ON OPERATION APPROACHES OF α-OPEN SETS AND SEMI-OPEN SETS

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Abstract. The aim of the present paper is to introduce and study the concept of operation-α-open sets, operation-semi-open sets and some operation-closures by using operations from $SO(X,\tau)$ into $P(X)$. We obtain some operation-closure formulas. As corollaries, well known formulas on α-closures and semi-closures are obtained.

1. Introduction

Throughout this paper, (X,τ) and (Y,σ) represent nonempty topological spaces on which no separation axioms are assumed, unless otherwise mentioned. A subset A of X is *semi-open* [10] in (X,τ) if there exists an open set O such that $O \subset A \subset Cl(O)$. It is well known that A is semi-open in (X,τ) if and only if $A \subset Cl(Int(A))$ holds. The complement of a semi-open set is called a *semi-closed* set. Let $SO(X,\tau)$ be the collection of all semi-open sets in (X,τ). For a subset B of X, let $sCl(B) := \bigcap\{F \mid B \subset F, X \setminus F \in SO(X,\tau)\}$ and $sInt(B) := \{x \in X \mid$ there exists a subset $U \in SO(X,\tau)$ such that $x \in U$ and $U \subset B\}$. It is proved that $sCl(B) = \{x \in X \mid U \cap B \neq \emptyset$ for any semi-open set U containing $x\}$. A subset A of X is said to be *preopen* [13] in (X,τ) if $A \subset Int(Cl(A))$ holds. We denote by $PO(X,\tau)$ the set of all preopen sets in (X,τ). Kasahara [7] defined and investigated the concept of operations on τ, i.e., the function $\alpha : \tau \to P(X)$ such that $U \subset U^{\alpha}$ for each $U \in \tau$, where U^{α} denotes the value $\alpha(U)$ of α at U and $P(X)$ the power set of X. He generalized the notion of compactness with help of operations. After the work of Kasahara, Janković [5] defined the concept of operation-closures and investigated some properties of functions with operation-closed graphs. In 1991, one of the present authors, Ogata, defined and investigated the concept of *operation-open sets* [15], say γ-open sets; he used the symbol $\gamma : \tau \to P(X)$ as an operation on τ. He avoided a confussion between the concept of α-sets [14] (sometimes α-open sets) and one of "α"-open set (where the latter symbol "α" is an operation in the sense of Kasahara [7]). Let $\gamma : \tau \to P(X)$ be an operation on τ. A nonempty subset A of X is said to be *γ-open* (*in the sense of Ogata*) [15] if for each point $x \in A$, there exists an open set U containing x such that $U^{\gamma} \subset A$. Recently, Krishnan et. al.[9] (resp. Tran Van, Dang Xuan et. al. [17]) investigated operations on the family $SO(X,\tau)$ (resp. $PO(X,\tau)$) for a topological space (X,τ).

In the present paper, we shall introduce the concept of an alternative operation-open sets, i.e., γ_s-α-open sets (cf. Definition 3.2(i)) using the concept of operations on $SO(X,\tau)$ due to [9, Definition 2.4]. Let $\gamma_s : SO(X,\tau) \to P(X)$ be a function from $SO(X,\tau)$ into $P(X)$ satisfying the following property: $V \subset V^{\gamma_s}$ for any $V \in SO(X,\tau)$, where V^{γ_s} denotes the value $\gamma_s(V)$ of γ_s at V. By [9, Definition 2.4], the function γ_s is an *operation on* $SO(X,\tau)$. In Section 2, we introduce and investigate the concepts of operations (cf. Definition 2.1) on

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a family, say ξ, of subsets of a topological space (X,τ) such that $\tau \subset \xi$; this operation is denoted by $\gamma_\xi : \xi \to P(X)$. Furthermore, we introduce the concept of an operaion-open sets and related concepts, say γ_ξ-open sets (cf. Definition 2.4 etc. below), and investigate some fundamental properties of them (cf. Theorem 2.10). In Section 3, especially we consider the case where $\xi := SO(X,\tau)$ and introduce the concept of γ_s-α-open sets (cf. Definition 3.2); and let $\gamma_s\tau^\alpha$ be the set of all γ_s-α-open sets in (X,τ). We also investigate a formula on $\gamma_s\tau^\alpha$-closures of subsets in (X,τ) (cf. Theorem 3.7(iii)). As corollary we get the well known formula [1] on α-closure of a subset (cf. Corollary 3.8). The concept of α-sets (sometimes, α-open sets) introducedd by Njåstad [14]; a subset A of X is said to be α-open in (X,τ) if $A \subset Int(Cl(Int(A)))$ holds in (X,τ). In Section 4, we investigate some properties on operation semi-open sets and operaton semi-closures $\gamma_s SO(X,\tau)$-$Cl(B)$ of a subset B of (X,τ).

2. Operations on a family and some operation-open sets

Wirst we recall a concept of operations on a family ξ of subsets of a space (X,τ) such that $\tau \subset \xi$. Kasahara [7] and Ogata [16] introduced and investigated a general theory on *operations on* τ, i.e., $\xi := \tau$. Throughout this section, for a family ξ we assume that $\tau \subset \xi$ holds.

Definition 2.1. (i) (cf. Kasahara [7], Ogata [16]) An *operation on* ξ, say γ_ξ, is a mapping from ξ to the power set $P(X)$ of X such that $U \subset U^{\gamma_\xi}$ for each $U \in \xi$, where U^{γ_ξ} denotes the value $\gamma_\xi(U)$ of γ_ξ at U and $\tau \subset \xi$. The operation on ξ is denoted by $\gamma_\xi : \xi \to P(X)$.

(ii) The *restriction of* $\gamma_\xi : \xi \to P(X)$ *to* τ, say $\gamma_\xi|\tau : \tau \to P(X)$, is well defined by $(\gamma_\xi|\tau)(V) := V^{\gamma_\xi}$ for every $V \in \tau$.

Remark 2.2. (i) The following function "Cl" : $P(X) \to P(X)$ is well defined as follows, i.e., "Cl"$(V) := Cl(V)$ for every subset $V \in P(X)$, where $Cl(E)$ is the closure of a subset E in a topological space (X,τ). Since $V \subset Cl(V)$ holds for every $V \in P(X)$, the function "Cl" : $P(X) \to P(X)$ is an example of *operations on* $P(X)$ (in the sense of Definition 2.1(i); and also for any subfamily ξ of $P(X)$, the restriction to ξ, say "Cl"$|\xi : \xi \to P(X)$, can be an *operation on* ξ if $\tau \subset \xi$. This operation is called the *closure operation on* ξ (cf. Remark 2.11(ii) below).

