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ESTIMATE ON DIFFUSION RATE OF CONTAMINANT IN RECYCLING LINE OF FOOD-TRAYS BY EPCO'S METHODS

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Abstract. Reduction of the amount of wastes coming from food containers and packaging is one of urgent issues for the humankind. Japanese manufacturers, including F. P. Corporation, are devising their own recycling system of disposable food containers for reusing resources in containers and packagings. Without waiting the Guidelines issued by the Ministry of Health, Labour and Welfare of Japanese Government, it is indispensable to ensure food safety when the manufacturer uses such recycled materials. This paper then intends to present methods for estimating a diffusion rate of contaminant if it is contained in post-consumer food containers and enters the recycling line. Our methods will be explained by applying them to the recycling line realized by F. P. Corporation. As our methods are quite general, they may easily be applied to any other recycling lines.

1 Introduction It is ordinarily seen that a large amount of household wastes is occupied by those which come from food containers and packagings. In order to reduce the amount of such wastes, the Recycling Law of Food Containers and Packaging has been established in Japan in 1995 for promoting more effective use of resources in containers and packagings.

F. P. Corporation (abbreviated to FPCO), a manufacturer of disposable food containers to be used in supermarkets, convenience stores and others, has been realizing an original recycling system since 1990.

Post-consumer food containers brought to supermarkets and others are gathered by collection boxes and are brought back to the recycling plants of FPCO by utilizing returning trucks which delivered their products as explained in [1]. FPCO's recycling process of foamed polystyrene containers consists of three main steps, namely, (1) sorting/crashing, (2) washing/dehydration, and (3) extrusion/pelletizing, in order to remove contaminators from the collected polystyrene containers. Using the regenerated polystyrene pellets, recycled foamed polystyrene containers are made via sheet formations. Its schematic diagram is sketched by Figure 1. For details, see the homepage [2].

Without waiting the Guidelines [3] issued by the Ministry of Health, Labour and Welfare of Japanese Government, it is indispensable to ensure food safety when the manufacturer uses such recycled materials for reproducing food containers. Careful and sufficient considerations must be taken for preventing any recycled containers containing adventitious chemical contaminant which may migrate into foods and influence human health from being distributed to the markets.

FPCO has received a non-objection letter on recycled foamed polystyrene containers from U. S. Food and Drug Administration. In addition, constant inspections are carried out in daily production activities in accordance with Japanese Food Sanitation Act.

Meanwhile, investigations on the worst case are always required in the field of food sanitation. One of these investigations, to know scientifically how contaminants diffuse through

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the recycling process is very important and to estimate reasonably the highest possible contaminant concentration is very crucial. By these reasons, a mathematical approach is proposed by the present authors and some analytical results are described in the paper. Specifically, we assume that a tray containing a unit amount of contaminant enters FPCO's recycling line. Then, under the worst external conditions to be considered, we analyse its diffusion rate. Finally, we compute the highest contaminant concentration by means of the random variable.

As our methods of estimation are very general, it is easy to know how the response is with respect to the change of controllable internal conditions. We then hope that the methods presented in this paper would play a meaningful role in order to establish safer and more reliable recycling processes for reusing more post-consumer food containers and packaging waste.

Finally, let us review FPCO's recycling line whose schematic diagram is sketched by Figure 1. The collected trays are crashed into small fragments. After being fully washed, the fragments are melted by a heater and the polystyrene in gel is pelletized by an extruding machine to yield numbers of pellets which are a unit grain of foamed polystyrene of a uniformed size in order to reproduce new food-trays. The pellets made from the used trays are packed in big boxes and are quadrupled by adding three times virgin pellets. After being entirely blended, the quadrupled pellets are laid in a thin layer, once again melted and are sheeted by another extruding machine to make polystyrene sheets. These sheets are laminated by a virgin film and cut into a unit size of tray. By these processes, the used trays are recycled to new ones.

