D. \tag{11}$$

Then, from (2), the new facility can capture customers on all DPs in D. In order to examine whether the location satisfied (11) exists or not, we solve the following problem for a given $\bar{l}_B \in \{1, \ldots, L\}$ with an auxiliary variable γ:

$$\begin{array}{rl} \text{minimize} & \gamma^2 \\ \text{subject to} & a_i(\boldsymbol{x}_B, \bar{l}_B) \geq \gamma \cdot a_i^A, \quad \forall i \in D, \\ & \boldsymbol{x}_B \in R^2, \gamma \geq 0. \end{array} \tag{12}$$

Let $(\gamma^{(D,\bar{l}_B)}, \boldsymbol{x}_B^{(D,\bar{l}_B)})$ be the optimal solution of (12). Then, $(\boldsymbol{x}_B^{(D,\bar{l}_B)}, \bar{l}_B)$ is satisfied (11) if $\gamma^{(D,\bar{l}_B)} \leq 1$.

We can find an optimal solution of (10) by examining whether the location satisfied (11) exists or not for all subsets in I and all levels in $\{1, \ldots, L\}$. Then (10) can be reformulated to the following combinational optimization problem:

$$\begin{array}{rl} \text{minimize} & d_1(\boldsymbol{x}_B^{(D,\bar{l}_B)}, \bar{l}_B) \\ \text{subject to} & \gamma^{(D,\bar{l}_B)} \leq 1, \\ & \bar{l}_B \in \{1, \ldots, L\},\ D \subseteq I. \end{array} \tag{13}$$

Since there are 2^n subsets of I, we would like to decrease the number of times of solving (12). Let $\boldsymbol{I}_3$ be the family of sets which have at most three DPs in I. Then, the following theorem is useful to decrease the number of the times.

Theorem 1 *An optimal solution of (13) can be found by solving (12) for all sets in* $\boldsymbol{I}_3$.

Proof: For any $D \in I$ *and* $\bar{l}_B \in \{1, \ldots, L\}$*, we prove the theorem by dividing the following three cases (i)-(iii) in (12).*

(i) Cases that one constraint is active for $(\boldsymbol{x}_B^{(D,\bar{l}_B)}, \bar{l}_B)$*: It only occurs that* D *is a singleton, which is included in* $\boldsymbol{I}_3$.

(ii) Cases that just two constraints are active for $(\boldsymbol{x}_B^{(D,\bar{l}_B)}, \bar{l}_B)$: Let i and j be the DPs in D whose constraints are active. Then, it follows that

$$\gamma^{(D,\bar{l}_B)} = \frac{a_i(\boldsymbol{x}_B^{(D,\bar{l}_B)}, \bar{l}_B)}{a_i^A} = \frac{a_j(\boldsymbol{x}_B^{(D,\bar{l}_B)}, \bar{l}_B)}{a_j^A}. \tag{14}$$

By transforming (14), $\boldsymbol{x}_B^{(D,\bar{l}_B)}$ satisfies the following equation:

$$\frac{||\boldsymbol{v}_i - \boldsymbol{x}_B^{(D,\bar{l}_B)}||}{||\boldsymbol{v}_j - \boldsymbol{x}_B^{(D,\bar{l}_B)}||} = \sqrt{\frac{a_j^A}{a_i^A}}. \tag{15}$$

Equation (15) represents a bisector between $\boldsymbol{v}_i$ and $\boldsymbol{v}_j$ if its right side is 1 and a circle otherwise. Moreover, from the optimality of (12), $\boldsymbol{x}_B^{(D,\bar{l}_B)}$ minimizes the sum of the distances to these two DPs. This means that $\boldsymbol{x}_B^{(D,\bar{l}_B)}$ is in the line segment between $\boldsymbol{v}_i$ and $\boldsymbol{v}_j$. From the line segment and equation (15), we can find $\boldsymbol{x}_B^{(D,\bar{l}_B)}$ such that

$$\boldsymbol{x}_B^{(D,\bar{l}_B)} = \frac{\sqrt{a_i^A}\boldsymbol{v}_i + \sqrt{a_j^A}\boldsymbol{v}_j}{\sqrt{a_i^A} + \sqrt{a_j^A}}. \tag{16}$$

Note that $\boldsymbol{x}_B^{(D,\bar{l}_B)}$ is only dependent on the sites of the competitive facility and these two DPs. This means that the solution of (12) for D is equivalent to that for $\{i, j\}$, which is included in $\boldsymbol{I}_3$.