(ii) The following function "Int" : $P(X) \to P(X)$ is well defined as follows, i.e., "Int"$(U) := Int(U)$ for every subset $U \in P(X)$, where $Int(E)$ is the interior of a subset E in a topological space (X,τ). If $U \subset Int(U)$ holds (i.e., $U = Int(U)$) in (X,τ) for every $U \in P(X)$, then the function "Int" : $P(X) \to P(X)$ can be called as *operation on* $P(X)$. However, there may be many topological spaces (X,τ) such that $U \not\subset Int(U)$ for some subset $U \in P(X)$ (cf. (iii) below); and so for such space (X,τ), the function "Int" : $P(X) \to P(X)$ is not an *operation on* $P(X)$. We see easily that the restriction of "Int" to τ, say "Int"$|\tau : \tau \to P(X)$, is an *operation on* τ, because $U = Int(U)$ holds for every subset $U \in \tau$ (cf. Definition 2.1(i)).

(iii) For the digital line $(\mathbb{Z},\kappa)$ (eg. [4]), the function "Int" : $SO(\mathbb{Z},\kappa) \to P(\mathbb{Z})$ is not an *operation on* $SO(\mathbb{Z},\kappa)$. Indeed, for a subset $\{2m, 2m+1\} \in SO(\mathbb{Z},\kappa)$, "$Int$"$(\{2m, 2m+1\}) = Int(\{2m, 2m+1\}) = \{2m+1\}$ and so $\{2m, 2m+1\} \not\subset$ "Int"$(\{2m, 2m+1\})$. The restriction of "Int" to κ, i.e., "Int"$|\kappa : \kappa \to P(\mathbb{Z})$ is an operation on κ (cf. (ii) above) (cf. [12]).

Remark 2.3. (i) For the case where $\xi := \tau$ in Definition 2.1(i), γ_ξ is identical to the *operation on* τ in the sense of [7] (e.g. [15]).

(ii) Krishnan et al. [9] introduced an operation on $SO(X,\tau)$, say $\gamma_s : SO(X,\tau) \to P(X)$. This operation γ_s is an *operation on* $\xi := SO(X,\tau)$ in the sense of Definition 2.1(i) above, because of $\tau \subset SO(X,\tau)$. Namely, we have the equality of two operations: $\gamma_{SO(X,\tau)} = \gamma_s : SO(X,\tau) \to P(X)$.

(iii) Tran An and Dang Xuan et al. [17] introduced an operation on $PO(X,\tau)$, say $\gamma_p : PO(X,\tau) \to P(X)$. This operation $\gamma_p : PO(X,\tau) \to P(X)$ is an *operation on* $\xi := PO(X,\tau)$ in the sense of Definition 2.1, because of $\tau \subset PO(X,\tau)$. Namely, we have the equality of two operations: $\gamma_{PO(X,\tau)} = \gamma_p : PO(X,\tau) \to P(X)$.

Secondly, we shall recall analogous fundamental concepts on operation $\gamma_\xi : \xi \to P(X)$ as follows (we note that $\tau \subset \xi$).

Definition 2.4. (i) Let τ_{γ_ξ} denotes the following collection of subsets of (X,τ):
• $\tau_{\gamma_\xi} := \{\emptyset\} \cup \{A|$ there exists a set $U \in \tau$ such that $x \in U$ and $U^{\gamma_\xi} \subset A$ for each point $x \in A\}$.
(ii) A subset A is said to be γ_ξ*-open* in (X,τ), if $A \in \tau_{\gamma_\xi}$. Namely, a subset A of (X,τ) is γ_ξ-open in (X,τ) if and only if, $A = \emptyset$ or, for each point $x \in A$ there exists an open set U containing x such that $U^{\gamma_\xi} \subset A$.
(ii)′ A subset A is said to be γ_ξ*-closed* in (X,τ), if $X \setminus A \in \tau_{\gamma_\xi}$, (i.e., $X \setminus A$ is γ_ξ-open in (X,τ) in the sense of (ii) above).

Using the concept of γ_ξ-open sets above, we define the operation-regularity (resp. operation-openness) of operation on ξ as follows:

Definition 2.5. Let $\gamma_\xi : \xi \to P(X)$ be an operation on ξ with $\tau \subset \xi$.
(i) $\gamma_\xi : \xi \to P(X)$ is said to be *regular on* ξ , if for each open neighbourhoods U and V of each $x \in X$, there exists an open neighbourhood W of x such that $W^{\gamma_\xi} \subset U^{\gamma_\xi} \cap V^{\gamma_\xi}$.
(ii) $\gamma_\xi : \xi \to P(X)$ is called to be *open on* ξ, if for each point $x \in X$ and any open neighbourhood U of x, there exists a γ_ξ-open set V containing x such that $V \subset U^{\gamma_\xi}$.

Definition 2.6. For a subset B of (X,τ), we define the following three subsets of (X,τ):
• $Cl_{\gamma_\xi}(B) := \{x \in X|\ U^{\gamma_\xi} \cap B \neq \emptyset$ for any open neighbourhood U of $x\}$;
• $Int_{\gamma_\xi}(B) := \{x \in X|$ there exists an open neighbourhood U of x such that $U^{\gamma^\xi} \subset B\}$;
• τ_{γ_ξ}-$Cl(B) := \bigcap\{F|\ B \subset F, X \setminus F \in \tau_{\gamma_\xi}\}$.

Remark 2.7. It is evident that for the subset $\emptyset$ (resp. X), $Cl_{\gamma_\xi}(\emptyset) = \tau_{\gamma_\xi}$-$Cl(\emptyset) = \emptyset$ (resp. $Cl_{\gamma_\xi}(X) = \tau_{\gamma_\xi}$-$Cl(X) = X$) hold, because $\tau \subset \xi$ and $\emptyset \in \tau_{\gamma_\xi}$ and $X \in \tau_{\gamma_s}$; furthermore, if $A \subset B$ in (X,τ), then $Cl_{\gamma_\xi}(A) \subset Cl_{\gamma_\xi}(B), Int_{\gamma_\xi}(A) \subset Int_{\gamma_\xi}(B)$ and τ_{γ_ξ}-$Cl(A) \subset \tau_{\gamma_\xi}$-$Cl(B)$ holds.

