TOTAL FACILITY CONSTRUCTION PLANNING PROBLEM

Hiroaki Ishii

Received April 3, 2019

Abstract

This paper consider the following construction problem with various type facilities, i.e., type 1: an emergency facility, type 2:semi-obnoxious one, type 3: welcome one, type 4: not so far but not so near one and type 5: supply center of school lunch. There are finite possible construction sites $F_1, F_2, \cdots, F_n$ in a rectangular area $U = \{a \leq x \leq b, c \leq y \leq d\}$ and construction cost c_{ij} depends on the construction site F_j and facility type $T_i, i = 1, 2, 3, 4, 5, j = 1, 2, \ldots, n$. We use A-distance and except construction cost, for T_i,weighted maximum distance from the emergency facility to the hospital via accident site $D_{1,j}$ to be minimized among $j = 1, 2, ..., n$ For $T_\ell, \ell = 2, 3, 4$. the minimal satisfaction degree μ_{ij} with respect to the membership function about A-distance from the facility site to be maximized among $j = 1, 2, \ldots, n$ and for T_5, the latest lunch delivery time t_{5j} of schools among possible construction site F_j should be minimized. Main problem is as follows.

Each facility $T_i, i = 1, 2, 3, 4, 5,$ is constructed at just one possible site so that total construction cost and weighted total sum $w_1 M_{1j_1} + w_2 M_{2j_2} + \cdots + w_5 M_{5j_5}$should be minimized where j_k is construction site of $T_k, k = 1, 2, 3, 4, 5$.. This problem becomes bi-criteria problem and we seek non-dominated solutions. Finally we discuss further research problems.

1 Introduction

There are huge amount of papers on facility location problems since Weber published his paper [?] and Hamacher et. al. [?] tried to classify them by using the similar codes to queuing and scheduling problems.

We considered many models on emergency facility location problem ([?], [?], [?], [?], [?]) and proposed an extended model of them. This paper is organized as follows. Section 2 formulates our model and defines non-dominated solutions. Section 3 proposes solution procedures to seek some non-dominated solutions. Section 4 summarizes results of our paper and discusses further research problems.

2 Problem formulation

We consider the following construction problem with various type facilities, i.e., type 1: an emergency facility, type 2:semi-obnoxious one (crematory, disposal center etc), type 3: welcome one (city hall), type 4: not so far but not so near one (shopping mall) and type 5: supply center of school lunch. There are finite possible construction sites $F_1, F_2, \cdots, F_n$ in a rectangular area $U = \{a \leq x \leq b, c \leq y \leq d\}$

and construction cost c_{ij} depends on the construction site F_j and facility type $T_i, i = 1,2,3,4,5, j = 1,2,\ldots,n$. We use A-distance and except construction cost for T_i,distance sum from the emergency facility to the hospital via accident site D_{1j} to be minimized among $j = 1,2,....,n$, For $T_\ell, \ell = 2,3,4$, the minimal satisfaction degree $\mu_{\ell j}$with respect to the membership function about A-distance from the facility site F_j to be maximized among $j = 1,2,...,n$,, and for T_5, latest lunch delivery time t_{5j} of schools among possible construction site F_j should be minimized. We denote D_{1j} as M_{1j}, $1 - \mu_{\ell j}, \ell = 2,3,,4$ and t_{5j} as M_{5j}. That is, corresponding to each facility type $k, k = 1,2,3,4,5$, we consider a sub-problem P_k and calculate $M_{kj}, j = 1,2,...,n$. First we review how to solve P_k and calculate $M_{kj}, j = 1,2,...,n$ in the next section. Main problem is as follows.

Each facility $T_i, i = 1,2,3,4,5$ is constructed at just one possible site so that total construction cost and weighted total sum $w_1 M_{1j_1} + w_2 M_{2j_2} + \cdots + w_5 M_{5j_5}$should be minimized where j_k is construction site of $T_k, k = 1,2,3,4,5$. This problem is formulated as the following bi-criteria problem P.

