CLASSIFICATION OF QUASI UNION HYPER K-ALGEBRAS OF ORDER 6

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Abstract. In this manuscript, we classify non isomorphic quasi union hyper K-algebras of order 6 and show that conjecture 4.2[6] is not true generally, and finally modify it.

1 Introduction

The study of BCK-algebra was initiated by Imai and Iséki[2] in 1966 as a generalization of the concept of set-theoretic difference and propositional calculi. The hyper structure theory (called also multi algebras) was introduced in 1934 by Marty[4] at the 8th congress of Scandinavian Mathematicians. Hyper structures have many applications to several sectors of both pure and applied sciences. Borzooei, et.al.[3] applied the hyper structure to BCK-algebras and introduced the concept of hyper BCK-algebra and hyper K-algebra in which, each of them is a generalization of BCK-algebra. Nasr-Azadani and Zahedi [5, 6] introduced quasi union hyper K-algebras and classified non isomorphic quasi union hyper K-algebras of order less than 6, and gave a conjecture for order n. Now we classify non isomorphic quasi union hyper K-algebras of order 6 and finally modify that conjecture.

2 Preliminaries

Let $H$ be a non-empty set, the set of all non-empty subset of $H$ is denoted by $P^*(H)$. A hyperoperation on $H$ is a map $\circ : H \times H \rightarrow P^*(H)$, where $(a, b) \mapsto a \circ b, \forall a, b \in H$. A set $H$, endowed with a hyperoperation, “ $\circ$”, is called a hyperstructure. If $A, B \subseteq H$, then $A \circ B = \bigcup_{a \in A, b \in B} a \circ b$.

Definition 2.1. [1, 3] Let $H$ be a non-empty set containing a constant “ 0 ” and “ $\circ$ ” be a hyperoperation on $H$. Then $H$ is called a hyper K-algebra if it satisfies HK1-HK5.

HK1: $(x \circ z) \circ (y \circ z) < x \circ y,$
HK2: $(x \circ y) \circ z = (x \circ z) \circ y,$
HK3: $x < x,$
HK4: $x < y, y < x$, then $x = y,$
HK5: $0 < x.$

for all $x, y, z \in H$, where $x < y$ means $0 \in x \circ y$. Moreover for any $A, B \subseteq H$, $A < B$ if $\exists a \in A, \exists b \in B$ such that $a < b$.

For briefly the readers could see some definitions and results about hyper K-algebra in[1, 3].

Let $H$ be a set containing “0”, $\mathcal{P}_0(H) = \{A \subseteq H : 0 \in A\}$ and $\mathcal{S} = \{f|f : H \rightarrow \mathcal{P}_0(H)\}$ is a function. For convenience we use $F^x$ instead of $f(x)$ for any $f \in \mathcal{S}$.

Theorem 2.2. [5] Let $X$ be a set and $H = X \cup \{0\}$. Then for any $f \in \mathcal{S}$, $\circ_f : H \times H \rightarrow \mathcal{P}^*(H)$ is defined by:

$x \circ_f y := \begin{cases} F^x & \text{if } x = y, \\ \{x\} & \text{otherwise.} \end{cases}$

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is a hyperoperation. Moreover the following statements are equivalent.

(i) \((H, \alpha_f, 0)\) is a hyper \(K\)-algebra,
(ii) \(F^x \circ_f y = F^x\) for all \(y \neq x, y \in H\).
(iii) if \(x \neq y\) and \(y \in F^x\), then \(y \in F^y\) and \(F^y \subseteq F^x\).

This hyper \(K\)-algebra is said to be a homomorphism if \(f(0) = 0\) and \(f(x \circ y) = f(x) \circ f(y)\), for all \(x, y \in H\). \(Ker f = \{x \in H : f(x) = 0\}\). Moreover if \(f\) is bijection this homomorphism is called an isomorphism.

**Definition 2.4.** [1] Let \(H_1\) and \(H_2\) be two hyper \(K\)-algebras. A mapping \(f : H_1 \rightarrow H_2\) is said to be a homomorphism if \(f(0) = 0\) and \(f(x \circ y) = f(x) \circ f(y)\), for all \(x, y \in H_1\). \(Ker f = \{x \in H_1 : f(x) = 0\}\). Moreover if \(f\) is bijection this homomorphism is called an isomorphism.