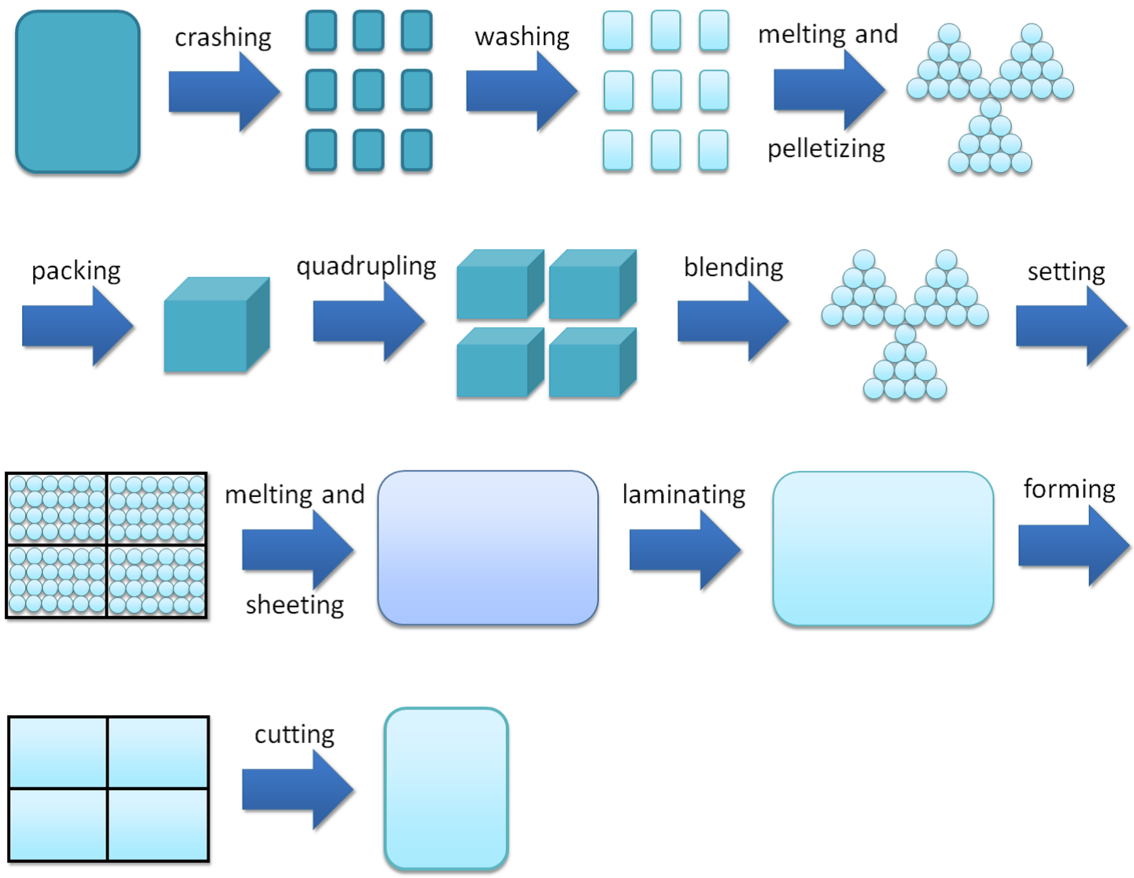

Fig. 1: FPCO's Recycling Methods

2 Material and Methods We first want to notice that through the recycling line sketched by Figure 1, the diffusion of contaminant consists of three independent kinds of diffusions.

First one is the temporal diffusion. Assume that one tray containing a unit amount of contaminant has entered the production line. Then, the contaminated tray is crashed into almost 250 fragments which contain as a result 4.0×10^{-3} unit of contaminant for each. Through melting and pelletizing, the 250 fragments are processed into numbers of pellets which contain a certain unit of contaminant. And these contaminated pellets together with other clear ones are packed in several boxes. Then, how do the contaminated pellets diffuse over the packing boxes?

Second one is the diffusion caused by combination which may be called the combinatorial diffusion. Consider a box of pellets which nearly consist of 1.0×10^7 pellets and assume that some of these, say n pellets, are contaminated. By addition of three boxes of virgin pellets, we have 4.0×10^7 pellets as a whole. These pellets are randomly divided into sets consisting of 100 pellets uniformly; consequently, we make 4.0×10^5 sets. Each set of pellets can yield just one new tray after melting, sheeting and cutting processes. Then, how do the n contaminated pellets included in 4.0×10^7 pellets in total diffuse over the dividing sets?

Third one is the diffusion caused by melting and extruding (here and after the word extruding will be used for two meanings: pelletizing by extrusion and sheeting by extrusion). The production line has two processes of melting and extruding. Naturally, through the two processes the contaminant in contaminated fragments or in contaminated pellets diffuses in the gel of polystyrene. Then, how does the contaminant diffuse in the gel spatially?

Let us next explain how we analysed these different kinds of diffusions.

As for the temporal diffusion, we made the following experiments. A certain number of colored fragments of tray were inserted in the recycling line and the arriving time of each fragment at the first melting stage was checked. Several times this trial was repeated. Through these experiments we know how long the fragments made of a contaminated tray entered in the line diffuse temporally before arriving at the first melting stage.

The combinatorial diffusion can be analysed exactly by using the theory of probability and combinatorics (e.g., see [4, 7]). Consider a collection of $N = 4.0 \times 10^7$ pellets which includes n contaminated pellets. We divide all the pellets randomly into 4.0×10^5 sets which consist uniformly of 100 pellets. Denote by X the maximum of contaminated pellets included in one set throughout the 4.0×10^5 sets. Of course X changes depending on how to divide, so X is considered as a random variable. The most favorable case is that the n contaminated pellets are completely divided into different sets, i.e., $X = 1$. On the contrary, the worst case is that the n pellets are divided into a single set, i.e., $X = n$, but the probability of such a division should be negligibly small. We will devise an easy way how to compute the probability such that $X = k$ for the variable $k = 1, 2, 3, \ldots, n$.

