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Two machine Flexible shop scheduling problem

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Abstract. We study two machine shop type scheduling problem in this paper. At shop type scheduling, one job is handled by plural machines. And it generally divided two types of problem. Flowshop type have been determined the order of operation at each machine. And it is not decided on Openshop type. In this paper, considering a problem with two machines and flexible jobs which have no strict order of machines, but have desirably order. The super shop problem which is mixed with flow shop and open shop is considered by Strusevich and suggests a solution method that based on 13 cases. In this paper, we extend this result and give more detailed condition on one case which include preemptive job. We also propose a solution to this problem by extending this result to flexible flowshop.

1 Introduction Shop type scheduling problem is one of the most major part of scheduling problem research. Especially the study for two machine flow shop scheduling problem by Johnson is one of the most famous results in scheduling problem. There are various types of constraints for shop scheduling problem. Two machine shop type scheduling problem is defined as follows.

- There are two machines M_1, M_2 and n jobs $1, \cdots, n$.
- Each job is processed by M_1 and M_2. And these processes are not allowed to overlap.
- There are constraints in the order of the processes for each job.
- The objective function is the completion time of all jobs.

Two machine flow shop problem is studied by Johnson [1]. In this problem the order of the processes for each job is fixed as $M_1 \to M_2$. He showed an optimizing procedure by sorting in processing time for each job as Johnson rule. The job shop type problem is defined as there are two types of processing order constraint as $M_1 \to M_2$ or $M_2 \to M_1$ for all jobs. This problem is studied by Jackson, J. R [2]. Also, there is no constraint in order of processes is called Open shop problem. This problem is studied by Gonzalez, T and Sahni, S [3]. Furthermore, it has been also considered a mixed problem that combines constraints of these three types. The problem including flow shop type and open shop type jobs has been studied by Masuda, T., Ishii, H. and Nishida, T [4] as a mixed shop problem. V. A. Strusevich studied two machine super shop including two types of flow shop jobs and open shop. He divided the problem to 13 cases based on the processing time and show the condition which optimal non-preemptive schedule is possibly different from the optimal preemptive schedule [5].

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2 Problem Definition In this paper, we consider the problem adding the flexible jobs to super shop problem. Flexible jobs are defined as : The order of processing by machine M_1, M_2 is unrestricted. However, the satisfaction degree is defined for the order. For example, in the painting jobs with two colors, any order of painting color is allowable, but there is the difference in the finish. Our problem settings are as follows;

- There exists a set of n jobs $N = J_{12} \cup J_{21} \cup O \cup F_{12} \cup F_{21}$.
- J_{12} : Flow shop type job set $M_1 \to M_2$.
- J_{21} : Flow shop type job set $M_2 \to M_1$.
- O : Open shop type job set which processing order is open i.e. either $M_1 \to M_2$ or $M_2 \to M_1$ is allowable.
- F_{12}: A flexible job preferably should be processed on M_1 but in some case first on M_2.
- F_{21} : A flexible job preferably should be processed on M_2 but in some case first on M_1.
- For each job, we define the two satisfaction degrees $\mu_1(j), \mu_2(j)$ of processing order on two machines.
- $\mu_1(j)$: the satisfaction degree in case that job j is processed M_1 first.
- $\mu_2(j)$: the satisfaction degree in case that job j is processed M_2 first.

The meanings of satisfaction degree are as follows;

- $j \in J_{12}$: $\mu_1(j) = 1, \mu_2(j) = 0$,
- $j \in J_{21}$: $\mu_1(j) = 0, \mu_2(j) = 1$,
- $j \in O$: $\mu_2(j) = 1, \mu_1(j) = 1$,
- $j \in F_{12}$: $\mu_1(j) = 1, 0 < \mu_2(j) < 1$,
- $j \in F_{21}$: $\mu_2(j) = 1, 0 < \mu_1(j) < 1$.

F_{12} and F_{21}: call flexible order job set.

For each job j, we define the processing times a_j, b_j on M_1, M_2 respectively. Each machine M_1 and M_2 processing at most one job at a time and each job is processed on at most one machine at a time. Under above setting, we seek a schedule minimizing the maximum completion time and maximizing the minimum satisfaction degree about processing order on machines, but usually there is no feasible schedule optimizing both criteria. We seek some non-dominated schedules after the definition of non-domination.