Definition 2.8. Let $\gamma_\xi : \xi \to P(X)$ be an operation, where $\tau \subset \xi$, and B subset of a topological space (X,τ).
(i) (X,τ) is said to be a γ_ξ*-regular space*, if for each point $x \in X$ and every open neighbourhood U of x there exists an open neighbourhood W of x such that $W^{\gamma_\xi} \subset U$.
(ii) B is called γ_ξ*-closed in the sense of Janković* if $Cl_{\gamma_\xi}(B) \subset B$, i.e., $B = Cl_{\gamma_\xi}(B)$ holds (cf. [5]).

Finally, the following Theorem 2.9 (resp. Theorem 2.10) is proved by definitions (resp. Theorem 2.9 and the corresponding theorems in [15]); hence the proofs are omitted.

Theorem 2.9. *Let $\gamma_\xi : \xi \to P(X)$ be an operation on ξ, where $\tau \subset \xi$.*
(i) *If a subset A is γ_ξ-open in (X,τ) if and only if A is $\gamma_\xi|\tau$-open in (X,τ) in the sense of* [15].
(ii) *For a subset B of (X,τ), $Cl_{\gamma_\xi}(B) = Cl_{\gamma_\xi|\tau}(B)$ and $Int_{\gamma_\xi}(B) = Int_{\gamma_\xi|\tau}(B)$ hold, where $Cl_{\gamma_\xi|\tau}(B), Int_{\gamma_\xi|\tau}(B)$ are defined by* [15].
(iii) *The operation $\gamma_\xi : \xi \to P(X)$ is regular on ξ if and only if $\gamma_\xi|\tau : \tau \to P(X)$ is regular in the sense of* [15].
(iv) *The operation $\gamma_\xi : \xi \to P(X)$ is open on ξ if and only if $\gamma_\xi|\tau : \tau \to P(X)$ is open in the sense of* [15].

(v) *A space* (X,τ) *is* γ_ξ*-regular if and only if* (X,τ) *is* $\gamma_\xi|\tau$*-regular in the sense of* [15]. □

Theorem 2.10. *Let* $\gamma_\xi : \xi \to P(X)$ *be an operation on* ξ*, where* $\tau \subset \xi$*. Then the following properties hold in* (X,τ)*.*

(i) (cf. [15, p.176]) *Every* γ_ξ*-open set is open in* (X,τ)*, i.e.,* $\tau_{\gamma_\xi} \subset \tau$*.*

(ii) (cf. [15, Proposition 2.3]) *The union of every family of* γ_ξ*-open sets is* γ_ξ*-open.*

(iii) (cf. [15, Proposition 2.9(i)(ii)]) *If* $\gamma_\xi : \xi \to P(X)$ *is regular on* ξ*, then the intersection of every finite family of* γ_ξ*-open sets is* γ_ξ*-open and hence* τ_{γ_ξ} *is a topology of* X*.*

(iv) (cf. [15, (3.3)]) *For a point* $x \in X$ *and a subset* A *of* $(X,\tau), x \in \tau_{\gamma_\xi}\text{-}Cl(A)$ *if and only if* $V \cap A \neq \emptyset$ *for any set* $V \in \tau_{\gamma_\xi}$ *such that* $x \in V$*.*

(v) (cf. [15, (3.4)]) *For a subset* A *of* (X,τ)*,* $A \subset Cl(A) \subset Cl_{\gamma_\xi}(A) \subset \tau_{\gamma_\xi}\text{-}Cl(A)$ *hold.*

(vi) (cf. [15, Theorem 3.6(i)]) *The subset* $Cl_{\gamma_\xi}(A)$ *is closed in* (X,τ) *and* $Int_{\gamma_\xi}(A)$ *is open in* (X,τ)*, where* A *is a subset of* (X,τ)*.*

(vii) $X \setminus Cl_{\gamma_\xi}(A) = Int_{\gamma_\xi}(X \setminus A)$ *holds.*

(viii) (cf. [15, Theorem 3.7]) *The following properties are equivalent for a subset* A *of* (X,τ) *:*

(1) A *is* γ_ξ*-open;*

(2) $X \setminus A$ *is* γ_ξ*-closed in the sense of Janković* (i.e., $Cl_{\gamma_\xi}(X \setminus A) = X \setminus A$ *holds);*

(3) $\tau_{\gamma_\xi}\text{-}Cl(X \setminus A) = X \setminus A$ *holds;*

(4) $Int_{\gamma_\xi}(A) = A$ *holds.*

(ix) (cf. [15, Theorem 3.6(iii)]),(v) (viii) above) *If* $\gamma_\xi : \xi \to P(X)$ *is open on* ξ*, then the following properties on a subset* A *of* (X,τ) *hold.*

(a) $Cl_{\gamma_\xi}(A) = \tau_{\gamma_\xi}\text{-}Cl(A)$ *and* $Cl_{\gamma_\xi}(Cl_{\gamma_\xi}(A)) = Cl_{\gamma_\xi}(A)$ *hold.*

(b) $Cl_{\gamma_\xi}(A)$ *is* γ_ξ*-closed in the sense of Janković* (cf. Definition 2.8(ii)).

(c) $Int_{\gamma_\xi}(A)$ *is* γ_ξ*-open.*

(x) (cf. [15, Lemma 3.10]) *If* $\gamma_\xi : \xi \to P(X)$ *is regular on* ξ*, then* $Cl_{\gamma_\xi}(A \cup B) = Cl_{\gamma_\xi}(A) \cup Cl_{\gamma_\xi}(B)$ *holds for any subsets* A *and* B *of* X*.*

(xi) (cf. [15, Corollary 3.11], (v) (iv) above) *If* $\gamma_\xi : \xi \to P(X)$ *is regular and open on* ξ*, then the operation* $Cl_{\gamma_\xi}(\bullet)$ *satisfies the Kuratowski closure axiom. Namely, for any subsets* A *and* B *of* $(X,\tau), A \subset Cl_{\gamma_\xi}(A), Cl_{\gamma_\xi}(A \cup B) = Cl_{\gamma_\xi}(A) \cup Cl_{\gamma_\xi}(B)$*,* $Cl_{\gamma_\xi}(Cl_{\gamma_\xi}(A)) = Cl_{\gamma_\xi}(A)$ *and* $Cl_{\gamma_\xi}(\emptyset) = \emptyset$ *and* $Cl_{\gamma_\xi}(X) = X$ *hold.*

