

POSITIVITY OF OPERATORS DERIVED FROM OPERATOR MONOTONE FUNCTIONS

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ABSTRACT. We pose an extension of the arithmetic-geometric mean inequality: For invertible positive operators A and B , if $AB^{-1} + BA^{-1}$ is selfadjoint, then $AB^{-1} + BA^{-1} \geq 2I$. As an application, we show that if $A(B + s)^{-1} + B(A + s)^{-1}$ is selfadjoint for all $s \geq 0$, then $(B - A)\{f(B) - f(A)\} \geq 0$ holds for all operator monotone function f on $(0, \infty)$. Next we prove that for $A, B \geq 0$, $AB + BA \geq 0$ if and only if $f(A) + f(B) \geq f(A + B) + f(0)I$ holds for operator monotone functions f on $[0, \infty)$. It is an extension of a theorem due to Moslehian and Najafi.