

# FUZZY BCC-SUBALGEBRAS OF BCC-ALGEBRAS WITH RESPECT TO A $t$ -NORM

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ABSTRACT. The notion of  $T$ -fuzzy BCC-subalgebras is introduced, and then some related results are obtained. Using a  $t$ -norm  $T$ , the direct product and  $T$ -product of  $T$ -fuzzy subalgebras are discussed, and their properties are investigated.

## 1. Introduction

A BCK-algebra is an important class of logical algebras introduced by K. Iséki and was extensively investigated by several researchers. The class of all BCK-algebras is a quasivariety. K. Iséki posed an interesting problem (solved by A. Wroński [20]) whether the class of BCK-algebras is a variety. In connection with this problem, Y. Komori ([18]) introduced a notion of BCC-algebras, and W. A. Dudek ([2, 3]) redefined the notion of BCC-algebras by using a dual form of the ordinary definition in the sense of Y. Komori. L. A. Zadeh [23] introduced the notion of fuzzy sets. At present this concept has been applied to many mathematical branches, such as group, functional analysis, probability theory, topology, and so on. In 1991, O. G. Xi [21] applied this concept to BCK-algebras, and he introduced the notion of fuzzy subalgebras(ideals) of the BCK-algebras with respect to minimum, and since then Y. B. Jun et al. studied fuzzy subalgebras and fuzzy ideals (see [11, 15, 16]), and moreover several fuzzy structures in BCC-algebras are considered (see [4, 5, 6, 7, 8]). In the present paper, we will redefine the fuzzy BCC-subalgebra of the BCC-algebras with respect to a  $t$ -norm  $T$  and hence generalize the notion in [4], and obtain some related results. we consider the direct product and  $t$ -normed product of fuzzy BCC-subalgebras of BCC-algebras with respect to a  $t$ -norm.

## 2. Preliminaries

By a *BCK-algebra* we mean an algebra  $(G, *, 0)$  of type  $(2, 0)$  satisfying the following conditions:

- (I)  $((x * y) * (x * z)) * (z * y) = 0$ ,
- (II)  $(x * (x * y)) * y = 0$ ,
- (III)  $x * x = 0$ ,
- (IV)  $0 * x = 0$ ,
- (V)  $x * y = 0$  and  $y * x = 0$  imply  $x = y$ ,

for all  $x, y, z \in G$ . We can define a partial ordering " $\leq$ " on  $G$  by  $x \leq y$  if and only if  $x * y = 0$ .

In any BCK-algebra  $G$ , the following hold:

- (P1)  $x * 0 = x$ ,
- (P2)  $x * y \leq x$ ,

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$$(P3) \quad (x * y) * z = (x * z) * y,$$

$$(P4) \quad (x * z) * (y * z) \leq x * y,$$

$$(P5) \quad x \leq y \text{ implies } x * z \leq y * z \text{ and } z * y \leq z * x.$$

By a *BCC-algebra* we mean a non-empty set  $G$  with a constant  $0$  and a binary operation  $*$  satisfying the following conditions:

$$(VI) \quad ((x * y) * (z * y)) * (x * z) = 0,$$

$$(III) \quad x * x = 0,$$

$$(IV) \quad 0 * x = 0,$$

$$(P1) \quad x * 0 = x,$$

$$(V) \quad x * y = 0 \text{ and } y * x = 0 \text{ imply } x = y.$$

for all  $x, y, z \in G$ . Any BCK-algebra is a BCC-algebra, but there are BCC-algebras which are not BCK-algebras. Note that a BCC-algebra is a BCK-algebra if and only if it satisfies:

$$(P3) \quad (x * y) * z = (x * z) * y.$$

On any BCC-algebra (similarly as in the case of BCK-algebras) one can define the natural ordering “ $\leq$ ” by putting

$$(1) \quad x \leq y \iff x * y = 0.$$

It is not difficult to verify that this order is partial and  $0$  is its smallest element. Moreover, in any BCC-algebra  $G$ , the following are true:

$$(2) \quad (x * y) * (z * y) \leq x * z,$$

$$(P5) \quad x \leq y \text{ implies } x * z \leq y * z \text{ and } z * y \leq z * x.$$

A non-empty subset  $S$  of a BCC-algebra  $G$  is called a *BCC-subalgebra* of  $G$  if  $x * y \in S$  for all  $x, y \in S$ .