**Theorem 2.8.** [6] Let \(H_f, H_g\) be two quasi union hyper \(K\)-algebras. Then we say that \(H_f\) is of type \((l_f^0, l_f^1, \ldots, l_f^n)\) and denote it by \(t(H_f)\), if \(l_f^i = |F^{x^i}|, 0 \leq i \leq n - 1\). Also \(H_f\) and \(H_g\) are co-type, if the type of \(H_f\) is a permutation of type \(H_g\).

**Theorem 2.9.** [6] Let \(H_f\) and \(H_g\) be quasi union hyper \(K\)-algebras and \(v\) be an isomorphism from \(H_f\) to \(H_g\). Then \(|F^x| = |v(F^x)| = |G^v(x)|\).

**Corollary 2.7.** [6] Let \(H_f\) and \(H_g\) be hyper \(K\)-algebras. Then \(H_f\) is not isomorphic to \(H_g\), if one of the following statement holds.

\(i) |0 \circ_f 0| \neq |0 \circ_g 0|,\)
\(ii) F^{x^i} = G^{x^i}\), for all \(1 \leq i \leq n\) except for some \(i\), (see Remark after Theorem 4.5),
\(iii) t_f^i(m) \neq t_g^j(m)\) for some \(m, 1 \leq m \leq n\), where \(t_f^i(m) = |\{x_i \in H - \{0\} : |F^{x^i}| = m\}|,\)
\(iv) N_n(H_f) \neq N_n(H_g)\), where \(N_n(H_f) = |\{F^{x^k} : x_k \in F^{x^k}, 0 \leq k \leq n\}|.\)
\(v) H_f\) and \(H_g\) are not co-type.

**Theorem 2.8.** [6] (Key Theorem) Let \(H_f\) be a quasi union hyper \(K\)-algebra of type \((l_f^0, l_f^1, \ldots, l_f^n)\). Then, there is a union hyper \(K\)-algebra \(H_g\) isomorphic to \(H_f\) such that \(t(H_g) = (l_g^0, l_g^1, \ldots, l_g^n)\) where \(l_g^0 \leq l_g^1 \leq \ldots \leq l_g^n\).

**Conjecture 2.10.** [6] Let \(|\mathcal{H}_n|, n \in \mathbb{N}\), be the number of non-isomorphic quasi union hyper \(K\)-algebras of order \(n\). Then \(|\mathcal{H}_1| = 1, |\mathcal{H}_2| = 3\) and for \(n > 2\)

\[|\mathcal{H}_n| = 2|\mathcal{H}_{n-1}| - |\mathcal{H}_{n-2}| + \left(\frac{2n - 2}{n - 2}\right)\]

3 review of the conjecture

In this section we show that the number of non isomorphic quasi union hyper \(K\)-algebras of order \(n\) and type \((l_0, l_1, \ldots, l_{n-1})\) such that \(l_0 = l_1\) is equal to \(2|\mathcal{H}_{n-1}| - |\mathcal{H}_{n-2}|\). So for counting the number of non isomorphic quasi union hyper \(K\)-algebras of order \(n\), it’s sufficient to count the number of non isomorphic quasi union hyper \(K\)-algebras of type \((l_0, l_1, \ldots, l_{n-1})\) such that \(l_0 = 1, l_1 \geq 2\).

Note: In this paper we denote the set of all non isomorphic quasi union hyper \(K\)-algebras of order \(n\) and type \((l_0, l_1, \ldots, l_{n-1})\) such that \(l_i \leq l_{i+1}\) where \(0 \leq i \leq n - 1\),
by $\mathcal{H}_n$, also $\mathcal{H}_n^{11} = \{ H_n \in \mathcal{H}_n | l_0 = l_1 = 1 \}$, $\mathcal{H}_n^{12} = \{ H_n \in \mathcal{H}_n | l_0 = 1, l_1 \geq 2 \}$ and $\mathcal{H}_n^{22} = \{ H_n \in \mathcal{H}_n | l_0 \geq 2 \}$.