(xii) (a)(cf. [15, Theorem 3.6(ii)], (v) above) *If* (X,τ) *is a* γ_ξ*-regular space, then* $Cl_{\gamma_\xi}(A) = Cl(A)$ *holds for a subset* A *of* (X,τ)*.*

(b) (cf. [15, Proposition 2.4], Theorem 2.9(v)(i) above) *A space* (X,τ) *is a* γ_ξ*-regular space if and only if* $\tau = \tau_{\gamma_\xi}$ *holds.* □

Remark 2.11. (i) The *identity operation on* ξ, say "*id*" : $\xi \to P(X)$, is well defined by $U^{\text{"}id\text{"}} = U$, where $U \in \xi$. Then, a subset A of (X,τ) is "*id*"-open in (X,τ) if and only if A is open in (X,τ), i.e., $\tau_{\text{"}id\text{"}} = \tau$ holds. Then, it is shown that $Cl_{\text{"}id\text{"}}(A) = Cl(A), Int_{\text{"}id\text{"}}(A) = Int(A)$ and every topological space (X,τ) is "*id*"-regular (cf. Theorem 2.10(xii)(b)). This operation "*id*":$\xi \to P(X)$ is an example of the open operation (cf. Definition 2.5(ii)).

(ii) The *closure operation* on ξ, "Cl" : $\xi \to P(X)$, is well defined by $U^{\text{"}Cl\text{"}} = Cl(U)$, where $U \in \xi$. Then, a subset A of (X,τ) is "Cl"-open in (X,τ) if and only if A is θ-open in (X,τ). That is, $\tau_{\text{"}Cl\text{"}} = \tau_{\text{"}Cl\text{"}|\tau} = \tau_\theta$ hold, where τ_θ is the collection of all θ-open sets of (X,τ) [18],(cf. Theorem 2.9(i)). Then, it is shown that $Cl_{\text{"}Cl\text{"}}(A) = Cl_\theta(A)$ and $Int_{\text{"}Cl\text{"}}(A) = Int_\theta(A)$ hold (cf. Theorem 2.9(ii), [18]) and (X,τ) is "Cl"-regular if and only if (X,τ) is regular (i.e., $\tau = \tau_\theta$). We note that, in general, $\tau_{\text{"}Cl\text{"}}\text{-}Cl(A) = \bigcap\{V|\ A \subset V, X \setminus V \in \tau_{\text{"}Cl\text{"}}\} = \bigcap\{V|\ A \subset V,\ X \setminus V \in \tau_\theta\} = \tau_\theta\text{-}Cl(A)$ hold for any subset A of (X,τ). Furthermore, we note that [15, Remark 3.19, Example 2.7] shows there exists a topological space (X,τ) such that $Cl_{\text{"}Cl\text{"}}(A) \neq \tau_{\text{"}Cl\text{"}}\text{-}Cl(A)$ (i.e., $Cl_\theta(A) \neq \tau_\theta\text{-}Cl(A)$)

for some set A of (X, τ). Indeed, let $X = \{a, b, c\}$ and $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$; then $\tau_{\text{“}Cl\text{”}} = \tau_\theta = \{\emptyset, X\}, \tau_{\text{“}Cl\text{”}}\text{-}Cl(\{a\}) = X$ and $Cl_{\text{“}Cl\text{”}}(\{a\}) = \{a, c\}$. However, it is known that, in general, for a subset A of $(X, \tau), Cl_{\text{“}Cl\text{”}}(A) = A$ (i.e., $Cl_\theta(A) = A$) holds if and only if $\tau_{\text{“}Cl\text{”}}\text{-}Cl(A) = A$ (i.e., $\tau_\theta\text{-}Cl(A) = A$) holds (cf. Theorem 2.10(viii)). The example [15, Example 2.7, Remark 3.19] shows that this operation “Cl” is not open.

(iii) For other examples of operations on $\xi = PO(X, \tau)$, we refer them in [17, Example 3.2 (iv)-(vi)].

A function, say “$Int \circ Cl$” : $\xi \to P(X)$, is well defined by $U^{\text{“}Int \circ Cl\text{”}} = Int(Cl(U))$, where $U \in \xi$.

Remark 2.12. Assume that $\tau \subset \xi$.

(i) The function “$Int \circ Cl$” : $\xi \to P(X)$ is an operation on ξ if and only if $\xi \subset PO(X, \tau)$ holds. (The operation “$Int \circ Cl$” is called the *“interior-closure” operation* on ξ.)

Proof. (Necessity) Put $\gamma :=$ “$Int \circ Cl$”. Let $V \in \xi$. Then, $V \subset V^\gamma$ holds, because $\gamma : \xi \to P(X)$ is an operation on ξ; and so $V \subset Int(Cl(V))$, i.e., $V \in PO(X, \tau)$. Thus, we have $\xi \subset PO(X.\tau)$.

(Sufficiency) Assume that $\xi \subset PO(X, \tau)$ holds. Let $U \in \xi$. Since $U \in PO(X, \tau), U \subset Int(Cl(U))$ holds, i.e., $U \subset U^{\text{“}Int \circ Cl\text{”}}$ holds. Hence, “$Int \circ Cl$” : $\xi \to P(X)$ is an operation on ξ.