A *fuzzy set* in a set  $G$  is a function  $\mu : G \rightarrow [0, 1]$ . Let  $\mu$  be a fuzzy set in a set  $G$ . For  $\alpha \in [0, 1]$ , the set  $U(\mu; \alpha) := \{x \in G \mid \mu(x) \geq \alpha\}$  is called a *level subset* of  $\mu$ . A fuzzy set  $\mu$  in a BCC-algebra  $G$  is called a *fuzzy BCC-subalgebra* of  $G$  if it satisfies the inequality:

$$\mu(x * y) \geq \min\{\mu(x), \mu(y)\}$$

for all  $x, y \in G$ .

### 3. $T$ -fuzzy BCC-subalgebras

**Definition 3.1** ([1]). By a  *$t$ -norm*  $T$ , we mean a function  $T : [0, 1] \times [0, 1] \rightarrow [0, 1]$  satisfying the following conditions:

$$(T1) \quad T(x, 1) = x,$$

$$(T2) \quad T(x, y) \leq T(x, z) \text{ if } y \leq z,$$

$$(T3) \quad T(x, y) = T(y, x),$$

$$(T4) \quad T(x, T(y, z)) = T(T(x, y), z),$$

for all  $x, y, z \in [0, 1]$ .

Every  $t$ -norm  $T$  has a useful property:

$$T(\alpha, \beta) \leq \min\{\alpha, \beta\}$$

for all  $\alpha, \beta \in [0, 1]$ .

**Definition 3.2.** A function  $\mu : G \rightarrow [0, 1]$  is called a *fuzzy BCC-subalgebra* of a BCC-algebra  $G$  with respect to a  $t$ -norm  $T$  (briefly, a  *$T$ -fuzzy BCC-subalgebra* of  $G$ ) if  $\mu(x * y) \geq T(\mu(x), \mu(y))$  for all  $x, y \in G$ .

**Example 3.3.** Consider a proper BCC-algebra  $G = \{0, 1, 2, 3, 4\}$  with the Cayley table as follows:

$*$	0	1	2	3	4
0	0	0	0	0	0
1	1	0	1	0	0
2	2	2	0	0	0
3	3	3	1	0	0
4	4	3	4	3	0

Define a fuzzy set  $\mu$  in  $G$  by

$$\mu(x) := \begin{cases} 1 & \text{if } x \in \{0, 1, 2\}, \\ 0 & \text{otherwise,} \end{cases}$$

and let  $T_m : [0, 1] \times [0, 1] \rightarrow [0, 1]$  be a function defined by

$$T_m(\alpha, \beta) = \max\{\alpha + \beta - 1, 0\}$$

for all  $\alpha, \beta \in [0, 1]$ . Then  $T_m$  is a  $t$ -norm ([22]). It is easy to check that  $\mu$  satisfies the inequality:

$$\mu(x * y) \geq T_m(\mu(x), \mu(y))$$

for all  $x, y \in G$ . Hence  $\mu$  is a  $T_m$ -fuzzy BCC-subalgebra of  $G$ .

**Theorem 3.4.** Let  $\mu$  be a  $T$ -fuzzy BCC-subalgebra of a BCC-algebra  $G$  and let  $\alpha \in [0, 1]$ . Then

- (i) if  $\alpha = 1$  then  $U(\mu; \alpha)$  is either empty or a BCC-subalgebra of  $G$ .
- (ii) if  $T = \min$ , then  $U(\mu; \alpha)$  is either empty or a BCC-subalgebra of  $G$ , and moreover  $\mu(0) \geq \mu(x)$  for all  $x \in G$ .