Finally, the spatial diffusion due to melting and extruding is analysed by the following experiments. A similar type of melting and sheeting machine was prepared. Among numbers of pellets, just one pellet which contains a material emitting fluorescent X-rays was put and passed through the heater and extruder. The resultant sheet was then carefully examined. How wide is the emitting material spread? What is magnitude of the X-ray in each part of sheet? Several times this experiment was repeated. Out of those data, we built a fitting function which describes the diffusion of the emitting material as a 3D graph, by using the techniques of implicit surface fitting (see [6, 10]). By these arguments we know how wide the contaminant in a pellet is spread and by what rate the contaminant diffuses through the melting and extruding processes.

It is, however, very difficult to analyse the spatial diffuses of contaminant in the first melting process, because the gel made from the fragments is immediately formed into numbers of pellets by a pelletizing extruder. So we want to introduce an imaginary process of sheeting and want to consider that the gel is once formed into sheets and then those sheets are formed into pellets.

3 Results

3.1 Temporal diffusion We inserted 50 colored fragments of tray at the end of crashing process and checked the arriving time of each fragment at the checking point which was set almost in the middle of crashing and melting stages. This trial was repeated 5 times. We could check for almost 30 fragments their arriving time for each trial. The result is graphed in Figure 2.

Here, $\Delta t = 1, 2, 3$ (min.) denotes a unit of time interval, the axis of abscissas $i = 1, 2, 3, \ldots$ denotes time $i\Delta t$ (min.), and the axis of ordinates denotes a number of fragments which arrived during the time from $(i-1)\Delta t$ to $i\Delta t$. From the data we observe that the range of arrival time is not so long and all the checked fragments arrived within 26 min. Indeed, we verify that, if the graphs in Figure 2 can be approximated by the normal distributions, then it is concluded that 95% of fragments arrive within 25 minutes (see [8]). Remembering that our cheking point is set at the middle of crashing and melting stages, we want to estimate that the temporal diffusion of contaminated fragments is about 1 hour.

After being melted and pelletized, the fragments are formed into pellets and the pellets are packed in big boxes. We know that each packing box is filled with pellets by just 1 hour. This means that the pellets made from the 250 contaminated fragments must be packed at most 2 boxes. In this way the n contaminated pellets can be included in a single packing box with a high probability, which means that the temporal diffusion must be disregarded.

Fig. 2: Experimental Data

3.2 Spatial diffusion We put one pellet which contains a material emitting fluorescent X-rays in a similar type of melting and sheeting machine. Magnitude of the X-ray in each part of the resultant sheet was measured by a photometer. The data is given by Table 1.

Table 1: Data

	1	2	3	4	5	6
1	0	0.027003484	0.031068525	0.024970964	0.022357724	0.019454123
2	0.013066202	0.028745645	0.030197445	0.026422764	0.022938444	0.019163763
3	0.007549361	0.034262485	0.042973287	0.033391405	0.022938444	0.014808362
4	0	0.020325203	0.041521487	0.030487805	0.025551684	0.018583043
5	0.006097561	0	0.009001161	0.012485482	0.032520325	0.025842044

7	8	9	10	11	12
0.012775842	0.019163763	0.008420441	0.007839721	0	0.008420441
0.018873403	0.013646922	0.009872242	0.007839721	0.006678281	0
0.012775842	0.008710801	0.010162602	0.007839721	0	0
0.016260163	0.012485482	0.011614402	0.010162602	0.007259001	0.005807201
0.031939605	0.022067364	0.021777003	0.016260163	0.012775842	0.011904762

13	14	15	16
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
0.011614402	0.011324042	0	0

The resultant sheet is of width 35cm $\times$ 480cm. This area is divided into 5×16 parts which are uniformly of width 7cm $\times$ 30cm. The numbers in Table 1 show the magnitude of the X-ray in these parts. The total magnitude is just 1. We see that the part $(3, 3)$ has the maximum magnitude. The data can also be illustrated by a rectangular graph drew in Figure 3.