Non-dominated schedule For each schedule s, we define schedule vector $v^s = (v_1^s, v_2^s) = (C_{\max}^s, \mu^s)$ where $C_{\max}^s$ is the maximum completion time of schedule s and $\mu^s = \min\{\min\{\mu_1(j), j \in A(s)\}, \min\{\mu_2(j), j \in B(s)\}\}$, where $A(s)$: set of jobs processed on M_1 first in schedule s, $B(s)$: set of jobs processed on M_2 first in schedule s. For schedules s^1, s^2, we call s^1 dominate s^2 if $v_1^{s^1} \leq v_1^{s^2}, v_2^{s^1} \geq v_2^{s^2}$ and $v^{s^1} \neq v^{s^2}$ and we call a schedule s non-dominated schedule if no schedule dominates s. We seek some non-dominated schedules $a(J) = \Sigma_{j \in J} a_j, b(J) = \Sigma_{j \in J} b_j, \pi(J)$: arbitrary schedule of job set J.

3 Super shop problem The procedure for our problem is based on reducing to super shop problem corresponding to the satisfaction degree. Super shop problem is considered as a mixed model with two flow shop model and open shop model. The definition for super shop problem is as follows.

For subset of jobs $Q \subseteq N$, $a(Q) = \Sigma_{j_i \in Q} a_i$, $b(Q) = \Sigma_{j_i \in Q} b_i$, $a(\emptyset) = b(\emptyset) = 0$, define the subscripts a or b on upside of arbitrary job set Q as $Q^a = \{j_i \in Q \mid a_i < b_i\}$ or $Q^b = \{j_i \in Q \mid a_i \geq b_i\}$ respectively.

- N_{12} : Flow shop type job set with job processing order $M_1 \to M_2$,
- N_{21} : Flow shop type job set with job processing order $M_2 \to M_1$,
- N_O : Open shop job set,
- j_k : The job $j_k \in N_O^a$ and has maximum processing time on machine M_2, i.e. $b_k = \max\{b_j \mid \text{for } j \in N_O^a\}$. Here $N_O^a = \{j \in N_O \mid a_j < b_j\}$,
- j_r : The job $j_r \in N_O^b$ and has maximum processing time on machine M_1, i.e. $a_r = \max\{a_j \mid \text{for } j \in N_O^b\}$. Here $N_O^b = \{j \in N_O \mid a_j \geq b_j\}$,
- $N = N_{12} \cup N_{21} \cup N_O$. Job set of all jobs.

Let $T = \max\{a(N), b(N)\}$. Let the permutation $\pi(Q)$ be an arbitrary permutation of jobs from Q, permutation $\pi(\emptyset)$ be dummy permutation, and $\varphi(N_{12})$, $\varphi(N_{21})$ be an optimal processing order applying Johnson rule to flow shop job set N_{12} and N_{21} respectively.

In our problem, let $C_{\max}(s)$ be a maximum completion time of super shop schedule s. Lower bound of maximum completion is as follows

$$C_{\max} \geq \max\{a(N), b(N), C_{\max}(s_{12}^*), C_{\max}(s_{21}^*), \max\{a_i + b_i \mid J_i \in N_O\} + \tau\}$$

Here s_{12}^* and s_{21}^* are the optimal schedules for jobs of N_{12} and N_{21} respectively by Johnson's rule, $\tau = \min\{a(N_{12}^a) + b(N_{12}^b), a(N_{21}^a) + b(N_{21}^b)\}$. Strusevich develop the solution algorithm for this problem. In this algorithm the problem divided to 13 cases based on the sum of processing time for each part.

Case 1 : $a(N_{12}^a) \geq b(N_{21} \cup N_O)$

Optimal schedule is constructed by following procedure, where N_{12} is an optimal processing order applying Johnson rule to flow shop job set N_{12}, and s_{12}^* is the corresponding optimal schedule.

1. M_1: processing order $\varphi(N_{12}) \to \pi(N_{21} \cup N_O)$ from 0.
2. M_2: first processing order $\pi(N_{21} \cup N_O)$ from 0 and second processing order $\varphi(N_{12})$ from $\max\{b(N_{21} \cup N_O), C_{\max}(s_{12}^*) - b(N_{12})\}$.

Case 2 : $b(N_{21}) \leq a(N_{12}) < b(N_{21} \cup N_O)$, and $a(N_O - \{j_k\}) \leq b(N_O - \{j_r\}), a(N - \{j_k\}) \geq b(N_{21} \cup N_O)$

Optimal schedule is constructed by following procedure. Let $\psi(N_O) = (j_r, \pi(N_O^b - \{j_r\}), \pi(N_O^a - \{j_k\}), j_k)$.