*Proof.* (i) Assume that  $\alpha = 1$  and let  $x, y \in U(\mu; \alpha)$ . Then  $\mu(x) \geq \alpha = 1$  and  $\mu(y) \geq \alpha = 1$ . It follows from Definitions 3.1 and 3.2 that

$$\mu(x * y) \geq T(\mu(x), \mu(y)) \geq T(1, 1) = 1$$

so that  $x * y \in U(\mu; 1)$ . Hence  $U(\mu; \alpha)$  is a BCC-subalgebra of  $G$ .

(ii) Assume that  $T = \min$  and let  $x, y \in U(\mu; \alpha)$ . Then

$$\mu(x * y) \geq T(\mu(x), \mu(y)) = \min(\mu(x), \mu(y)) \geq \min(\alpha, \alpha) = \alpha$$

for all  $\alpha \in [0, 1]$ , and so  $x * y \in U(\mu; \alpha)$ . Thus  $U(\mu; \alpha)$  is a BCC-subalgebra of  $G$ . Moreover, since  $x * x = 0$  for all  $x \in G$ , we have

$$\mu(0) = \mu(x * x) \geq T(\mu(x), \mu(x)) = \min(\mu(x), \mu(x)) = \mu(x).$$

This completes the proof.  $\square$

**Theorem 3.5.** Let  $\mu$  be a  $T$ -fuzzy BCC-subalgebra of a BCC-algebra  $G$ . If there is a sequence  $\{x_n\}$  in  $G$  such that  $\lim_{n \rightarrow \infty} T(\mu(x_n), \mu(x_n)) = 1$ , then  $\mu(0) = 1$ .

*Proof.* Let  $x \in G$ . Then  $\mu(0) = \mu(x * x) \geq T(\mu(x), \mu(x))$ . Therefore  $\mu(0) \geq T(\mu(x_n), \mu(x_n))$  for each  $n \in \mathbf{N}$ . Since  $1 \geq \mu(0) \geq \lim_{n \rightarrow \infty} T(\mu(x_n), \mu(x_n)) = 1$ , it follows that  $\mu(0) = 1$ , ending the proof.  $\square$

If  $\mu$  is a fuzzy set in a BCC-algebra  $G$  and  $\theta$  is a mapping from  $G$  into itself, we define a mapping  $\mu[\theta] : G \rightarrow [0, 1]$  by  $\mu[\theta](x) = \mu(\theta(x))$  for all  $x \in G$ .

**Proposition 3.6.** *If  $\mu$  is a  $T$ -fuzzy BCC-subalgebra of a BCC-algebra  $G$  and  $\theta$  is an endomorphism of  $G$ , then  $\mu[\theta]$  is a  $T$ -fuzzy BCC-subalgebra of  $G$ .*

*Proof.* For any  $x, y \in G$ , we have

$$\begin{aligned}\mu[\theta](x * y) &= \mu(\theta(x * y)) = \mu(\theta(x) * \theta(y)) \\ &\geq T(\mu(\theta(x)), \mu(\theta(y))) = T(\mu[\theta](x), \mu[\theta](y)).\end{aligned}$$

Hence  $\mu[\theta]$  is a  $T$ -fuzzy BCC-subalgebra of  $G$ .  $\square$

Let  $f$  be a mapping defined on a BCC-algebra  $G$ . If  $\nu$  is a fuzzy set in  $f(G)$  then the fuzzy set  $\mu = \nu \circ f$  in  $G$  (i.e., the fuzzy set defined by  $\mu(x) = \nu(f(x))$  for all  $x \in G$ ) is called the *preimage* of  $\nu$  under  $f$ .