In order to use these data more conveniently, it is necessary to describe the graph by a suitable fitting surface. Several methods are known how to fit a function $f(x, y)$ to a given rectangular graph. We here use the normal distribution for the variable x and the Johnson Sb distribution for the variable y due to [6], that is,

$$f(x,y) = \frac{b-a}{2\pi\sigma(b-y)(y-a)} \exp\left\{-\frac{(x-\mu)^2}{2\sigma^2} - \frac{1}{2}\left[\gamma + \delta \log\left(\frac{y-a}{b-y}\right)\right]^2\right\}, \tag{3.1}$$

where $a, b, \gamma, \delta, \mu$ and σ are parameters to be determined, see [10]. Some optimization

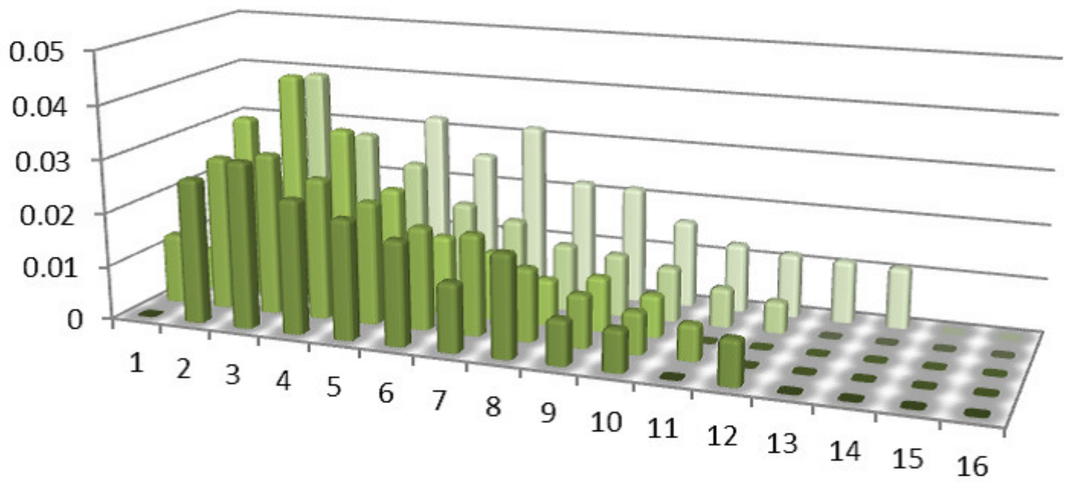

Fig. 3: Rectangular Graph

arguments owing to [5] yield that, under

$$\begin{cases} a = -10.1864 \\ b = 15.9004 \\ \gamma = 0.6660 \\ \delta = 0.6671 \\ \mu = 2.3588 \\ \sigma = 1.8366, \end{cases} \tag{3.2}$$

its fitting becomes the maximum, for the details see [9].

We also impose a condition that the numerical integral of $f(x, y)$ is nearly equal to 1. The graph of the function (3.1) with parameters (3.2) is given by Figure 4.

It is possible to derive many properties of the spatial diffusion through the melting and sheeting processes by using this fitting function.

Assume that one contaminated pellet containing, say a unit amount of, contaminant is put in the second melting process. The contaminant in the pellet diffuses, after melting and sheeting, over the sheet to be laminated and cut according to the function obtained by Figure 4. Noticing that a reproduced tray is of width 12cm $\times$ 20cm, we can compute the maximum amount of contaminant in a tray as

$$\mathit{SDR} = 0.037264 \tag{3.3}$$

(for the details see [9]), which is called *the Spatial Diffusion Rate*.

Let us now estimate the spatial diffusion in the first melting process. As discussed above, we should disregard the temporal diffusion of contaminated fragments. So we assume that 250 contaminated fragments are put simultaneously in the first melting stage. In addition,