(ii) Let “$Int \circ Cl$” : $\xi \to P(X)$ be the *“interior-closure” operation* on ξ, where $\tau \subset \xi \subset PO(X, \tau)$ (cf. (i) above). A subset A of (X, τ) is “$Int \circ Cl$”-open in (X, τ) if and only if A is δ-open in (X, τ). That is, $\tau_{\text{“}Int \circ Cl\text{”}} = \tau_{\text{“}Int \circ Cl\text{”}|\tau} = \tau_\delta$ hold, where τ_δ is the collection of all δ-open sets of (X, τ) [18]. Then, it is shown that $Cl_{\text{“}Int \circ Cl\text{”}}(A) = Cl_\delta(A)$ and $Int_{\text{“}Int \circ Cl\text{”}}(A) = Int_\delta(A)$ hold and (X, τ) is “$Int \circ Cl$”-regular if and only if $\tau = \tau_\delta$ holds (cf. Theorem 2.10(xii)(b)) ((X, τ) is called *semi-regular* if $\tau = \tau_\delta$ holds, eg. [3, Lemma 2.1]). This operation “$Int \circ Cl$” : $\xi \to P(X)$ is an example of the open operation on ξ (cf. Definition 2.5(ii)), under the assumption that $\tau \subset \xi \subset PO(X, \tau)$. Indeed, let $x \in X$ and U be any open neighbourhood of x. Then, since $x \in Int(Cl(U))$ and $\{Int(Cl(U))\}^{\text{“}Int \circ Cl\text{”}} = Int(Cl(U))$ hold, the subset $Int(Cl(U))$ is an “$Int \circ Cl$”-open set containing x; furthermore, $Int(Cl(U)) \subset U^{\text{“}Int \circ Cl\text{”}}$ holds. Thus, “$Int \circ Cl$” : $\xi \to P(X)$ is open on ξ. Therefore, by Theorem 2.10(ix), $Cl_{\text{“}Int \circ Cl\text{”}}(A) = \tau_{\text{“}Int \circ Cl\text{”}}\text{-}Cl(A)$ (i.e., $Cl_\delta(A) = \tau_\delta\text{-}Cl(A)$) holds for any subset A of (X, τ).

3. Operation-α-open sets

In the present section, we introduce and study the concept of *operation-α-open sets* in a topological space (X, τ). We study a formula on operation-α-closuress of subsets of (X, τ) (cf. Theorem 3.7(iii)) As a corollary of Theorem 3.7(iii), we get the well known formula of $\tau^\alpha\text{-}Cl(E)$ of a subset E of (X, τ). Firstly, we recall the concept of the *semi-operation-open sets* and two kinds of *operation-semi-closures* due to Krishnan et al. [9].

Definition 3.1. ([9]) Let $\gamma_s : SO(X, \tau) \to P(X)$ be an operation on $SO(X, \tau)$ and A a subset of X.

(i) A subset A is said to be *semi-γ_s-open* [9, Definition 2.5] in (X, τ), if for each $x \in A$ there exists a subset $U \in SO(X, \tau)$ such that $x \in U$ and $U^{\gamma_s} \subset A$.

• The collection of all semi-γ_s-open sets is denoted as $SO(X, \tau)_{\gamma_s}$.

(ii) The following two *operation-semi-closures of A* are defined as follows:

• $SO(X, \tau)_{\gamma_s}\text{-}Cl(A) := \bigcap\{F |\ A \subset F,\ X \setminus F \in SO(X, \tau)_{\gamma_s}\}$ ([9, Definition 2.24]);

• $sCl_{\gamma_s}(A) := \{x \in X | U^{\gamma_s} \cap A \neq \emptyset$ for every semi-open set U (i.e., $U \in SO(X, \tau)$) containing $x\}$ ([9, Definition 2.23]).

(iii) For a subset A of (X, τ), an operation semi-interior is defined:

• $sInt_{\gamma_s}(A) := \{x \in X|$ there exists a subset $U \in SO(X,\tau)$ such that $x \in U$ and $U^{\gamma_s} \subset A\}$.

Definition 3.2. Let $\gamma_s : SO(X,\tau) \to P(X)$ be an operation on $SO(X,\tau)$ and A a subset of (X,τ).

(i) A subset A of (X,τ) is called a γ_s-α-*open set* in (X,τ), if there exists a γ_s-open set U (i.e., $U \in \tau_{\gamma_s}$) such that $U \subset A \subset \mathrm{sCl}_{\gamma_s}(U)$. The complement of a γ_s-α-open set is called γ_s-α-*closed* in (X,τ).

(ii) • The collection of all γ_s-α-open sets in (X,τ) is denoted by $\gamma_s\tau^\alpha$.

Remark 3.3. (i) For the identity operation "*id*" : $SO(X,\tau) \to P(X)$, it is well known that [9] $SO(X,\tau)_{\text{"}id\text{"}} = SO(X,\tau)$ and $SO(X,\tau)_{\text{"}id\text{"}}\text{-}Cl(A) = sCl_{\text{"}id\text{"}}(A) = sCl(A)$ hold for any subset A of (X,τ). Furthermore, for a subset A of (X,τ), A is "*id*"-α-open in (X,τ) if and only if A is α-open in (X,τ). Namely, $_{\text{"}id\text{"}}\tau^\alpha = \tau^\alpha$ holds for any topology τ of X. Indeed, for the identity operation $\gamma_s =$ "*id*" : $SO(X,\tau) \to P(X)$, a subset A is "*id*"-α-open in (X,τ) if and only if $(*)$ there exists an open set U such that $U \subset A \subset sCl(U)$ (cf. Remark 2.11(i)); it is well known that the above property $(*)$ holds if and only if A is α-open (cf. [11], [6] and the contents of MR914467(88i:54009) in Math.R.AMS).

(ii) By Definition 3.1(ii) and (iii), it is shown that $sInt_{\gamma_s}(A) = X \setminus sCl_{\gamma_s}(X \setminus A)$ holds in (X,τ); furthermore, for the identity operation "*id*" : $SO(X,\tau) \to P(X), sInt_{\text{"}id\text{"}}(A) = sInt(A)$ holds, where A is any subset of (X,τ).

Theorem 3.4. *Let A be a subset of (X,τ) and $\gamma_s : SO(X,\tau) \to P(X)$ an operation on $SO(X,\tau)$.*

(i) *If $A \in \gamma_s\tau^\alpha$, then $A \subset sCl_{\gamma_s}(Int_{\gamma_s}(A))$ holds.*

(ii) *Suppose that $\gamma_s : SO(X;\tau) \to P(X)$ is open on $SO(X,\tau)$ or $\gamma_s|\tau : \tau \to P(X)$ is open. If $A \subset sCl_{\gamma_s}(Int_{\gamma_s}(A))$ holds, then $A \in \gamma_s\tau^\alpha$.*

(iii) *Suppose that $\gamma_s : SO(X;\tau) \to P(X)$ is open on $SO(X,\tau)$* (cf. Definition 2.5(ii)). *For a subset A of $(X,\tau), A$ is γ_s-α-open in (X,τ) if and only if $A \subset sCl_{\gamma_s}(Int_{\gamma_s}(A))$ holds. Furthermore, for a subset B of $(X,\tau), B$ is γ_s-α-closed in (X,τ) if and only if $sInt_{\gamma_s}(Cl_{\gamma_s}(B)) \subset B$ holds.*

Proof. **(i)** It follows from assumption that there exists a set $V \in \tau_{\gamma_s}$ such that $V \subset A \subset sCl_{\gamma_s}(V)$. Using Theorem 2.10(viii), we have that $V = Int_{\gamma_s}(V) \subset Int_{\gamma_s}(A)$ holds and so $A \subset sCl_{\gamma_s}(V) \subset sCl_{\gamma_s}(Int_{\gamma_s}(A))$ holds.