**Theorem 3.7.** *An onto homomorphic preimage of a  $T$ -fuzzy BCC-subalgebra is a  $T$ -fuzzy BCC-subalgebra.*

*Proof.* Let  $f : G \rightarrow G'$  be an onto homomorphism of BCC-algebras,  $\nu$  a  $T$ -fuzzy BCC-subalgebra of  $G'$ , and  $\mu$  the preimage of  $\nu$  under  $f$ . Then

$$\begin{aligned}\mu(x * y) &= \nu(f(x * y)) = \nu(f(x) * f(y)) \\ &\geq T(\nu(f(x)), \nu(f(y))) = T(\mu(x), \mu(y))\end{aligned}$$

for all  $x, y \in G$ . Hence  $\mu$  is a  $T$ -fuzzy BCC-subalgebra of  $G$ .  $\square$

If  $\mu$  is a fuzzy set in a BCC-algebra  $G$  and  $f$  is a mapping defined on  $G$ . The fuzzy set  $\mu^f$  in  $f(G)$  defined by  $\mu^f(y) = \sup_{x \in f^{-1}(y)} \mu(x)$  for all  $y \in f(G)$  is called the *image* of  $\mu$  under  $f$ . A fuzzy set  $\mu$  in  $G$  is said to have *sup property* if, for every subset  $T \subseteq G$ , there exists  $t_0 \in T$  such that  $\mu(t_0) = \sup_{t \in T} \mu(t)$ .

**Proposition 3.8.** *An onto homomorphic image of a fuzzy BCC-subalgebra with sup property is a fuzzy BCC-subalgebra.*

*Proof.* Let  $f : G \rightarrow G'$  be an onto homomorphism of BCC-algebras and let  $\mu$  be a fuzzy BCC-subalgebra of  $G$  with sup property. Given  $x', y' \in G'$ , let  $x_0 \in f^{-1}(x')$  and  $y_0 \in f^{-1}(y')$  such that  $\mu(x_0) = \sup_{t \in f^{-1}(x')} \mu(t)$  and  $\mu(y_0) = \sup_{t \in f^{-1}(y')} \mu(t)$ , respectively. Then

$$\begin{aligned}\mu^f(x' * y') &= \sup_{z \in f^{-1}(x' * y')} \mu(z) \\ &\geq \min\{\mu(x_0), \mu(y_0)\} \\ &= \min\left\{\sup_{t \in f^{-1}(x')} \mu(t), \sup_{t \in f^{-1}(y')} \mu(t)\right\} \\ &= \min\{\mu^f(x'), \mu^f(y')\}.\end{aligned}$$

Hence  $\mu^f$  is a fuzzy BCC-subalgebra of  $G'$ .  $\square$

Proposition 3.8 can be strengthened in the following way. To do this we need the following definition.

**Definition 3.9** ([22]). A  $t$ -norm  $T$  on  $[0, 1]$  is called a *continuous  $t$ -norm* if  $T$  is a continuous function from  $[0, 1] \times [0, 1]$  to  $[0, 1]$  with respect to the usual topology.

Note that the function “min” is a continuous  $t$ -norm.

**Theorem 3.10.** *Let  $T$  be a continuous  $t$ -norm and let  $f$  be a homomorphism on a BCC-algebra  $G$ . If  $\mu$  is a  $T$ -fuzzy BCC-subalgebra of  $G$ , then  $\mu^f$  is a  $T$ -fuzzy BCC-subalgebra of  $f(G)$ .*

*Proof.* Let  $A_1 = f^{-1}(y_1)$ ,  $A_2 = f^{-1}(y_2)$  and  $A_{12} = f^{-1}(y_1 * y_2)$ , where  $y_1, y_2 \in f(G)$ . Consider the set

$$A_1 * A_2 := \{x \in G \mid x = a_1 * a_2 \text{ for some } a_1 \in A_1 \text{ and } a_2 \in A_2\}.$$

If  $x \in A_1 * A_2$ , then  $x = x_1 * x_2$  for some  $x_1 \in A_1$  and  $x_2 \in A_2$  and so

$$f(x) = f(x_1 * x_2) = f(x_1) * f(x_2) = y_1 * y_2,$$

i.e.,  $x \in f^{-1}(y_1 * y_2) = A_{12}$ . Thus  $A_1 * A_2 \subseteq A_{12}$ . It follows that

$$\begin{aligned} \mu^f(y_1 * y_2) &= \sup_{x \in f^{-1}(y_1 * y_2)} \mu(x) = \sup_{x \in A_{12}} \mu(x) \\ &\geq \sup_{x \in A_1 * A_2} \mu(x) \geq \sup_{x_1 \in A_1, x_2 \in A_2} \mu(x_1 * x_2) \\ &\geq \sup_{x_1 \in A_1, x_2 \in A_2} T(\mu(x_1), \mu(x_2)). \end{aligned}$$