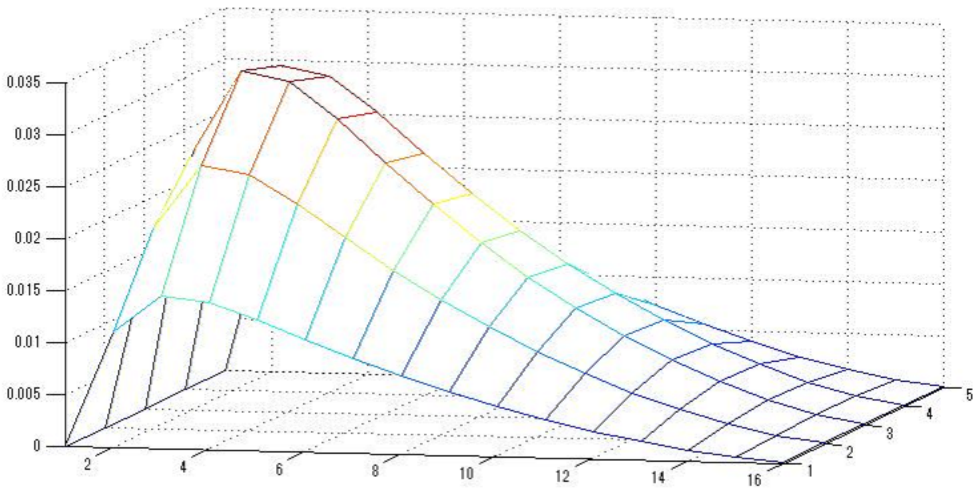

Fig. 4: Johnson Sb Distribution

we set an imaginary process of sheeting, namely, we consider that the fragments are once melted by a heater, the gel is extruded to form it into a sheet, and the sheet is processed into pellets. We therefore assume that a unit amount of contaminant is put in the melting and sheeting processes. Then its diffusion can estimated as above. The contaminant spreads over a sheet of width 35cm $\times$ 480cm and its distribution is given by the function (3.1) with parameters (3.2). Since one tray measures 12cm $\times$ 20cm and consists of almost 100 pellets, this sheet yields 70 trays, i.e., 7.0×10^3 pellets which are contaminated. In this way, a unit amount of contaminant diffuses over 7×10^3 pellets with some rate which depends on each pellet. It is, however, very difficult to estimate a distribution of rates over such a large number of pellets. So, considering the fact that the gel of polystyrene is stirred harder by the pelletizing extruder, we want to take a homogeneous distribution but over a little bit smaller number of pellets. In this paper, we set 6.0×10^3 contaminated pellets which contain a uniform amount of contaminant, namely,

$$n = 6.0 \times 10^3 \tag{3.4}$$

and all these pellets contain uniformly a $1/[6.0 \times 10^3]$ unit of contaminant.

3.3 Combinatorial diffusion Consider a collection of $N = 4.0 \times 10^7$ pellets which includes, according to (3.4), $n = 6.0 \times 10^3$ contaminated pellets. We divide these pellets randomly into $q = 4.0 \times 10^5$ sets of pellets which consist uniformly of $p = 100$ pellets.

More precisely, we study dispositions of the N pellets into the $q \times p$ sites described by Figure 5. Let X be a random variable which is defined as the maximum number of contaminated pellets through the all dividing sets for each disposition. That is, X is a random variable defined on the sample space

$$\Omega = \{\text{all the permutations of the } N \text{ pellets into the } q \times p \text{ sites}\}.$$

The probability such that $X = k$, where $k = 1, 2, 3, \dots, n$, can be computed by the following methods.

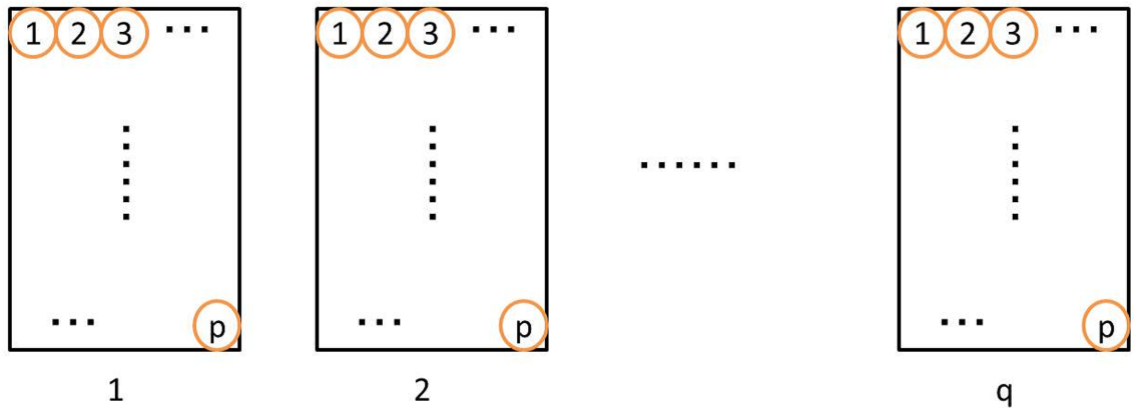

Fig. 5: Division

I. *Probability of* $X = 1$. The total number of elements of Ω, namely, the total number of permutations of N pellets is of course $N!$.

In the meantime, the number of permutations such that $X = 1$, namely, the number of permutations in which the n contaminated pellets are completely disposed in different sets is computed by the following procedure:

1. First, we count the number of choice of n sites for contaminated pellets. As for sets, we have ${}_qC_n$. For such a choice, each set has ${}_pC_1$ sites for a contaminated pellet. Therefore, it counts ${}_qC_n[{}_pC_1]^n$.

2. Let the n sites for contaminated pellets be fixed as (1). Then there are $n!$ permutations of the contaminated pellets.

3. Let the sites for contaminated pellets be fixed as (1) and let the contaminated pellets be disposed as (2). Then the non-contaminated pellets are disposed by $(N-n)!$ ways.