(ii) We have $Int_{\gamma_s}(A) \subset A \subset sCl_{\gamma_s}(Int_{\gamma_s}(A))$ hold. Since $Int_{\gamma_s}(A)$ is γ_s-open under assumption that γ_s is open (cf. Theorem 2.10(ix)(c)), it is shown that A is γ_s-α-open in (X,τ).

(iii) For the proof of the former, it is obvious from (i) and (ii). For the proof of the latter, it is obtained by using the former, Remark 3.3(ii) and Theorem 2.10(vii) for $\xi = SO(X,\tau)$. □

Proposition 3.5. *Let $\gamma_s : SO(X,\tau) \to P(X)$ be an operation on $SO(X,\tau)$. If $A_i \in \gamma_s\tau^\alpha$ for each $i \in \Omega$, then $\bigcup\{A_i|\ i \in \Omega\} \in \gamma_s\tau^\alpha$, where Ω is any index set.*

Proof. It follows from Definition 3.2(i) that, for each $i \in \Omega$, there exists a γ_s-open set U_i such that $U_i \subset A_i \subset sCl_{\gamma_s}(U_i)$. Then, $\bigcup\{U_i|i \in \Omega\} \subset \bigcup\{A_i|i \in \Omega\} \subset \bigcup\{sCl_{\gamma_s}(U_i)|\ i \in \Omega\} \subset sCl_{\gamma_s}(\bigcup\{U_i|\ i \in \Omega\})$ hold and so $\bigcup\{A_i|\ i \in \Omega\} \in \gamma_s\tau^\alpha$, because $\bigcup\{U_i|\ i \in \Omega\} \in \tau_{\gamma_s}$ (cf. Theorem 2.10(ii)). □

Definition 3.6. Let $\gamma_s : SO(X,\tau) \to P(X)$ be an operation and a subset B of (X,τ). The following closure of B, say $\gamma_s\tau^\alpha\text{-}Cl(B)$, is defined:

• $\gamma_s\tau^\alpha\text{-}Cl(B) := \bigcap\{F|\ B \subset F, X \setminus F \in \gamma_s\tau^\alpha\}$.

Theorem 3.7. *Let $\gamma_s : SO(X,\tau) \to P(X)$ be an operation and A a subset of (X,τ). Then, the following properties hold.*

(i) *The closure $\gamma_s\tau^\alpha$-$Cl(A)$ is γ_s-α-closed in (X,τ).*

(ii) $A \cup sInt_{\gamma_s}(Cl_{\gamma_s}(A)) \subset \gamma_s\tau^\alpha$-$Cl(A)$.

(iii) *If $\gamma_s : SO(X,\tau) \to P(X)$ is open on $SO(X,\tau)$* (cf. Definiton 2.5(ii)), *then $\gamma_s\tau^\alpha$-$Cl(A) \subset A \cup sInt_{\gamma_s}(Cl_{\gamma_s}(A))$ holds and so*
$\gamma_s\tau^\alpha$-$Cl(A) = A \cup sInt_{\gamma_s}(Cl_{\gamma_s}(A))$ *holds.*

Proof. **(i)** Using Proposition 3.5, the intersection of every family of γ_s-α-closed sets is γ_s-α-closed. Thus, $\gamma_s\tau^\alpha$-$Cl(B)$ is γ_s-α-closed in (X,τ).

(ii) Since $\gamma_s\tau^\alpha$-$Cl(A)$ is a γ_s-α-closed set, $sInt_{\gamma_s}(Cl_{\gamma_s}(\gamma_s\tau^\alpha\text{-}Cl(A))) \subset \gamma_s\tau^\alpha$-$Cl(A)$ holds by Theorem 3.4(i), Definition 3.1 and Remark 3.3(ii). Since $A \subset \gamma_s\tau^\alpha$-$Cl(A)$, we have that $A \cup sInt_{\gamma_s}(Cl_{\gamma_s}(A)) \subset \gamma_s\tau^\alpha$-$Cl(A)$ holds.

(iii) We first claim that a subset $A \cup (sInt_{\gamma_s}(Cl_{\gamma_s}(A)))$ is γ_s-α-closed. We put $B := A \cup (sInt_{\gamma_s}(Cl_{\gamma_s}(A)))$. Then, we have $sInt_{\gamma_s}(Cl_{\gamma_s}(B)) \subset sInt_{\gamma_s}(Cl_{\gamma_s}(A \cup Cl_{\gamma_s}(A))) = sInt_{\gamma_s}(Cl_{\gamma_s}(Cl_{\gamma_s}(A))) = sInt_{\gamma_s}(Cl_{\gamma_s}(A)) \subset A \cup sInt_{\gamma_s}(Cl_{\gamma_s}(A))$ and so $sInt_{\gamma_s}(Cl_{\gamma_s}(B)) \subset B$. Using Theorem 3.4(iii), the set B is γ_s-α-closed. Thus we prove that $\gamma_s\tau^\alpha$-$Cl(A) \subset B$ holds, because $\gamma_s\tau^\alpha$-$Cl(A)$ is the smallest γ_s-α-closed set including the set A (cf. Theorem 3.4); and so we have the required equality using (ii). □

By Theorem 3.7(iii), the explicit form of the operation-closure $\gamma_s\tau^\alpha$-$Cl(\bullet)$ is obtained, under assumption that γ_s is open on $SO(X,\tau)$. Let $\gamma_s =$ "*id*" $: SO(X,\tau) \to P(X)$ in the theorem above. Then, we have the following well known formula.