Since  $T$  is continuous, for every  $\varepsilon > 0$  there exists a number  $\delta > 0$  such that if  $\sup_{x_1 \in A_1} \mu(x_1) - x_1^* \leq \delta$  and  $\sup_{x_2 \in A_2} \mu(x_2) - x_2^* \leq \delta$  then

$$T(\sup_{x_1 \in A_1} \mu(x_1), \sup_{x_2 \in A_2} \mu(x_2)) - T(x_1^*, x_2^*) \leq \varepsilon.$$

Choose  $a_1 \in A_1$  and  $a_2 \in A_2$  such that  $\sup_{x_1 \in A_1} \mu(x_1) - \mu(a_1) \leq \delta$  and  $\sup_{x_2 \in A_2} \mu(x_2) - \mu(a_2) \leq \delta$ .

Then

$$T(\sup_{x_1 \in A_1} \mu(x_1), \sup_{x_2 \in A_2} \mu(x_2)) - T(\mu(a_1), \mu(a_2)) \leq \varepsilon.$$

Consequently

$$\begin{aligned} \mu^f(y_1 * y_2) &\geq \sup_{x_1 \in A_1, x_2 \in A_2} T(\mu(x_1), \mu(x_2)) \\ &\geq T(\sup_{x_1 \in A_1} \mu(x_1), \sup_{x_2 \in A_2} \mu(x_2)) \\ &= T(\mu^f(y_1), \mu^f(y_2)), \end{aligned}$$

which shows that  $\mu^f$  is a  $T$ -fuzzy BCC-subalgebra of  $f(G)$ .  $\square$

**Lemma 3.11** ([1]). *Let  $T$  be a  $t$ -norm. Then*

$$T(T(\alpha, \beta), T(\gamma, \delta)) = T(T(\alpha, \gamma), T(\beta, \delta))$$

for all  $\alpha, \beta, \gamma, \delta \in [0, 1]$ .

**Theorem 3.12.** *Let  $T$  be a  $t$ -norm and let  $G = G_1 \times G_2$  be the direct product BCC-algebra of BCC-algebras  $G_1$  and  $G_2$ . If  $\mu_1$  (resp.  $\mu_2$ ) is a  $T$ -fuzzy BCC-subalgebra of  $G_1$  (resp.  $G_2$ ), then  $\mu = \mu_1 \times \mu_2$  is a  $T$ -fuzzy BCC-subalgebra of  $G$  defined by*

$$\mu(x_1, x_2) = (\mu_1 \times \mu_2)(x_1, x_2) = T(\mu_1(x_1), \mu_2(x_2))$$

for all  $(x_1, x_2) \in G_1 \times G_2$ .

*Proof.* Let  $x = (x_1, x_2)$  and  $y = (y_1, y_2)$  be any elements of  $G = G_1 \times G_2$ . Then

$$\begin{aligned} \mu(x * y) &= \mu((x_1, x_2) * (y_1, y_2)) = \mu(x_1 * y_1, x_2 * y_2) \\ &= T(\mu_1(x_1 * y_1), \mu_2(x_2 * y_2)) \\ &\geq T(T(\mu_1(x_1), \mu_1(y_1)), T(\mu_2(x_2), \mu_2(y_2))) \\ &= T(T(\mu_1(x_1), \mu_2(x_2)), T(\mu_1(y_1), \mu_2(y_2))) \\ &= T(\mu(x_1, x_2), \mu(x_2, y_2)) \\ &= T(\mu(x), \mu(y)). \end{aligned}$$