1. M_1: processing order $\pi(N_{12}) \to \pi(N_{21}) \to \psi(N_O)$ from 0
2. M_2: processing order $\pi(N_{21}) \to \psi(N_O) \to \pi(N_{12})$ from 0, where $\psi(N_O) = (j_r, \pi(N_O^b - \{j_r\}), \pi(N_O^a - \{j_k\}), j_k)$ and the corresponding maximum completion time of optimal schedule in this case is T.

Case 3 : $b(N_{21}) \leq a(N_{12}) < b(N_{21} \cup N_O), a(N_O - \{j_k\}) \leq b(N_O - \{j_r\}), b(N - \{j_k\}) \geq b(N_{21} \cup N_O)$.

Optimal schedule is constructed by following procedure.

1. M_1: processing order $\psi(N_O - \{j_k\}) \to \pi(N_{12}) \to \pi(N_{21}) \to J$ from 0.
2. M_2: processing order $\pi(N_{21}) \to j_k \to \psi(N_O - \{j_k\}) \to \pi(N_{12})$ from 0, where $\psi(N_O - \{j_k\}) = (\pi(N_O^a - \{j_k\}), \pi(N_O^b - \{j_r\}))$ and the corresponding maximum completion time of optimal schedule in this case is T.

Case 4 : $b(N_{21}) \leq a(N_{12}) < b(N_{21} \cup N_O), a(N_O - \{j_k\}) > b(N_O - \{j_r\}), a(N_{12} \cup N_{21}) \geq b(N \cup \{j_r\})$, corresponding maximum completion time of optimal schedule is T.

Case 5 : $b(N_{21} \cup N_O) > a(N_{12}) \geq b(N_{21}), a(N_O - \{j_r\}) \geq b(N_{21} \cup \{j_r\}) > a(N_{12} \cup N_{21})$, corresponding maximum completion time of optimal schedule is T.

Case 6 : first set m, $b(N_{21} \cup N_O) > a(N_O - \{j_k\})$, and $a(N_O - \{j_k\}) \leq b(N_O - \{j_r\}) \Rightarrow j_m = j_k$,

$a(N_{21} \cup N_{12}) < b(N_{21} \cup \{j_r\})$, and $a(N_O - \{j_k\}) > b(N_O - \{j_r\}) \Rightarrow j_m = j_r$, $b(N_{21}) \leq a(N_{21}) < b(N_{21} \cup N_O), a(N - \{m\}) < b(N_{12} - \{j_m\}), a_m \leq b(N_{12} \cup N_O - \{j_m\})$ corresponding maximum completion time of optimal schedule is $b(N)$.

Case 7 : $b(N_{21}) \leq a(N_{12}) < b(N_{21} \cup N_O), a(N - \{j_m\}) < b(N_{21} \cup \{j_m\}), a_m > b(N_{12} \cup N_O - \{j_m\}), a(N_{12}^a) + b(N_{12}^b) \leq a(N_{21}^a) + b(n_{21}^b), a(N_{12}^b) < b_m$ corresponding maximum completion time of optimal schedule is $\max\{T, a_m + b_m + a(N_{12}^a) + b(N_{12}^b)\}$.

Case 8 : $b(N_{21}) \leq a(N_{12}) < b(N_{21} \cup N_O), a(N - \{j_m\}) < b(N_{21} \cup \{j_m\}), a_m > b(N_{12} \cup N_O - \{j_m\})$, $a(N_{12}^a) + b(N_{12}^b) \leq a(N_{21}^a) + b(n_{21}^b), a(N_{12}^b) < b_m, a(N_{12} \cup N_O) > b(N_{21} \cup \{j_m\})$, corresponding maximum completion time of optimal schedule is T.

Case 9 : $b(N_{21}) \leq a(N_{12}) < b(N_{21} \cup N_O), a(N - \{j_m\}) < b(N_{21} \cup \{j_m\}), a_m > b(N_{12} \cup N_O - \{j_m\})$, $a(N_{12}^a) + b(N_{12}^b) \leq a(N_{21}^a) + b(N_{21}^b), a(N_{12}^b) < b_m, a(N_{12} \cup N_O) > b(N_{21} \cup \{j_m\})$, corresponding maximum completion time of optimal schedule is T.