**Theorem 3.1.** (Going down)

Let $H_n = \{ x_0 = 0, x_1, x_2, \ldots, x_{n-1} \}$ and $(H_n, \circ, 0)$ be a quasi union hyper $K$-algebra. Then:

i) If $(H_n, \circ, 0) \in \mathcal{H}_n^{11}$, then there exists a quasi union hyper $K$-algebra, $D_1(H_n)$, of order $n-1$ and type $(l_0^1, l_1^1, \ldots, l_{n-2}^1)$ such that $l_i^1 = l_{i+1}$, where $0 \leq i \leq n-2$.

ii) If $H_n \in \mathcal{H}_n^{22}$, then there exists a quasi union hyper $K$-algebra, $D_2(H_n)$, of order $n-1$ and type $(l_0^2, l_1^2, \ldots, l_{n-2}^2)$ such that $l_i^2 = l_{i+1} - 1$, where $0 \leq i \leq n-2$.

**Proof.** i) Let $H_n \in \mathcal{H}_n^{11}$, $H_{n-1} = \{ x_0 = 0, x_2, \ldots, x_{n-1} \}$ and “$\circ_d$” be a restriction “$\circ$” to $H_{n-1}$. Then it’s clear that $(H_{n-1}, \circ_d, 0)$ is a quasi union hyper $K$-algebra and $l_i^{d} = l_{i+1}$ where $0 \leq i \leq n-2$.

ii) Let $H_n \in \mathcal{H}_n^{22}$, and $H_{n-1} = \{ x_0 = 0, x_2, \ldots, x_{n-1} \}$. Then $(H_{n-1}, \circ_{d_2}, 0)$ is a quasi union hyper $K$-algebra where “$\circ_{d_2}$” is defined as follows:

$$x_i \circ_{d_2} x_j = \begin{cases} x_i \circ x_j - \{ x_1 \} & \text{if } i = j, \\ \{ x_i \} & \text{if } i \neq j. \end{cases}$$

for all $x_i, x_j \in H_{n-1}$. By Theorem 2.2, it’s sufficient to show that if $x_k \in x_i \circ_{d_2} x_j$, then $x_k \in x_k \circ_{d_2} x_k$ and $x_k \circ_{d_2} x_k \subseteq x_i \circ_{d_2} x_i$, for all $x_i, x_k \in H_{n-1}$. Let $x_k \in x_i \circ_{d_2} x_i$. Since $(H_n, \circ, 0)$ is a quasi union hyper $K$-algebra, then $x_k \in x_k \circ x_k - \{ x_1 \}$ and $x_k \circ x_k - \{ x_1 \} \subseteq x_i \circ x_i - \{ x_1 \}$. These imply that $x_k \in x_k \circ_{d_2} x_k$ and $x_k \circ_{d_2} x_k \subseteq x_i \circ_{d_2} x_i$. Moreover $|x_i \circ_{d_2} x_i| = |x_i \circ x_i| - 1$, i.e., $l_i^{d} = l_{i+1} - 1$, and these complete the proof.

**Theorem 3.2.** (Going up)

Let $H_{n-1} = \{ x_0 = 0, x_1, x_2, \ldots, x_{n-2} \}$ and $(H_{n-1}, \circ, 0)$ be a quasi union hyper $K$-algebra. Then:

i) If $(H_{n-1}, \circ, 0) \in \mathcal{H}_n^{11} \cup \mathcal{H}_n^{12}$, then there exists a quasi union hyper $K$-algebra, $U_1(H_{n-1}) \in \mathcal{H}_n^{11}$, of order $n$ and type $(l_0^1, l_1^1, \ldots, l_{n-1}^1)$ such that $l_0 = l_1 = 1$.

ii) If $(H_{n-1}, \circ, 0) \in \mathcal{H}_n^{22}$, then there exists a quasi union hyper $K$-algebra, $U_2(H_{n-1})$, of order $n$ and type $(l_0^2, l_1^2, \ldots, l_{n-1}^2)$ such that $l_0 \geq 2$, i.e., $U(H_{n-1}) \in \mathcal{H}_n^{22}$.