Hence  $\mu$  is a  $T$ -fuzzy BCC-subalgebra of  $G$ .  $\square$

We will generalize the idea to the product of  $n$   $T$ -fuzzy BCC-subalgebras. We first need to generalize the domain of  $t$ -norm  $T$  to  $\prod_{i=1}^n [0, 1]$  as follows:

**Definition 3.13** ([1]). The function  $T_n : \prod_{i=1}^n [0, 1] \rightarrow [0, 1]$  is defined by

$$T_n(\alpha_1, \alpha_2, \dots, \alpha_n) = T(\alpha_i, T_{n-1}(\alpha_1, \dots, \alpha_{i-1}, \alpha_{i+1}, \dots, \alpha_n))$$

for all  $1 \leq i \leq n$ , where  $n \geq 2$ ,  $T_2 = T$  and  $T_1 = \text{id}$  (identity).

**Lemma 3.14** ([1]). For a  $t$ -norm  $T$  and every  $\alpha_i, \beta_i \in [0, 1]$  where  $1 \leq i \leq n$  and  $n \geq 2$ , we have

$$\begin{aligned} T_n(T(\alpha_1, \beta_1), T(\alpha_2, \beta_2), \dots, T(\alpha_n, \beta_n)) \\ = T(T_n(\alpha_1, \alpha_2, \dots, \alpha_n), T_n(\beta_1, \beta_2, \dots, \beta_n)). \end{aligned}$$

**Theorem 3.15.** *Let  $T$  be a  $t$ -norm and let  $\{G_i\}_{i=1}^n$  be the finite collection of BCC-algebras and  $G = \prod_{i=1}^n G_i$  the direct product BCC-algebra of  $\{G_i\}$ . Let  $\mu_i$  be a  $T$ -fuzzy BCC-subalgebra of  $G_i$ , where  $1 \leq i \leq n$ . Then  $\mu = \prod_{i=1}^n \mu_i$  defined by*

$$\begin{aligned} \mu(x_1, x_2, \dots, x_n) &= \left( \prod_{i=1}^n \mu_i \right)(x_1, x_2, \dots, x_n) \\ &= T_n(\mu_1(x_1), \mu_2(x_2), \dots, \mu_n(x_n)) \end{aligned}$$

is a  $T$ -fuzzy BCC-subalgebra of the BCC-algebra  $G$ .

*Proof.* Let  $x = (x_1, x_2, \dots, x_n)$  and  $y = (y_1, y_2, \dots, y_n)$  be any elements of  $G = \prod_{i=1}^n G_i$ . Then

$$\begin{aligned}
 \mu(x * y) &= \mu(x_1 * y_1, x_2 * y_2, \dots, x_n * y_n) \\
 &= T_n(\mu_1(x_1 * y_1), \mu_2(x_2 * y_2), \dots, \mu_n(x_n * y_n)) \\
 &\geq T_n(T(\mu_1(x_1), \mu_1(y_1)), T(\mu_2(x_2), \mu_2(y_2)), \dots, T(\mu_n(x_n), \mu_n(y_n))) \\
 &= T(T_n(\mu_1(x_1), \mu_2(x_2), \dots, \mu_n(x_n)), T_n(\mu_1(y_1), \mu_2(y_2), \dots, \mu_n(y_n))) \\
 &= T(\mu(x_1, x_2, \dots, x_n), \mu(y_1, y_2, \dots, y_n)) \\
 &= T(\mu(x), \mu(y)).
 \end{aligned}$$

Hence  $\mu$  is a  $T$ -fuzzy BCC-subalgebra of  $G$ .  $\square$

**Definition 3.16.** Let  $T$  be a  $t$ -norm and let  $\mu$  and  $\nu$  be fuzzy sets in a BCC-algebra  $G$ . Then the  $T$ -product of  $\mu$  and  $\nu$ , written  $[\mu \cdot \nu]_T$ , is defined by  $[\mu \cdot \nu]_T(x) = T(\mu(x), \nu(x))$  for all  $x \in G$ .