(iii) Cases that more than three constraints are active for $(\boldsymbol{x}_B^{(D,\bar{l}_B)}, \bar{l}_B)$: Let i, j, and k be the DPs in D whose constraints are active. Then, it follows that

$$\gamma^{(D,\bar{l}_B)} = \frac{a_i(\boldsymbol{x}_B^{(D,\bar{l}_B)}, \bar{l}_B)}{a_i^A} = \frac{a_j(\boldsymbol{x}_B^{(D,\bar{l}_B)}, \bar{l}_B)}{a_j^A} = \frac{a_k(\boldsymbol{x}_B^{(D,\bar{l}_B)}, \bar{l}_B)}{a_k^A}. \tag{17}$$

By transforming (17), $\boldsymbol{x}_B^{(D,\bar{l}_B)}$ satisfies the following two equations:

$$\frac{||\boldsymbol{v}_i - \boldsymbol{x}_B^{(D,\bar{l}_B)}||}{||\boldsymbol{v}_j - \boldsymbol{x}_B^{(D,\bar{l}_B)}||} = \sqrt{\frac{a_j^A}{a_i^A}}, \quad \frac{||\boldsymbol{v}_j - \boldsymbol{x}_B^{(D,\bar{l}_B)}||}{||\boldsymbol{v}_k - \boldsymbol{x}_B^{(D,\bar{l}_B)}||} = \sqrt{\frac{a_k^A}{a_j^A}}. \tag{18}$$

As (15), each of the equations in (18) represents a bisector or a circle. Then, there are at most two intersection points for the two equations, and $\boldsymbol{x}_B^{(D,\bar{l}_B)}$ is uniquely decided because $\gamma^{(D,\bar{l}_B)}$ is the minimum of the objective value of (12) for D. Note that $\boldsymbol{x}_B^{(D,\bar{l}_B)}$ is only dependent on the sites of the competitive facility and these three DPs. This means that the solution of (12) for D is equivalent to that for $\{i, j, k\}$, which is included in $\boldsymbol{I}_3$.

Therefore, the set of solutions of (12) for all subsets in I is equivalent to that for all subsets in $\boldsymbol{I}_3$. This means that an optimal solution of (13) can be found by solving (12) for all sets in $\boldsymbol{I}_3$.

Moreover, from the proof of Theorem 1, the following useful corollary is obtained.

Corollary 2 *The site $\boldsymbol{x}_B^{(D,\bar{l}_B)}$ is independent of $\bar{l}_B$.*
Proof: For Case (i), let $D := \{i\}$. Then, one of the optimal solutions of problem (12) for D is obviously $(\boldsymbol{v}_i, 0)$, which is independent of $\bar{l}_B$. For Case (ii) and (iii), (16) and (18) do not include $\bar{l}_B$. Therefore, $\boldsymbol{x}_B^{(D,\bar{l}_B)}$ is independent of $\bar{l}_B$.

Let (D^*, l_B^*) be the optimal solution of (13). From Corollary 2, $\boldsymbol{x}_B^{(D^*,l_B^*+1)} = \boldsymbol{x}_B^{(D^*,l_B^*)}$ and $\gamma^{(D^*,l_B^*+1)} \leq \gamma^{(D^*,l_B^*)} \leq 1$. Because of the building cost, $d_1(\boldsymbol{x}_B^{(D^*,l_B^*)}, l_B^*) \leq d_1(\boldsymbol{x}_B^{(D^*,l_B^*+1)}, l_B^*+1)$. Moreover, from the optimality of (D^*, l_B^*), $\gamma^{(D^*,l_B^*-1)} > 1$. Then, l_B^* is the minimum of the level satisfied $\gamma^{(D^*,l_B^*)} \leq 1$.

Let l_B^D be the minimum of the level satisfied $\gamma^{(D,l_B^D)} \leq 1$, where we set $l_B^D = L$ if there is no level satisfied $\gamma^{(D,l_B^D)} \leq 1$. From Corollary 2, we denote $\boldsymbol{x}_B^{(D,\bar{l}_B)}$ by $\boldsymbol{x}_B^D$ simply. Then, from Theorem 1 and Corollary 2, (13) can be reduced to the following combinational optimization problem:

$$\begin{array}{rl} \text{minimize} & d_1(\boldsymbol{x}_B^D, l_B^D) \\ \text{subject to} & D \in \boldsymbol{I}_3. \end{array} \tag{19}$$

We can find an optimal solution of (19) by solving (12) at $(n +_n C_2 +_n C_3)$ times.

3.3 Solving method at the second stage At the second stage of the goal programming, we need to solve the following problem:

$$\begin{array}{rl} \text{minimize} & d_2(\boldsymbol{x}_B, l_B) \\ \text{subject to} & d_1(\boldsymbol{x}_B, l_B) \leq \beta \\ & \boldsymbol{x}_B \in R^2,\ l_B \in \{1, \ldots, L\}, \end{array} \tag{20}$$

where β is an aspiration level for the first objective.