$P : minimize \Sigma_{j=1}^n \Sigma_{i=1}^5 c_{ij} x_{ij}, : minimize \Sigma_{j=1}^n \Sigma_{i=1}^5, w_i M_{ij} x_{ij}$
subject to $\Sigma_{i=1}^5 x_{ij} = 1, j = 1,2,...,n, \Sigma_{j=1}^n x_{ij} = 1, i = 1,2,3,4,5,$
$x_{ij} = 0$ or $1, i = 1,2,3,4,5, j = 1,2,...,n$

3 Solution Procedure

(**A distance**) ([**?**])There exists a set of directions $A = \{\alpha_1, \alpha_2, \cdots, \alpha_a\}$ where $\alpha_i, i = 1,2,....,a$ is an angle from x axis in an orthogonal coordinate and let $0 \le \alpha_1 < \alpha_2 < \cdots < \alpha_a < 180^o$. Hereafter if no confusion occurs, directions $\alpha_i, i = 1,2,...,a$ and angles $\alpha_i, i = 1,2,...,a$ are used as the same meaning. Directions $\alpha_j.\alpha_{j+1}$ are called neighboring where $\alpha_a, \alpha_1$are also called neighboring, that is, α_{a+1} is interpreted as α_1.Further A line, a half line and a line segment are called A-directional (or A-oriented) if their directions are ones of $\alpha_i, i = 1,2,...,a$. Then A distance between two points$(p^1, p^2) \in R^2$ are defined as follows

$$d_A(p^1,p^2) = \begin{cases} d_2(p^1,p^2) & \text{if direction } \overline{p^1p^2} \text{ is A-oriented} \\ min\{d_A(p^1,p^3) + d_A(p^3,p^2) | p^3 \in R^2\} & \text{Otherwise} \end{cases} \quad (1)$$

where $d_2(p^1,p^2)$is the Euclidian distance between (p^1,p^2). That is, according to the results in [**?**], when
α_j <an angle of the line connecting demand point i with the facility site $(x,y) < \alpha_{j+1}$,

$d_i = M_1|m_2(p_i - x) - (q_i - y)| + M_2|m_1(p_i - x) - (q_i - y)|$ where $m_1 = max(tan\alpha_j, tan\alpha_{j+1}), m_2 = min(tan\alpha_j, tan\alpha_{j+1}), M_1 = \frac{\sqrt{1+m_1^2}}{m_1-m_2}, M_2 = \frac{\sqrt{1+m_2^2}}{m_1-m_2}$. If either α_j or α_{j+1} is 90^o, then we interpret $M_1 = \lim_{m_1 \to \infty} \frac{\sqrt{1+m_1^2}}{m_1-m_2} = 1, M_2|m_1(p_i-x)-(q_i-y)| = \lim_{m_1\to\infty} \frac{\sqrt{1+m_2^2}}{m_1-m_2}|m_1(p_i-x)-(q_i-y)| = \sqrt{1+m_2^2}|p_i - x|$.

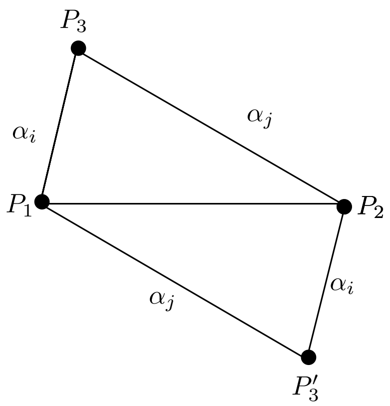

Fig.1 A-distance between P^1 and P^2

First we show the facility location of type 1. That is, an emergency facility problem P_1 as follows: There exist m hospitals $H_1, H_2, \cdots, H_m$. If an accident occurs, the ambulance cars in the facility site p rushes to the scene of accident and bring the injured persons to the nearest hospitals as soon as possible. Demand points (possible accident occurrence points) are distributed uniformly in U in a rectangular area. Let $S(Q)$ denotes the nearest hospital to the point $Q \in U$. Then the distance sum from p is $R(p, Q) = d_A(p, Q) + d_A(Q, S(Q))$ and $R(p) = max\{R(p, Q)|Q \in U\}$ should be minimized among $p \in F_1.F_2, \cdots, F_n$. $d_A(Q, S(Q))$ is calculated as below using Voronoi diagram with respect to hospitals $H_1, H_2, \cdots, H_m$.