We therefore conclude that

$$P(X=1) = \frac{{}_qC_n \cdot [{}_pC_1]^n \cdot n! \cdot (N-n)!}{N!} = \frac{p^n \cdot q! \cdot (N-n)!}{(q-n)! \cdot N!}. \tag{3.5}$$

By some calculations,

$$P(X=1) = \frac{pq}{N} \cdot \frac{p(q-1)}{N-1} \cdot \frac{p(q-2)}{N-2} \cdots \frac{p(q-n+1)}{N-n+1}.$$

This provides us a practical scheme for computing $P(X=1)$ such that

$$\begin{cases} P_0 = \dfrac{pq}{N} = 1, \\ P_i = \dfrac{p(q-i)}{N-i} \cdot P_{i-1} \qquad (i = 1, 2, 3, \ldots, n-1). \end{cases}$$

It then results in

$$P(X=1) \approx 3.60565 \times 10^{-5}. \tag{3.6}$$

II. *Probability of* $X = 2$. Let us compute $P(X = 2)$. To this end, we introduce another random variable X_2 which denotes the number of sets including just two contaminated pellets for each permutation of Ω. Let x_2 be a variable running from 1 to $\frac{n}{2}$. It is clear that

$$P(X=2) = \sum_{x_2=1}^{\frac{n}{2}} P(X=2,\, X_2=x_2). \tag{3.7}$$

So it suffices to compute $P(X = 2,\, X_2 = x_2)$.

Then each $P(X = 2,\, X_2 = x_2)$ can be obtained by the following procedure:

1. First, compute the number of choice of $2x_2$ sites at which the double contaminated pellets are disposed. Of course, the choice of x_2 sets in which two contaminated pellets are disposed is ${}_qC_{x_2}$. For such a choice, the choice of two sites for contaminated pellets is ${}_pC_2$ per each set. Therefore, it counts ${}_qC_{x_2}[{}_pC_2]^{x_2}$.
2. Under (1), the permutations of n pellets into the chosen $2x_2$ sites is ${}_nP_{2x_2}$.
3. Under (1) and (2), a collection of $N - 2x_2$ pellets (including $n - 2x_2$ contaminated ones) remains to be divided into q sets. But any set other than those chosen in (1) must include at most one contaminated pellet. Then an analogous procedure to that explained above is available to compute the number of such permutations. Indeed, we have ${}_{q-x_2}C_{n-2x_2} \cdot [{}_pC_1]^{n-2x_2} \cdot (n - 2x_2)! \cdot (N - n)!$.

It then follows that

$$\begin{aligned} &P(X=2,\, X_2=x_2)\\ &\quad= \frac{{}_qC_{x_2}\cdot[{}_pC_2]^{x_2}\cdot{}_nP_{2x_2}\cdot{}_{q-x_2}C_{n-2x_2}\cdot[{}_pC_1]^{n-2x_2}\cdot(n-2x_2)!\cdot(N-n)!}{N!}\\ &\quad= \frac{p^{n-x_2}\cdot(p-1)^{x_2}\cdot q!\cdot n!\cdot(N-n)!}{2^{x_2}\cdot x_2!\cdot(q-n+x_2)!\cdot(n-2x_2)!\cdot N!}. \end{aligned}$$

It is easy to verify the following recurrence formula for x_2:

$$\begin{cases} P(X=2,\, X_2=0) = P(X=1), \\ P(X=2,\, X_2=x_2) = \dfrac{(p-1)(n-2x_2+2)(n-2x_2+1)}{2px_2(q-n+x_2)} \\ \qquad\qquad \times P(X=2,\, X_2=x_2-1) \qquad \left(x_2 = 1,2,3,\ldots,\frac{n}{2}\right). \end{cases}$$

Using this formula we can compute $P(X = 2,\, X_2 = x_2)$ for all $x_1 = 1, 2, 3, \ldots, \frac{n}{2}$. Then $P(X = 2)$ is obtained by the summation (3.7). Indeed,

$$P(X=2) \approx 8.05853\times10^{-1}. \tag{3.8}$$

III. *Probability of* $X = 3$. We introduce a further random variable X_3 which denotes the number of sets including just three contaminated pellets for each permutation of Ω. Let x_3 be a variable running from 1 to $\frac{n}{3}$. Then,

$$P(X=3) = \sum_{\substack{1\le x_3\le\frac{n}{3}\\ 3\le 2x_2+3x_3\le n}} P(X=3, X_3=x_3,\, X_2=x_2). \tag{3.9}$$

So let us compute $P(X=3,\, X_3=x_3,\, X_2=x_2)$ for every pair (x_3, x_2) such that $1 \le x_3 \le \frac{n}{3}$ and $3 \le 2x_2 + 3x_3 \le n$.