The set of the solutions which are given at the first stage and satisfied the aspiration level is denoted by follows:

$$\bar{X}^* := \{(\boldsymbol{x}_B^D, l_B^D) \mid D \in \boldsymbol{I}_3,\ d_1(\boldsymbol{x}_B^D, l_B^D) \leq \beta\}. \tag{21}$$

At the second stage, we move the new facility from each location in X^* so as to maximize the second objective value, with keeping the first objective value. Then, (20) can be divided into the problems for all solutions in X^*. The problem for solution $(\boldsymbol{x}_B^D, l_B^D) \in X^*$ is formulated as follows:

$$\begin{array}{rl} \text{minimize} & d_2(\boldsymbol{x}_B, l_B^D) \\ \text{subject to} & a_i(\boldsymbol{x}_B, l_B^D) \geq a_i^A, \quad \forall i \in D, \\ & \boldsymbol{x}_B \in R^2. \end{array} \tag{22}$$

Note that the first objective value is equal or more than $d_2(\boldsymbol{x}_B^D, l_B^D)$ if the constraints of (22) is satisfied.

Next we consider the method for solving (22). The first constraints of (22) is translated as follows:

$$\max\{||\boldsymbol{x}_B - \boldsymbol{v}_i||, \varepsilon\} \leq \sqrt{\frac{q(l_B^D)}{a_i^A}}, \quad \forall i \in D. \tag{23}$$

Because the right side of (23) is constant, the constraint of (22) is convex. Moreover, from (5) and (8), the objective function of (22) is also convex. Therefore, (22) is a convex programming problem. We can solve (22) by using the solution algorithms for convex programming problems, such as successive quadratic programming (SQP) method; for details of the SQP method, the reader can refer to the book of Nocedal and Wright [14].

Table 1: Distribution of customers

i	$\boldsymbol{v}_i$	w_i
1	(0.00, 2.00)	300
2	(2.00, 5.00)	300
3	(4.00, 0.00)	200
4	(5.00, 3.00)	400
5	(8.00, 5.00)	100
6	(9.00, 2.00)	200

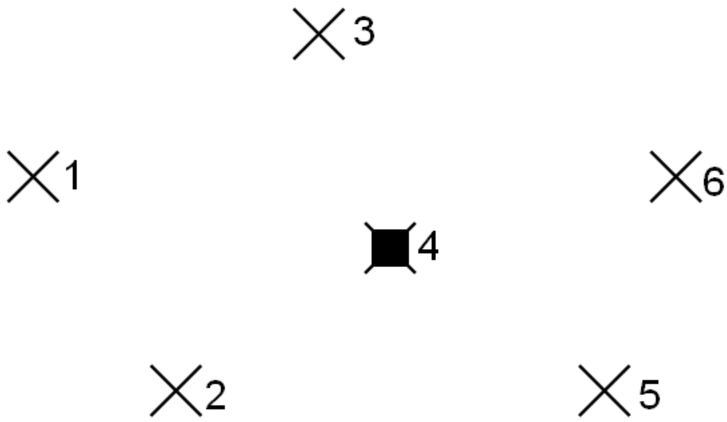

Figure 1: Sites of the DPs and the competitive facility

4 Numerical Example In this section, we illustrate the solution algorithm described in Section 3 by applying it to a numerical example of the CFLP. In the example, the numbers of customers on DPs and their sites are given in Table 1, and one competitive facility has already been located on $\boldsymbol{v}_4$ and its level $l_A = 2$. Then, the sites of the six DPs and the competitive facility is shown in Fig. 1.

In the plane shown in Fig. 1, the DM can locate one new facility whose quality level is high or low, that is $L = 2$. The quality and cost of building for each level are given in Table 2. For (1) and the criterion of customers, we set $\varepsilon = 10^{-4}$ and $\epsilon = 10^{-4}$.

Table 2: Quality and cost of building

l	$q(l)$	$c(l)$
1	1.00	100
2	2.00	200

For the above example, we solve the multiobjective CFLP (6). For (9) in Section 3.1, the DM sets her/his goals such that $\hat{f}_1 = 1000$ and $\hat{f}_2 = 0$.