(**Voronoi diagram**)

For a set of s points $V_1, V_2, \cdots, V_s$, Voronoi polygon $V_A(V_i)$ on point V_i with respect to A-distance on X is defined as follows:

$$V_A(V_i) = \cap_{j \neq i}\{Q|d_A(Q, V_i) \leq d_A(Q, V_j), Q \in X\}$$

The set of all Voronoi polygons for the points in V is a partition of some region on a plane X. Edge of Voronoi polygon is called Voronoi edge. Then we construct Voronoi diagram $VD_A(\mathbf{H})$ with respect to the set of hospital points $\mathbf{H} = H_1, H_2, \cdots, H_m$ and A-distance on the area X. To construct Voronoi diagram is done in $O(mlogm)$ computational time ([**?**]). Figure 2 illustrates Voronoi diagram in this case. According to the Theorem in [[**?**]], maximizer among Q with respect to $R(p)$ is one of the following points (a),(b) in Theorem 1.

Theorem 1.

(a) The intersection points of boundary of U and the Voronoi diagram

(b) Vertices of Voronoi diagram.

Let these points of Theorem 1 be $p_1^1, p_2^1, \cdots, p_{n_i}^1$

Then we can calculate $R(F_j), j = 1, 2, ..., n$ as follows.

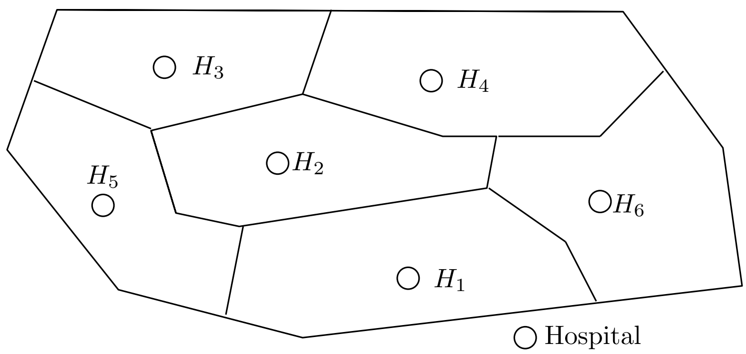

Fig.2 Voronoi diagram with respect to hospitals $H_1, H_2, H_3, H_4, H_5, H_6$

$R(F_j) = Max\{R(F_j, p_k^1)|k = 1, 2, ..., n_1\}, j = 1, 2, ..., n$

The optimal solution of P_1 is a minimizer of $min\{R(F_j), j = 1, 2, ..., n\}$. Next for type 2, 3, 4 facility, we consider A-distance $d_A(i, p)$ from demand point $p = (x, y) \in U$ to the facility site $F_i = (p_i, q_i)$ which is calculated as follows:

From the above results, when α <an angle of the line connecting demand point (x, y) with the facility site $F_i < \alpha_{i+1}$, $d_A(i, p) = M_1|m_2(p_i - x) - (q_i - y)| + M_2|m_1(p_i - x) - (q_i - y)|$ where $m_1 = max(tan\alpha_j, tan\alpha_{j+1}), m_2 = min(tan\alpha_j, tan\alpha_{j+1}), M_1 = \frac{\sqrt{1+m_1^2}}{m_1-m_2}, M_2 = \frac{\sqrt{1+m_2^2}}{m_1-m_2}$. Therefore A-distance $d_A(i, p)$ from demand points to facility site $F_i = min\{z_1^i, z_2^i, \cdots, z_a^i\}$ where $z_j^i, j = 1, 2, ..., a$ is the optimal value of following problem Z_j:

$Z_j : Minimizez$

subject to $M_1|m_2(p_i - x) - (q_i - y)| + M_2|m_1(p_i - x) - (q_i - y)| < z$

$m_2|x - p_i| \leq |y - q_i| \leq m_1|x - p_i|, (x, y) \in U$

This is a linear programming problem basically. For T_2, we consider the following membership function.

$$\mu_2(p) = \begin{cases} 0 & (d_A(i,p) \leq a_i) \\ \frac{d_A(i,p)-a_i}{b_i-a_i} & (a_i \leq d_A(i,p) \leq b_i) \\ 1 & (d_A(i,p) \geq b_i) \end{cases} \tag{2}$$

where $a_i < b_i$and subproblem is to maximize $\mu_{2i} = min\{\mu_{2i}(p)|p \in U\}$ with respect to $F_i, i = 1, 2, ..., n$. Optimal solution is a maximizer of μ_{2i}. For T_3, we consider the following membership function.

$$\mu_{3i}(p) = \begin{cases} 0 & (d_A(i,p) \geq e_i) \\ 1 - \frac{d_A(i,p)-c_i}{e_i-c_i} & (c_i \leq d_A(i,p) \leq e_i) \\ 1 & (d_A(i,p) \leq c_i) \end{cases} \tag{3}$$

where $c_i < e_i$ and subproblem is to maximize $\mu_{3i} = min\{\mu_{3i}(p)|p \in U\}$ with respect to $F_i, i = 1, 2, ..., n$. Optimal solution is a maxmizer of μ_{3i}. For T_4, we consider the following membership function.

$$\mu_{4i}(p) = \begin{cases} 0 & (d_A(i,p) \leq a_i) \\ \frac{d_A(i,p)-a_i}{b_i-a_i} & (a_i \leq d_A(i,p) \leq b_i) \\ 1 & (b_i \leq d_A(i,p) \leq c_i) \\ 1 - \frac{d_A(i,p)-c_i}{e_i-c_i} & (c_i \leq d_A(i,p) \leq e_i) \\ 0 & (d_A(i,p) \geq e_i) \end{cases} \tag{4}$$

where $a_i < b_i < c_i < e_i$ and subproblem is to maximize $\mu_{4i} = min\{\mu_{4i}(p)|p \in U\}$ with respect to $F_i, i = 1, 2, ..., n$. Optimal solution is a maximizer of μ_{4i}. Finally consider the supply center of school lunch T_5 given as follows. There are s schools $S_1.S_2, \cdots, S_s$ in urban area U. We consider the construction site of new supply center providing lunch for these schools among n possible sites $F_1, F_2, \cdots, F_n$ The trader delivers ingredients to the supply center every morning. After receiving these ingredients,the supply center starts to make lunch. Lunch for all schools should be ready on delivery time. The delivery cars must deliver lunch to be in lunch time of each school. For that purpose, we divide schools into groups corresponding to r delivery cars. We choose the best site of the center by minimizing the latest delivery time of lunch among all schools. First we calculate A-distances $d_A(i, j)$ from each possible site $F_j, i = 1, 2, ..., n$ to each school $.S_j, j = 1, 2, ..., s$. Sorting $d_A(i, j)$ for each F_i let the result be $d_A(i, i(1) \le d_A(i, i(2)) \le \cdots d_A(i, I(s))$. Then for eacf F_i, we divide schools into r trucks as follows.Choose r longest distances and assign school $S_{i(is-t+1)} todeliverycars$TR(t),t=1,2,...,r.$Thenletbe \tilde{d}_A(, k) = 2d_A(i, i(k)), k = 1, 2, ..., s - r$. Next we divide schools to r group by the following steps.