**Proof.** i) Let $(H_{n-1}, \circ, 0) \in \mathcal{H}_n^{11} \cup \mathcal{H}_n^{12}$. Then we set $H_n = \{ x_0 = 0, x_{n-1}, x_1, \ldots, x_{n-2} \}$ and we show that $(H_n, \circ_{u_1}, 0)$ is a quasi union hyper $K$-algebra, where

$$x_i \circ_{u_1} x_j = \begin{cases} x_i \circ x_j & \text{if } i = j \neq n - 1, \\ \{ 0 \} & i = j = n - 1, \\ \{ x_i \} & i \neq j. \end{cases}$$

for all $x_i, x_j \in H_n$. It’s clear that $(H_n, \circ_{u_1}, 0)$ is a quasi union hyper $K$-algebra of type $(l_0^1, l_1^1, \ldots, l_{n-1}^1)$ such that $l_0 = l_1 = 1$.

ii) Let $(H_{n-1}, \circ, 0) \in \mathcal{H}_n^{22}$ and $H_{n-1} = \{ x_0 = 0, x_1, \ldots, x_{n-2} \}$. Then we set $H_n = \{ x_0 = 0, x_{n-1}, x_1, \ldots, x_{n-2} \}$ and we show that $(H_n, \circ_{u_2}, 0)$ is a quasi union hyper $K$-algebra, where

$$x_i \circ_{u_2} x_j = \begin{cases} x_i \circ x_j \cup \{ x_{n-1} \} & \text{if } i = j \neq n - 1, \\ x_0 \circ x_0 \cup \{ x_{n-1} \} & i = j = n - 1, \\ \{ x_i \} & i \neq j. \end{cases}$$
for all \(x_i, x_j \in H_n\). Since \(H_{n-1}\) is a quasi union hyper K-algebra and \(x_0 \circ x_0 \subseteq x_i \circ x_i\) for all \(0 \leq i \leq n - 2\), if \(x_k \in x_i \circ u_2 x_i\), then \(x_k \in x_k \circ u_2 x_k\) for all \(x_i, x_k \in H_n\). Therefore \((H_n, \circ u_2, 0)\) is a quasi union hyper K-algebra, of type \((l_0 + 1, l_0 + 1, l_1 + 1, \ldots, l_{n-2} + 1)\) and we denote this quasi union hyper K-algebra by \(U_2(H_{n-1})\) and it’s type is equal to \((l_0^n, l_1^n, \ldots, l_{n-1}^n)\) where \(l_0^n \leq l_{1}^n \leq l_{2}^n \cdots l_{n-1}^n \geq 2\).

**Note:** By attention to the proof of these theorems, \(U_i(H_{n-1}), i = 1, 2\), is obtained by inserting \(x_n\) between 0 and 1 and \(D_i(H_n), i = 1, 2\), is obtained by deleting \(x_1\) from \(H_n\).

**Theorem 3.3.** \(D_2 U_2 = 1_{H_{n-1}}, U_2 D_2 = 1_{H_{n}^{12}}, D_1 U_1 = 1_{H_{n-1}^{11} \cup H_{n-1}^{12}}\) and \(U_1 D_1 = 1_{H_{n}^{11}}\).

**Proof.** Let \(H_{n-1} = \{x_0, x_1, \ldots, x_{n-2}\}\). Then \(U_2(H_{n-1}) = H_n = H_{n-1} \cup \{x_{n-1}\}\) and

\[
x_i \circ u_2 x_j = \begin{cases} x_i \circ x_i \cup \{x_{n-1}\} & \text{if } i = j \neq n - 1, \\ x_0 \circ x_0 \cup \{x_{n-1}\} & \text{if } i = j = n - 1, \\ \{x_i\} & \text{if } i \neq j. \end{cases}
\]

By definition of \(D_2\) we have \(D_2(U_2(H_{n-1})) = H_{n-1}\), also we have:

\[
x_i \circ d_2 x_j = \begin{cases} x_i \circ u_2 x_i - \{x_{n-1}\} & \text{if } i = j, \\ \{x_i\} & \text{if } i \neq j. \end{cases}
\]

\[
= \begin{cases} x_i \circ x_i \cup \{x_{n-1}\} - \{x_{n-1}\} & \text{if } i = j, \\ \{x_i\} & \text{if } i \neq j. \end{cases}
\]

\[
= \begin{cases} x_i \circ x_i & \text{if } i = j, \\ \{x_i\} & \text{if } i \neq j. \end{cases}
\]