1. First, as before, compute the number of choice of $3x_3$ sites at which the triple contaminated pellets are disposed. The choice of x_3 sets in which three contaminated pellets are disposed is ${}_qC_{x_3}$. For such a choice, the choice of three sites for contaminated pellets is ${}_pC_3$ per each set. Therefore, it counts ${}_qC_{x_3}[{}_pC_3]^{x_3}$.
2. Under (1), the permutations of n pellets into the chosen $3x_3$ sites is ${}_nP_{3x_3}$.
3. Under (1) and (2), a collection of $N-3x_3$ pellets (including $n-3x_3$ contaminated ones) remains to be divided into q sets. But any set other than those chosen in (1) must include at most two contaminated pellets. Then an analogous procedure to that for the case where $X=2$ is available to compute the number of such permutations. Indeed, we have

$$
\begin{aligned}
{}_{q-x_3}C_{x_2} \cdot [{}_pC_2]^{x_2} \cdot {}_{n-3x_3}P_{2x_2} \cdot {}_{q-x_3-x_2}C_{n-3x_3-2x_2}[{}_pC_1]^{n-3x_3-2x_2} \\
\times (n-3x_3-2x_2)! \cdot (N-n)!.
\end{aligned}
$$

It then follows that

$$
\begin{aligned}
&P(X=3,\, X_3=x_3,\, X_2=x_2) \\
&\quad = \bigl\{{}_qC_{x_3}[{}_pC_3]^{x_3} \cdot {}_nP_{3x_3} \cdot {}_{q-x_3}C_{x_2} \cdot [{}_pC_2]^{x_2} \cdot {}_{n-3x_3}P_{2x_2} \cdot {}_{q-x_3-x_2}C_{n-3x_3-2x_2} \\
&\qquad\quad \times [{}_pC_1]^{n-3x_3-2x_2} \cdot (n-3x_3-2x_2)! \cdot (N-n)!\bigr\}/N! \\
&\quad = \frac{p^{n-2x_3-x_2} \cdot (p-1)^{x_3+x_2} \cdot (p-2)^{x_3} \cdot q! \cdot n! \cdot (N-n)!}{6^{x_3} \cdot 2^{x_2} \cdot x_3! \cdot x_2! \cdot (q-n+2x_3+x_2)! \cdot (n-3x_3-2x_2)! \cdot N!}.
\end{aligned}
$$

To compute $P(X=3)$ in an easy way, we rewrite (3.9) into

$$
P(X=3) = \sum_{x_2=0}^{\frac{n}{2}-2} \sum_{x_3=1}^{\left[\frac{n-2x_2}{3}\right]} P(X=3, X_3=x_3,\, X_2=x_2), \tag{3.10}
$$

where $\left[\frac{n-2x_2}{3}\right]$ denotes the integer part of $\frac{n-2x_2}{3}$, i.e., $0 \le \frac{n-2x_2}{3} - \left[\frac{n-2x_2}{3}\right] < 1$. Then, for each fixed $x_2 = 0, 1, 2, \ldots, \frac{n}{2}-2$, we verify the following recurrence formula for x_3:

$$
\begin{cases}
P(X=3,\, X_3=0,\, X_2=x_2) = P(X=2,\, X_2=x_2), \\
P(X=3,\, X_3=x_3,\, X_2=x_2) \\
\quad = \dfrac{(p-1)(p-2)(n-3x_3-2x_2+1)(n-3x_3-2x_2+2)(n-3x_3-2x_2+3)}{6p^2x_3(q-n+2x_3+x_2-1)(q-n+2x_3+x_2)} \\
\quad \times P(X=3,\, X_3=x_3-1,\, X_2=x_2) \qquad \left(x_3 = 1, 2, 3, \ldots, \left[\frac{n-2x_2}{3}\right]\right).
\end{cases}
$$

For each fixed $0 \le x_2 \le \frac{n}{2}-2$, we first compute the summation of the probabilities $P(X=3,\, X_3=x_3,\, X_2=x_2)$ for $1 \le x_3 \le \left[\frac{n-2x_2}{3}\right]$. Then by the formula (3.10), we compute $P(X=3)$. It then results in

$$
P(X=3) \approx 1.93364 \times 10^{-1}. \tag{3.11}
$$

IV. *Probability of $X = k$ for $k \geq 4$.* By the similar procedures, we can develop our methods of computation for the cases where $k = 4, 5, 6, \dots, p$, and using those we can in fact compute $P(X = k)$ for all these k. For instance, we have

$$P(X = 4) \approx 1.07083 \times 10^{-3}. \tag{3.12}$$

By the way, in view of (3.6), (3.8), (3.11) and (3.12), we immediately verify that

$$P(X = 5) < 1 - \sum_{k=1}^{4} P(X = k) \approx 7.13 \times 10^{-4}. \tag{3.13}$$