Step 1:Set $B_{i(t)} = d_A(i, i(s-t+1)) + \tilde{d}_A(i, t), t = 1, 2, ..., r-1, B_{i(r)} = d_A(i, i(s-r+1)) + \tilde{d}_A(i, i(s-r)), , k = s - r$ and $G(t) = S_{i(s-t+1)}, t = 1, 2, ..., r-1, G(r) = \{S_{i(s-r+1}, S_{i(s-r)}\}$

Step 2: Let $k = k+!$. if $k = 0$,terminates .Otherwise go to Step3.

Step 3: Let $B(s) \leftarrow min\{B_{i(u)}|u = 1, 2, ..., r\}$ and its minimizer be $t(k)$. Then $B_{i(t(k))} = B_{i(t(k))} + \tilde{d}_A(i, i(k)), G(t(k)) = G(t(k)) \cup S_{i(k)}$. Return to Step2.

Note that final $B_{i(u)}$ divided by the standard speed describes the total delivery time using TR_u to group of schools $G(u), u = 1, 2, \cdots, r$. Though heuristic, the above dividing method tries to make burden even, that is, minimizing the maximum burden among delivery cars. Let the maximum burden using the above dividing method for candidate site $F_i$be $BM(i)$. Here we assume the staring time of making lunch is and so finishing time preparing lunch is also fixed and so minimizer of $BM(i), i = 1, 2, ..., n$ is an optimal solution of P_5.Here $M_{5i} = \frac{BM(i)}{SP} + CT$ where CT is a starting time to making school lunch and SP is a standard speed of delivery truck. Based on the above discussion, we have the main problem P.

$P : minimize \Sigma_{j=1}^{n} \Sigma_{i=1}^{5} c_{ij} x_{ij}, : minimize \Sigma_{j=1}^{n} \Sigma_{i=1}^{5}, w_i M_{ij} x_{ij}$

subject to $\Sigma_{i=1}^{5} x_{ij} = 1, j = 1, 2, ..., n, \Sigma_{j=1}^{n} x_{ij} = 1, i = 1, 2, 3, 4, 5,$

$x_{ij} = 0 or 1, i = 1, 2, 3, 4, 5, j = 1, 2, ..., n$ But this problem has bi-criteria and so we define non-dominated solution now,

(**Non-dominated solution**)

For two solutions, $\mathbf{F}^1 = (F_{j_1^1}, F_{j_2^1}, F_{j_3^1}, F_{j_4^1}, F_{j_5^1},)$, that is,$(x_{ij_i^1} = 1, i = 1, 2, ..., 5, other x_{ij} = 0)$ and $\mathbf{F}^2 = (F_{j_1^2}, F_{j_2^2}, F_{j_3^2}, F_{j_4^2}, F_{j_5^2},)$, that is, $(x_{ij_i^2} = 1, i = 1, 2, ..., 5, other x_{ij} = 0)$ if

$\Sigma_{j=1}^{n} \Sigma_{i=1}^{5} c_{ij} x_{j_j^1} \le \Sigma_{j=1}^{n} \Sigma_{i=1}^{5} c_{ij} x_{j_j^2}, \Sigma_{j=1}^{n} \Sigma_{i=1}^{5} W_i M_{ij} x_{j_j^1} \le \Sigma_{j=1}^{n} \Sigma_{i=1}^{5} W_i M_{ij} x_{j_j^2}$ and at least one inequality holds strictly inequality, then we call solution $\mathbf{F^1}$ dominates $\mathbf{F^2}$. If there exists no solution dominates $\mathbf{F}$, $\mathbf{F}$ is called non-dominated solution.