Corollary 3.8. ([1, Theorems 2.2,2.3, Corollary 2.4], [2, Theorem 1.5(c)]) *For any subset E of (X,τ), τ^α-$Cl(E) = E \cup Cl(Int(Cl(E)))$ holds.*

Proof. Put $\gamma_s =$ "*id*" $: SO(X,\tau) \to P(X)$ in Theorem 3.7(iii) and put $\xi = SO(X,\tau)$ in Remark 2.11(i) and Remark 3.3. It is shown that τ^α-$Cl(E) = E \cup sInt(Cl(E)) = E \cup \{Cl(E) \cap Cl(Int(Cl(E)))\} = E \cup Cl(Int(Cl(E)))$ hold, because the identity operation is open on $SO(X,\tau)$ and $sInt(A) = A \cap Cl(Int(A))$ and $Cl(Int(Cl(A))) \subset Cl(A)$ hold for any subset A of (X,τ). □

4. Remarks on operation-semi-closures

In the end of the present paper, for an operation $\gamma_s : SO(X,\tau) \to P(X)$ on $SO(X,\tau)$, we define the concept of γ_s-semi-open sets (cf. Definition 4.1) and an operation-semi-closure , say $\gamma_s SO(X,\tau)$-$Cl(E)$, (cf. Definition 4.6) of a subset E of a topological space (X,τ). In 2006, Krishnan and Balachandran [8] introduced and investigated the concept of *γ-semi-open sets* for an operation $\gamma : \tau \to P(X)$; by [8, Definition 3.1], a subset A of (X,τ) is said to be *γ-semi-open* in (X,τ), if there exists a γ-open set U such that $U \subset A \subset \tau_\gamma$-$Cl(U)$, where τ_γ denotes the set of all γ-open sets in (X,τ).

In the present paper, we attempt to define analogously *γ_s-semi-open sets* for an operation $\gamma_s : SO(X,\tau) \to P(X)$ as follows.

Definition 4.1. Let $\gamma_s : SO(X,\tau) \to P(X)$ be an operation on $SO(X,\tau)$. A subset A of (X,τ) is called *γ_s-semi-open* in (X,τ), if there exists a γ_s-open set U such that $U \subset A \subset \mathrm{Cl}_{\gamma_s}(U)$ (cf. Definition 3.1(i), Definition 3.2). The complement of a γ_s-semi-open set is called *γ_s-semi-closed in (X,τ)*.

• The collection of all γ_s-semi-open sets (resp. γ_s-semi-closed sets) in (X,τ) is denoted by $\gamma_s SO(X,\tau)$ (resp. $\gamma_s SC(X,\tau)$).

Remark 4.2. For a subset A of (X,τ), $A \in$ "*id*"$SO(X,\tau)$ if and only if $A \in SO(X,\tau)$. Indeed, for the identity operation $\gamma_s =$ "*id*" $: SO(X,\tau) \to P(X)$, a subset A is "*id*"-semi-open if and only if there exists a set $U \in \tau$ such that $U \subset A \subset Cl(U)$ (i.e., A is semi-open in (X,τ)).

Theorem 4.3. *Let A be a subset of (X,τ).*

(i) (a) *If $A \in \gamma_s SO(X,\tau)$, then $A \subset Cl_{\gamma_s}(Int_{\gamma_s}(A))$ holds.*

(b) *If If $A \in \gamma_s SC(X,\tau)$, then $Int_{\gamma_s}(Cl_{\gamma_s}(A)) \subset A$ holds.*

(ii) *Suppose that $\gamma_s : SO(X,\tau) \to P(X)$ is open on $SO(X,\tau)$. If $A \subset Cl_{\gamma_s}(Int_{\gamma_s}(A))$ holds, then $A \in \gamma_s SO(X,\tau)$.*

(iii) *Suppose that $\gamma_s : SO(X,\tau) \to P(X)$ is open on $SO(X,\tau)$* (cf. Definition 2.5(ii)). *For a subset A of (X,τ), $A \in \gamma_s SO(X,\tau)$ if and only if $A \subset Cl_{\gamma_s}(Int_{\gamma_s}(A))$ holds. For a subset B of $(X,\tau), B \in \gamma_s SC(X,\tau)$ if and only if $Int_{\gamma_s}(Cl_{\gamma_s}(B)) \subset B$ holds.*

Proof. **(i) (a)** It follows from assumption that there exists a set $V \in \tau_{\gamma_s}$ such that $V \subset A \subset Cl_{\gamma_s}(V)$. Using Theorem 2.10(viii) for $\xi = SO(X,\tau)$, we have that $V = Int_{\gamma_s}(V) \subset Int_{\gamma_s}(A)$ holds and so $A \subset Cl_{\gamma_s}(V) \subset Cl_{\gamma_s}(Int_{\gamma_s}(A))$ holds.

(b) It is obtained by (a) above and Theorem 2.10(vii) for $\xi = SO(X,\tau)$.

(ii) We have $Int_{\gamma_s}(A) \subset A \subset Cl_{\gamma_s}(Int_{\gamma_s}(A))$ hold. By Theorem 2.10(ix), it is shown that $Int_{\gamma_s}(A)$ is γ_s-semi-open and so $A \in \gamma_s SO(X,\tau)$.

(iii) It is obvious from (i), (ii) and Theorem 2.10(vii) for $\xi = SO(X,\tau)$. □

Theorem 4.4. *Suppose that $\gamma_s : SO(X,\tau) \to P(X)$ is open on $SO(X,\tau)$* (cf. Definition 2.5(ii)). *If $A \in \gamma_s SO(X,\tau)$ and $Int_{\gamma_s}(A) \subset B \subset Cl_{\gamma_s}(A)$, then $B \in \gamma_s SO(X,\tau)$ and $Cl_{\gamma_s}(Int_{\gamma_s}(A)) = Cl_{\gamma_s}(Int_{\gamma_s}(B))$ holds.*