Case 10 : $b(N_{21}) \leq a(N_{12}) < b(N_{21} \cup N_O), a(N - \{j_m\}) < b(N_{21} \cup \{j_m\}), a_m > b(N_{12} \cup N_O - \{j_m\})$, $a(N_{12}^a) + b(N_{12}^b) > a(N_{21}^a) + b(N_{21}^b), a_m \geq b(N_{21}^a)$, corresponding maximum completion time of optimal schedule is $\max\{T, a_m + b_m + a(N_{12}^a) + b(N_{12}^b)\}$.

Case 11 : $b(N_{21}) \leq a(N_{12}) < b(N_{21} \cup N_O)$, $a(N - \{j_m\}) < b(N_{21} \cup \{j_m\})$, $b(N_{21}^a) > a_m > b(N_{12} \cup N_O - \{j_m\})$, $a(N_{12}^a) + b(N_{12}^b) > a(N_{21}^a) + b(N_{21}^b)$, $a(N_{12} \cup j_m) \geq b(N_{21} \cup N_O)$, the corresponding maximum completion time of optimal schedule in this case is $b(N)$.

Case 12 : $b(N_{21}) \leq a(N_{12}) < b(N_{21} \cup N_O)$, $a(N - \{j_m\}) < b(N_{21} \cup \{j_m\})$, $b(N_{21}^a) > a_m > b(N_{12} \cup N_O - \{j_m\})$, $a(N_{12}^a) + b(N_{12}^b) > a(N_{21}^a) + b(N_{21}^b)$, $a(N_{12} \cup \{j_m\}) \geq b(N_{21} \cup N_O)$, corresponding maximum completion time of optimal schedule in this case is T.

Strusevich proposed the 13 cases for Super shop problem. At only one case, the schedule includes the nonpreemptive jobs. We divide this case to two cases by precisely condition.

Case 13(i) : $b(N_{21}) \leq a(N_{12}) < b(N_{21} \cup N_O)$, $a(N - \{j_m\}) < b(N_{21} \cup \{j_m\})$, $a_m > b(N_{12} \cup N_O - \{j_m\})$, $b(N_{21}) + b_m > a(N_{21} \cup N_O - \{j_m\})$, the corresponding maximum completion time of optimal schedule in this case is $a(N)$.

Case 13(ii) : $b(N_{21}) \leq a(N_{12}) < b(N_{21} \cup N_O)$, $a(N - \{j_m\}) < b(N_{21} \cup \{j_m\})$, $a_m > b(N_{12} \cup N_O - \{j_m\})$, $b(N_{21}) + b_m \leq a(N_{21} \cup N_O - \{j_m\})$, the corresponding maximum completion time of optimal schedule in this case is $a(N)$. Only in this case, optimal non-preemptive schedule is possibly different from the optimal preemptive schedule. But in this case also optimal maximum completion time is one of $a_m + b_m, a(N), b(N)$. We should check only some cases among 14 cases.

4 Solution procedure of flexible shop scheduling problem In this section, we propose the solution procedure for our flexible shop model. This procedure is based on super shop problem. There are multiple optimal solutions for the value of satisfaction degree. Therefore, we seek the non-domination solution. The detail of the procedure is as follows.

Assignment of processing order and solve the super shop problems

1. Sort $\mu_2(j), j \in F_{12}, \mu_1(j), j \in F_{21}$ and result be $\mu(0) = 1 > \mu(1) > \mu(2) > \mu(3) > \cdots > \mu(u) > \mu(u+1) = 0$.
2. Consider the super shop problem $P(t)$ with parameter $\mu(t), t = 0, 1, 2, \cdots, u+1$ as the subproblem where u: the number of different values in $\mu_2(j), j \in F_{12}, \mu_1(j), j \in F_{21}$,
3. $P(t)$:the super shop problem with
   (a) $N_{12} = J_{12} \cup \{j \in F_{12} \mid \mu_2(j) < \mu(t)\}, N_{21} = N_{12} = J_{21} \cup \{j \in F_{12} \mid \mu(j) < \mu(t)\}$
   (b) $N_O = O \cup \{j \in F_{12} \mid \mu_2(j) \geq \mu(t)\} \cup \{j \in F_{21} \mid \mu_1(j) \geq \mu(t)\}$
   (c) Apply the super shop scheduling algorithm by checking 14 cases and obtain optimal scheduling $s(t)$. Note that $P(0)$: $N_{12} = J_{12} \cup F_{12}$, $N_{21} = J_{21} \cup F_{21}$, $N_O = O$, $P(u+1)$: $N_{12} = J_{12}$, $N_{21} = J_{21}$, $N_O = O \cup F_{12} \cup F_{21}$.
4. From $s(0), s(1), \cdots, s(u+1)$, choose non-dominated schedules. Note that N_O is non-decreasing about t and N_{12}, N_{21} is non-increasing.