So \(D_2 U_2 = 1_{H_{n-1}}\). Similarly we can show that \(U_2 D_2 = 1_{H_{n}^{22}}, D_1 U_1 = 1_{H_{n-1}^{11} \cup H_{n-1}^{12}}\) and \(U_1 D_1 = 1_{H_{n}^{11}}\). \(\square\)

**Lemma 3.4.** i) \(|H_{n}^{22}| = |H_{n-1}|\), ii) \(|H_{n}^{11}| = |H_{n-1}^{11}| + |H_{n-1}^{12}|\).

**Proof.** i) We define functions \(\psi\) and \(\phi\) as follows:

\[
\psi : H_{n-1} \longrightarrow H_{n}^{22}, \quad \psi(H_{n-1}) = U_2(H_{n-1}), \\
\phi : H_{n-1}^{11} \longrightarrow H_{n-1}^{11} \cup H_{n-1}^{12}, \quad \phi(H_{n-1}^{11}) = D_1(H_{n-1}).
\]

By Theorem 3.3, it’s clear that \(\psi\) and \(\phi\) are bijection functions. So \(|H_{n}^{22}| = |H_{n-1}|\) and since \(H_{n-1}^{11}\) and \(H_{n-1}^{12}\) are distinct sets we have \(|H_{n}^{11}| = |H_{n-1}^{11}| + |H_{n-1}^{12}|\). \(\square\)

By attention to the last results, the Conjecture 2.10 is modified as follows:

**Theorem 3.5.** \((\text{modified conjecture})\) \(|H_n| = 2|H_{n-1}| - |H_{n-2}| + |H_{n}^{12}|\).
Proof. Since $\mathcal{H}_n = \mathcal{H}_n^{11} \cup \mathcal{H}_n^{12} \cup \mathcal{H}_n^{22}$ and $\mathcal{H}_n^{11}$, $\mathcal{H}_n^{12}$ and $\mathcal{H}_n^{22}$ are distinct sets, then $|\mathcal{H}_n| = |\mathcal{H}_n^{11}| + |\mathcal{H}_n^{12}| + |\mathcal{H}_n^{22}|$. By Lemma 3.4, we have $|\mathcal{H}_n^{22}| = |\mathcal{H}_{n-1}|$ and $|\mathcal{H}_n^{11}| = |\mathcal{H}_{n-1}^{11}| + |\mathcal{H}_{n-1}^{12}|$.

So

$$|\mathcal{H}_n| = |\mathcal{H}_n^{11}| + |\mathcal{H}_n^{12}| + |\mathcal{H}_n^{22}|$$

$$= (|\mathcal{H}_{n-1}^{11}| + |\mathcal{H}_{n-1}^{12}|) + |\mathcal{H}_n^{12}| + |\mathcal{H}_{n-1}|$$

$$= (|\mathcal{H}_{n-1}^{11}| - |\mathcal{H}_{n-2}^{11}|) + |\mathcal{H}_n^{12}| + |\mathcal{H}_{n-1}|$$

$$= 2|\mathcal{H}_{n-1}| + |\mathcal{H}_n^{12}| - |\mathcal{H}_{n-2}|$$

$$= 2|\mathcal{H}_{n-1}| - |\mathcal{H}_{n-2}| + |\mathcal{H}_{n-1}^{12}|$$

$\square$

Open problem: What’s the number of $|\mathcal{H}_n^{12}|$ respect to $n$?

4 non isomorphic quasi union hyper $K$-algebras of order 6 In this section at first we prove some theorems that help us to distinguish which quasi union hyper $K$-algebras are non isomorphic. Then we introduce the elements of $\mathcal{H}_6^{12}$, and show that $|\mathcal{H}_6^{12}| = 346$ which is not equal to $\left(\begin{array}{c}2 \times 6 - 2 \\ 6 - 2\end{array}\right)$. So the Conjecture 2.10 is not true in general.

Note. Let $H_n = \{x_0 = 0, x_1, x_2, \ldots, x_{n-1}\}$ and $(H_n, \circ, 0)$ is a quasi union hyper $K$-algebra. For simplicity, we show $(H_n, \circ, 0)$ by ordered set $\{F^x_i : 0 \leq i \leq n - 1\}$, where $F^x_i = x_i \circ x_i$.