Proof. Since A is γ_s-semi-open in (X,τ), there exists a γ_s-open set U such that $U \subset A \subset Cl_{\gamma_s}(U)$. Then, we have $Int_{\gamma_s}(U) \subset Int_{\gamma_s}(A) \subset B \subset Cl_{\gamma_s}(A) \subset Cl_{\gamma_s}(Cl_{\gamma_s}(U))$ and so $Int_{\gamma_s}(U) \subset B \subset Cl_{\gamma_s}(Cl_{\gamma_s}(U))$.
By using Theorem 2.10 (viii) and (ix) for $\xi = SO(X,\tau)$, it is shown that $U = Int_{\gamma_s}(U) \subset B \subset Cl_{\gamma_s}(Cl_{\gamma_s}(U)) = Cl_{\gamma_s}(U))$ hold. Thus, B is γ_s-semi-open in (X,τ). It follows from assumption, Theorem 4.3(ii) and Theorem 2.10(ix) and Remark 2.7 for $\xi = SO(X,\tau)$ that $Cl_{\gamma_s}(Int_{\gamma_s}(B)) \subset Cl_{\gamma_s}(Int_{\gamma_s}(Cl_{\gamma_s}(A))) \subset Cl_{\gamma_s}(Cl_{\gamma_s}(A)) \subset Cl_{\gamma_s}(Cl_{\gamma_s}(Cl_{\gamma_s}(Int_{\gamma_s}(A)))) = Cl_{\gamma_s}(Int_{\gamma_s}(A))$ hold and so $Cl_{\gamma_s}(Int_{\gamma_s}(B)) \subset Cl_{\gamma_s}(Int_{\gamma_s}(A))$ holds. The converse implication is similarly proved as follows: $Cl_{\gamma_s}(Int_{\gamma_s}(A)) \subset Cl_{\gamma_s}(B) \subset Cl_{\gamma_s}(Cl_{\gamma_s}(Int_{\gamma_s}(B)))$ $=Cl_{\gamma_s}(Int_{\gamma_s}(B))$ hold. Therefore we have the required equality: $Cl_{\gamma_s}(Int_{\gamma_s}(A)) = Cl_{\gamma_s}(Int_{\gamma_s}(B))$.
□

Proposition 4.5. *t460* (i) *If $A_i \in \gamma_s SO(X,\tau)$ for each $i \in \Sigma$, then $\bigcup_{i\in\Sigma} A_i \in \gamma_s SO(X,\tau)$, where Σ is any index set.*

(ii) *If $F_i \in \gamma_s SC(X,\tau)$ for each $i \in \Omega$, then $\bigcap\{F_i|\ i \in \Omega\} \in \gamma_s SC(X,\tau)$, where Ω is any index set.*

Proof. **(i)** For each $i \in \Sigma$, there exists a set $U_i \in \tau_{\gamma_s}$ such that $U_i \subset A_i \subset Cl_{\gamma_s}(U_i)$. Then, $\bigcup\{U_i|\ i \in \Sigma\} \subset \bigcup\{A_i|\ i \in \Sigma\} \subset \bigcup\{Cl_{\gamma_s}(U_i)|\ i \in \Sigma\} \subset Cl_{\gamma_s}(\bigcup\{U_i|\ i \in \Sigma\})$ hold and so $\bigcup\{A_i|\ i \in \Sigma\}$ is a γ_s-semi-open set, because $\bigcup\{U_i|\ i \in \Sigma\} \in \tau_{\gamma_s}$ (cf. Theorem 2.10 for $\xi = SO(X,\tau)$).

(ii) It is obvious from (i). □

Definition 4.6. For an operation $\gamma_s : SO(X,\tau) \to P(X)$ and a subset B of (X,τ), the following operation-closure of B, say $\gamma_s SO(X,\tau)$-$Cl(B)$ (shortly $\gamma_s sCl(B)$), is defined as follows:

•$\gamma_s SO(X,\tau)$-$Cl(B)(= \gamma_s sCl(B))$:=$\bigcap\{F|\ B \subset F, X \setminus F \in \gamma_s SO(X,\tau)\}$.

It is evident that, for $\gamma =$ "id" $: SO(X,\tau) \to P(X)$ and a subset B of (X,τ), "id" $SO(X,\tau)$-$Cl(B) =$ "id" $sCl(B) = sCl(B)$ hold.

Theorem 4.7. *Let $\gamma_s : SO(X,\tau) \to P(X)$ be an operation and A a subset of (X,τ).*
(i) *The closure $\gamma_s sCl(A)$ is γ_s-semi-closed in (X,τ).*
(ii) *The following implication $A \cup Int_{\gamma_s}(Cl_{\gamma_s}(A)) \subset \gamma_s sCl(A)$ holds.*
(iii) *If $\gamma_s : SO(X,\tau) \to P(X)$ is open on $SO(X,\tau)$* (cf. Definition 2.5(ii))*, then $\gamma_s sCl(A) \subset A \cup Int_{\gamma_s}(Cl_{\gamma_s}(A))$ holds and so $\gamma_s sCl(A) = A \cup Int_{\gamma_s}(Cl_{\gamma_s}(A))$ holds.*

Proof. **(i)** By Proposition 4.5, it is shown that the intersection of every family of γ_s-semi-closed sets is γ_s-semi-closed. Thus, $\gamma_s sCl(B)$ is γ_s-semi-closed in (X,τ).

(ii) Since $\gamma_s sCl(A)$ is a γ_s-semi-closed set, $Int_{\gamma_s}(Cl_{\gamma_s}(\gamma_s sCl(A))) \subset \gamma_s sCl(A)$ holds by Theorem 4.3(i)(b). Since $A \subset \gamma_s sCl(A)$, we have that $A \cup Int_{\gamma_s}(Cl_{\gamma_s}(A)) \subset \gamma_s sCl(A)$ holds.

(iii) We first claim that a subset $A \cup (Int_{\gamma_s}(Cl_{\gamma_s}(A)))$ is γ_s-semi-closed. Let $B := A \cup (Int_{\gamma_s}(Cl_{\gamma_s}(A)))$. Then, we have that $Int_{\gamma_s}(Cl_{\gamma_s}(B)) \subset Int_{\gamma_s}(Cl_{\gamma_s}(A \cup Cl_{\gamma_s}(A))) = Int_{\gamma_s}(Cl_{\gamma_s}(Cl_{\gamma_s}(A))) = Int_{\gamma_s}(Cl_{\gamma_s}(A)) \subset A \cup Int_{\gamma_s}(Cl_{\gamma_s}(A)) = B$ and so $Int_{\gamma_s}(Cl_{\gamma_s}(B)) \subset B$. Using Theorem 4.3 (iii), the set B is γ_s-semi-closed. Thus we prove the implication $\gamma_s sCl(A) \subset B$, because $\gamma_s sCl(A)$ is the smallest γ_s-semi-closed set including the set A and $A \subset B$. We have the required equality using (ii). □

In the above theorem, let $\gamma_s =$ "id" $: SO(X,\tau) \to P(X)$. Then, we have the following well known formula:

Corollary 4.8. ([6]) *For any subset E of (X,τ), $sCl(E) = E \cup Int(Cl(E))$ holds.* □

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