Finally, let us consider the worst disposition that the n contaminated pellets are divided into just $r = n/p = 60$ sets which therefore consist of entirely contaminated pellets. First, compute the number of choice of sites. Clearly, the number of choice of sets is ${}_qC_r$ which equals to that of choice of sites. The permutation of n pellets to these chosen suites is $n!$. The permutation of non contaminated pellets is $(N - n)!$. Therefore,

$$P(X = p,\, X_p = r,\, X_{p-1} = \cdots = X_2 = 0) = \frac{{}_qC_r \cdot n! \cdot (N-n)!}{N!} = \frac{q! \cdot n! \cdot (N-n)!}{r! \cdot (q-r)! \cdot N!}.$$

In view of (3.5) we have

$$P(X = p,\, X_p = r,\, X_{p-1} = \cdots = X_2 = 0) = \frac{n! \cdot (q-n)!}{p^n \cdot r! \cdot (q-r)!}\, P(X = 1).$$

Here,

$$\frac{n! \cdot (q-n)!}{r! \cdot (q-r)!} = \frac{n(n-1)(n-2)\cdots[n-(n-r-1)]}{(q-r)(q-r-1)(q-r-2)\cdots[q-r-(n-r-1)]}$$

and

$$\frac{n}{q-r} > \frac{n-1}{q-r-1} > \frac{n-2}{q-r-2} > \cdots > \frac{n-(n-r-1)}{q-r-(n-r-1)}.$$

Since $\frac{n}{q-r} = \frac{600}{39994} < \frac{1}{60}$, we see that

$$P(X = p,\, X_p = r,\, X_{p-1} = \cdots = X_2 = 0) < \frac{1}{6^{n-r} \times 10^{3n-r}}\, P(X = 1), \tag{3.14}$$

which is an extremely small number.

4 Conclusion We have obtained the following results on diffusion rate of contaminant in the recycling line sketched by Figure 1.

Assume that one tray containing a unit amount of contaminant has entered the production line. Through the crashing, washing, melting and pelletizing processes, the contaminant diffuses into a certain number of pellets which is a unit grain of polystyrene of uniformed size to reproduce the new trays. By the experiment of pursuing some number of colored fragments of tray inserted in the line (Figure 2), we know that the temporal diffusion must be disregarded, although the contaminant spreads over a certain number, say n, of pellets. The n contaminated pellets must be packed in a single packing box.

By the experiment of measuring magnitude of the X-ray in each part of the resultant sheet formed by a heating and sheeting machine (Figure 3), we know that it is reasonable to assume that n is 6×10^3 and the n contaminated pellets have a unified amount of contaminant, namely, $1/[6 \times 10^3]$ unit.

By the addition of three boxes of virgin pellets, we have a collection of $N = 4.0 \times 10^7$ pellets which includes the n contaminant pellets. Through the blending and setting processes, these pellets are randomly divided into q sets which consist uniformly of $p = 100$ pellets and yield just one new tray. Consequently, we have $q = 4.0 \times 10^5$, i.e., $N = pq$. Diffusion of the n contaminated pellets over the q sets can be known by the using the theory of combinatorial probability. Introduce a random variable X which denotes the maximum number of contaminated pellets in a set through the q sets in these divisions. Of course, X takes a value k from 1 to p. The probability of $X = k$ which is denoted by $P(X = k)$ can exactly be computed. For $k = 1, 2, 3, 4$ and 5, its approximate value or its estimate of value is given by (3.6), (3.8), (3.11), (3.12) and (3.13), respectively.

Consider a case of $X = k$ which takes place at probability $P(X = k)$. Then the sets containing k contaminated pellets yield one recycling tray through the melting and sheeting processes. According to (3.3), the contaminant in a pellet diffuses in an area of sheet which corresponds to one tray at most with rate $\mathit{SDR} = 0.037264$. Therefore the recycling trays yielded by these sets are feared to contain at most contaminant of amount

$$\mathit{TDR} = \frac{1}{6.0 \times 10^3} \times 0.037264 \times k = \frac{k}{1.6101 \times 10^5}$$

unit. We then want to call this rate *the Total Diffusion Rate.*

The most favorable case is that $X = 1$. In this case, TDR takes its minimum $1/[1.6101 \times 10^5]$, but as seen by (3.6) the probability is very small. The probability that either $X = 2$ or $X = 3$ takes place reaches to higher than 0.999. In these cases we have $\mathit{TDR} = 1/[8.0505 \times 10^4]$ or $1/[5.3670 \times 10^4]$, respectively. The worst case with realistic occurring probability might be, in view of (3.13), the case of $X = 5$. In this case, we have $\mathit{TDR} = 1/[3.2202 \times 10^4]$. To the contrary, the theoretically worst case is that $X = p$ $(= 100)$. In such a case, TDR attains its minimum $1/[1.6101 \times 10^3]$, but as seen by (3.14), its occurring probability is extremely small.

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