5 Numerical Example In this section, we consider the some numerical example. The following jobs are considered.

	N_{12}	F_{12}		N_{21}	F_{21}		N_O	
i	1	2	3	4	5	6	7	8
a_i	3	4	1	1	2	2	17	2
b_i	2	5	4	2	3	1	12	1
μ_i^A	1	1	1	0	0.6	0.8	1	1
μ_i^B	0.3	0.7	0.5	1	1	1	1	1

Table 1: Numerical Example

We obtain 5 cases ($\mu = 1.0, 0.8, 0.7, 0.6, 0.5, 0.3$) in non-increasing order of satisfaction, and seek the optimal schedule in each case.

For $\mu = 1.0$: In this constraint, the processing order of flexible jobs is fixed.

This case corresponds the case 10 of super-shop from the following checking:

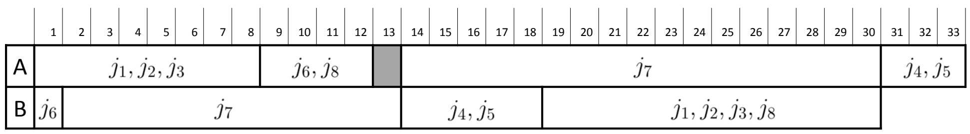
$N_O^a = \emptyset$, $N_O^b = \{j_7, j_8\}$, $j_r = j_7$, j_k is not defined. Therefore $a(N_o - \emptyset) = 19 > b(N_a - \{j_7\}) = 1$, $a(N_{12} \cup N_{21}) = 15 < b(N_{21} \cup \{j_7\}) = 18$ holds and so we set $j_m = j_7$. $a_7 = 17 > b(N_{12} \cup N_O - \{j_7\}) = 12$, $a(N_{12}^a) + b(N_{12}^b) + 4 + 1 + 2 = 7 > a(N_{21}^a) + b(N_{21}^b) = 1 + 2 + 1 = 4$, $a_7 = 17 > b(N_{21}^a) = 5$

	N_{12}			N_{21}			N_O	
i	1	2	3	4	5	6	7	8
a_i	3	4	1	1	2	2	17	2
b_i	2	5	4	2	3	1	12	1

Table 2: $\mu = 1.0$

Here $N_{12} = \{j_1, j_2, j_3\}$, $N_{21}^b \cup N_O - \{j_m\} = \{j_6, j_8\}$, $j_m = j_7$, $N_{21}^b = \{j_4, j_5\}$, and $N_{21}^b = \{j_6\}$, $N_{21}^b = \{j_4, j_5\}$, $N_{12} \cup N_O - \{j_m\} = \{j_1, j_2, j_3, j_8\}$.

If preemption of the processing is not allowed, the optimal completion time $C_{\max}(s^*) = 33$.

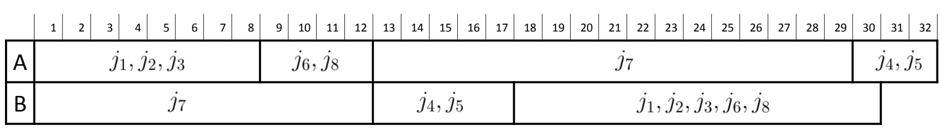


For $\mu = 0.8$: In this constraint, the processing order of flexible jobs is fixed.

	N_{12}			N_{21}		N_O		
i	1	2	3	4	5	6	7	8
a_i	3	4	1	1	2	2	17	2
b_i	2	5	4	2	3	1	12	1

Table 3: $\mu = 0.8$

We obtain the two optimal schedule which completion time $C_{\max}(s^*) = 32$ without preemption.



Since completion time 32 is a lower bound, we need not check $\mu = 0.7, \mu = 0.6, \mu = 0.5, \mu = 0.3$.

6 Discussion and Conclusion If case 13(ii) does not occur, we can obtain non-dominated solutions without preemption. Since $a(N), b(N), T = \max\{a(N), b(N)\}$ are constant independent from processing order of any jobs, we can utilize this fact to make our algorithm efficient. Anyway, we must consider the efficient method to solve each super shop scheduling problem using some sensitivity of the conditions about change on processing order of F_{12}, F_{21}. For that purpose, we should simplify the cases of the solution method due to Strusevich including investigation of further division in case(ii) though we divide case 13 into two subcases 13(i) and 13(ii).

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