At the first stage of the goal programming, we solve (19) for finding an optimal solution of (10). If $\alpha \geq 1/2$, then the optimal solution of (19) $(D^*, l_B^*) = (\{2,3\}, 2)$, and $\boldsymbol{x}_B^{D^*} =$

$(3.07, 2.66)$ in the line segment between $\boldsymbol{v}_2$ and $\boldsymbol{v}_3$. Then the new facility can capture the customers on DPs 1 to 3, and its objective value is $800\alpha - 200$. If $1/6 < \alpha \leq 1/2$, then the optimal solution of (19) $(D^*, l_B^*) = (\{1,2\}, 1)$, and $\boldsymbol{x}_B^{D^*} = (1.17, 3.76)$ in the line segment between $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$. Then the new facility can capture the customers on DPs 1 and 2, and its objective value is $600\alpha - 100$. If $\alpha \leq 1/6$, the DM does not locate her/his facility because the objective value is equal or less than 0 for any location. Fig. 2 shows the sites of the new facility at the first stage of the goal programming.

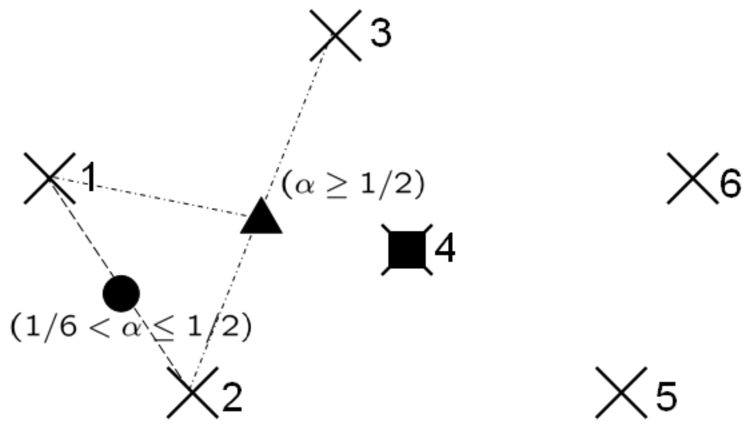

Figure 2: Sites of the new facility at the first stage of the goal programming

At the second stage of the goal programming, we solve (20). If $\alpha \geq 1/2$, the optimal solution of (20) is $\boldsymbol{x}_B^* = (1.72, 2.19)$, and its objective value is 9.99×10^2, which is improved from the solution given at the first stage whose objective value is 1.15×10^3. If $1/6 < \alpha < 1/2$, the optimal solution of (20) is $\boldsymbol{x}_B^* = (1.17, 3.76)$. Moreover, points in the line segment between v_1 and v_2 are optimal sites for (20) because $w_1 = w_2$. Then its objective value is 1.08×10^3. Fig. 3 shows the sites of the new facility at the second stage of the goal programming.

From the result in cases that $\alpha \geq 1/2$, the DM can improve the convenience for customers without decreasing her/his profit. Then, the solution algorithm can find a good location from the view of points of both the profit and the convenience for customers.

5 Conclusions and future studies In this paper, we have extended a CFLP with the single objective function to a multiobjective CFLP by introducing an objective about the convenience for customers. For the formulated multiobjective CFLP, we have proposed the solution algorithm based upon the goal programming. We have illustrated the solution algorithm by applying it to numerical examples of the CFLP. Then, it is shown that the solution algorithm can find a good location from the view of points of both the profit and the convenience for customers.

This multiobjective CFLP deals with the location of one facility. If there are many facilities to locate simultaneously, it is expected that finding a strict optimal solution of the multiobjective CFLP is difficult. It is a future study to propose an efficient solution algorithm based upon heuristic algorithms, like the genetic algorithm [7] and the tabu search [16]. Moreover, in the real CFLPs, customers subjectively decide the demand and the convenience of the facilities. This means that the data of the demand and the convenience

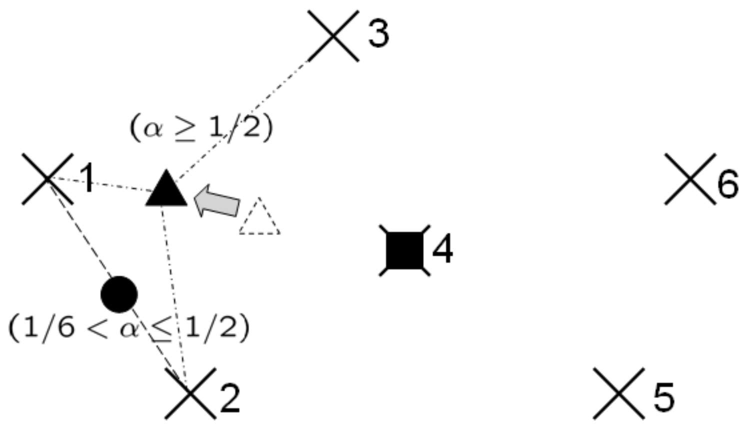

Figure 3: Sites of the new facility at the second stage of the goal programming

include vagueness. In another future study, we are going to introduce the fuzzy theory [1] to the multiobjective CFLPs and extend it to a fuzzy multiobjective CFLP.

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