We seek some non-dominated solutions. First non-dominated solution is obtained from the solution minimizing the total construction cost, that is, optimal solution of the following assignment problem AP:

minimize $\Sigma_{j=1}^{n} \Sigma_{i=1}^{5} c_{ij} x_{ij}$

subject to $\Sigma_{i=1}^{5} x_{ij} = 1, j = 1, 2, ..., n, \Sigma_{j=1}^{n} x_{ij} = 1, i = 1, 2, 3, 4, 5,$

$x_{ij} = 0 or 1, i = 1, 2, 3, 4, 5, j = 1, 2, ..., n$

This problem is a special transportation problem with 5 supply nodes and n demand nodes where upper supply quantity is 1 and each demand quantity is at most 1. Unit transportation cost is 1. Usually dummy n-5 supply nodes with big transportation cost to demand nodes and only one possible supply quantity, Or among $O(n^5)$ solutions, we find an optimal solution, that is minimizer of $\Sigma_{j=1}^{n}\Sigma_{i=1}^{5}c_{ij}x_{ij}$ is a non-dominated solution.Similarly an optimal solution of the following another assignment problem AM.

AM: minimize $\Sigma_{j=1}^{n}\Sigma_{i=1}^{5}, w_i M_{ij} x_{ij}$

subject to $\Sigma_{i=1}^{5}x_{ij} = 1, j = 1, 2, ..., n, \Sigma_{j=1}^{n}x_{ij} = 1, i = 1, 2, 3, 4, 5,$

$x_{ij} = 0 or 1, i = 1, 2, 3, 4, 5, j = 1, 2, ..., n$

Another one is given as follows:

Let $WM_{ij} = w_i M_{ij}, i = 1, 2, ..., 5, j = 1, 2, ..., n$ and sort them for each i=1,2,...,5. Then results be set $\mathbf{WM^i} = \{WM_i(i(1)), WM_i(i(2)), WM_i(i(3)), \cdots, WM_i(i(n_i))\}$ where $WM(i(i(1) < WM(i(i(2) < WM(i(i3) < \cdots < WM(i(i(n_i)), i = 1, 2, ..., 5, n$ is the number of different ones. We choose cheaper disjoint five construction sites from the list $\mathbf{WM^i} i = 1, 2, ..., 5$,and resulting one is a non-dominated solution. Of course, if there exists the same value $WM_{i_\ell}(i(k)) = WM_{i_\varsigma}(i(\tau))$, we prefer cheaper construction cost one.

4 Conclusion

We have a total construction planning model of various type facilities. Of course many other type facilities should be considered and also the case of given facilities are important to total planning including some case closing old ones. Further in a financial aspect, some facilities will be establish in a same place and in turn, in urban area, many barriers exist where inside we cannot pass and so make detours. Therefore more realistic situation should considered when we total planning. Further scenario analysis that considers near future situation including enviromental aspects.

References

[1] W. H. Hamacher and N. Stefan, Classification of location models, Location Science, Vol.6, pp.229-242, 1998

[2] H. C. Hsia, H. Ishii and K. Y. Yeh, Ambulance service facility problem , Journal of the Operations Research Society of Japan, Vol.52, pp.338-354, 2009.

[3] H. Ishii, Y.L. Lee and K. Y. Yeh, Facility location problem with preference of candidate sites, Fuzzy Sets and Systems, Vol.158, pp.1922-1930, 2007.

[4] H. Ishii, Y.L. Lee and K. Y. Yeh, Mathematical ranking method for facility location problem, International Journal of Association for Management Systems, Vol. 4, pp.73-76, 2012.

[5] H. Ishii and Y.L. Lee, Mathematical ranking method for emergency facility location problem with block-wisely defferent accident occurrence probabilities, Procdia Computer Science Vol.22, pp.1065-1072, 2000.

[6] H. Ishii and Y. Sasaki, Facility Facility location Problem for supply center of school lunch, Scienticae

Mathematicae Japonicae Vol.30, online, 2005.

[7] T.Matutomi and H. Ishii, Minimax location problem wit A-distance, European Journal of Operational Research, Vol. 41, pp181-195, 2008.

[8] A, Weber, *Ü ber Den Stand Der Industrien 1. Teil : Reine Theorie Des standortes.* 1909.

[9] P.Widnayer et. al., On some distance problems in fixed orientations, SIAM J.. on Computing, Vol.16, pp.728-746, 1987.

Communicated by *Koyu Uematsu*

Email: ishiroaki@yahoo.co.jp