**Theorem 3.17.** Let  $T$  be a  $t$ -norm and let  $\mu$  and  $\nu$  be  $T$ -fuzzy BCC-subalgebras of a BCC-algebra  $G$ . If  $T^*$  is a  $t$ -norm which dominates  $T$ , i.e.,

$$T^*(T(\alpha, \beta), T(\gamma, \delta)) \geq T(T^*(\alpha, \gamma), T^*(\beta, \delta))$$

for all  $\alpha, \beta, \gamma, \delta \in [0, 1]$ , then the  $T^*$ -product of  $\mu$  and  $\nu$ ,  $[\mu \cdot \nu]_{T^*}$ , is a  $T$ -fuzzy BCC-subalgebra of  $G$ .

*Proof.* For any  $x, y \in G$  we have

$$\begin{aligned}
 [\mu \cdot \nu]_{T^*}(x * y) &= T^*(\mu(x * y), \nu(x * y)) \\
 &\geq T^*(T(\mu(x), \mu(y)), T(\nu(x), \nu(y))) \\
 &\geq T(T^*(\mu(x), \nu(x)), T^*(\mu(y), \nu(y))) \\
 &= T([\mu \cdot \nu]_{T^*}(x), [\mu \cdot \nu]_{T^*}(y)).
 \end{aligned}$$

Hence  $[\mu \cdot \nu]_{T^*}$  is a  $T$ -fuzzy BCC-subalgebra of  $G$ .  $\square$

Let  $f : G \rightarrow G'$  be an onto homomorphism of BCC-algebras. Let  $T$  and  $T^*$  be  $t$ -norms such that  $T^*$  dominates  $T$ . If  $\mu$  and  $\nu$  are  $T$ -fuzzy BCC-subalgebras of  $G'$ , then the  $T^*$ -product of  $\mu$  and  $\nu$ ,  $[\mu \cdot \nu]_{T^*}$ , is a  $T$ -fuzzy BCC-subalgebra of  $G'$ . Since every onto homomorphic preimage of a  $T$ -fuzzy BCC-subalgebra is a  $T$ -fuzzy BCC-subalgebra, the preimages  $f^{-1}(\mu)$ ,  $f^{-1}(\nu)$  and  $f^{-1}([\mu \cdot \nu]_{T^*})$  are  $T$ -fuzzy BCC-subalgebras of  $G$ . The next theorem provides that the relation between  $f^{-1}([\mu \cdot \nu]_{T^*})$  and the  $T^*$ -product  $[f^{-1}(\mu) \cdot f^{-1}(\nu)]_{T^*}$  of  $f^{-1}(\mu)$  and  $f^{-1}(\nu)$ .

**Theorem 3.18.** Let  $f : G \rightarrow G'$  be an onto homomorphism of BCC-algebras. Let  $T$  and  $T^*$  be  $t$ -norms such that  $T^*$  dominates  $T$ . Let  $\mu$  and  $\nu$  be  $T$ -fuzzy BCC-subalgebras of  $G'$ . If  $[\mu \cdot \nu]_{T^*}$  is the  $T^*$ -product of  $\mu$  and  $\nu$  and  $[f^{-1}(\mu) \cdot f^{-1}(\nu)]_{T^*}$  is the  $T^*$ -product of  $f^{-1}(\mu)$  and  $f^{-1}(\nu)$ , then

$$f^{-1}([\mu \cdot \nu]_{T^*}) = [f^{-1}(\mu) \cdot f^{-1}(\nu)]_{T^*}.$$

*Proof.* For any  $x \in G$  we get

$$\begin{aligned}
 [f^{-1}([\mu \cdot \nu]_{T^*})](x) &= [\mu \cdot \nu]_{T^*}(f(x)) \\
 &= T^*(\mu(f(x)), \nu(f(x))) \\
 &= T^*([f^{-1}(\mu)](x), [f^{-1}(\nu)](x)) \\
 &= [f^{-1}(\mu) \cdot f^{-1}(\nu)]_{T^*}(x),
 \end{aligned}$$

ending the proof.  $\square$

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