Theorem 4.1. Let $H_n$ be a quasi union hyper $K$-algebra and $x \sim y$ if and only if $|F^x| = |F^y|$. Then $\sim$ is an equivalence relation on $H_n$.

Proof. Straightforward. $\square$

Definition 4.2. Let $H_n$ be a quasi union hyper $K$-algebra and $x, y \in H_n$. We say that $x$ and $y$ are co-class if and only if $x \sim y$. We denote the equivalence classes of $\sim$ by $E_0, E_1, \ldots, E_k, 1 \leq k \leq n$.

Definition 4.3. Let $H_n = \{x_0 = 0, x_1, \ldots, x_{n-1}\}$ be a quasi union hyper $K$-algebra. We relate matrix $A(H_n) = (a_{ij})$ to $H_n$ such that $a_{ij} = \chi_{F^x_i}(x_i), i = 0, \ldots, n - 1$, where $\chi_{F^x_i}$ is a characteristic function. The column that consists of sum of row’s elements is called last column and so the row that consists of sum of column’s elements is called last row. If $H_f$ and $H_g$ be two quasi union hyper $K$-algebras of order $n$, then two row’s of $A(H_f)$ and $A(H_g)$ are called co-row if one is a permutation of the other and If each row of $A(H_f)$ is co-row with a row of $A(H_g)$, then we say $A(H_f)$ is co-matrix with $A(H_g)$.

Example 4.4. Let $H_f = \{\{0\}, \{0, 1\}, \{0, 2\}, \{0, 1, 3, 4\}\}$ and $H_g = \{\{0\}, \{0, 1\}, \{0, 2\}, \{0, 2, 3, 4\}\}$ be two isomorphic quasi union hyper $K$-algebras on $H = \{0, 1, 2, 3, 4\}$. Then $A(H_f)$ and $A(H_g)$ are as follows:

<table>
<thead>
<tr>
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<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>last column</th>
</tr>
</thead>
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<td>0 ∘ 0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
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<td>1</td>
<td>0</td>
<td>0</td>
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<td>2</td>
</tr>
<tr>
<td>2 ∘ 2</td>
<td>1</td>
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<td>0</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>3 ∘ 3</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>4 ∘ 4</td>
<td>1</td>
<td>1</td>
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<td>1</td>
<td>1</td>
<td>4</td>
</tr>
</tbody>
</table>

| last row | 5 | 3 | 1 | 2 | 2 |

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### Theorem 4.5
Let $H_f$ and $H_g$ be two isomorphic quasi union hyper K-algebras of type $(l_0, l_1, \ldots, l_{n-1})$. Then:

1. The last column of $A(H_f)$ and $A(H_g)$ are the same, and they are types of $H_f$ and $H_g$.
2. The last row of $A(H_f)$ and $A(H_g)$ are co-row.
3. $A(H_f)$ is co-matrix with $A(H_g)$.
4. If $\sigma$ is isomorphism between $H_f$ and $H_g$, then $\sigma(E_i) = E_i$.

**Proof.** By Theorem 2.6, the proof is clear. \qed

### Remark
The corollary 2.7(ii) is not true generally. See next example.

### Example 4.6
Consider two quasi union hyper K-algebras $H_f$ and $H_g$ on $H = \{0, 1, 2, 3, 4, 5\}$ as follows:

$$
H_f = \{\{0\}, \{0,1\}, \{0,2\}, \{0,1\}, \{0,2\}, \{0,1\}\},
H_g = \{\{0\}, \{0,1\}, \{0,2\}, \{0,1\}, \{0,2\}, \{0,2\}\}.
$$

$F^{x_i} = G^{x_i}$ for $i = 0, 1, 2, 3, 4$ and $F^5 \neq G^5$, but $H_f$ and $H_g$ are isomorphic to each other by permutation $(12)(34)$.

In the next, we classify non isomorphic quasi union hyper K-algebras of order 6, and also we show that the number of non isomorphic quasi union hyper K-algebras of order 5 gave in [6] is not true. Then we give quasi union hyper K-algebras of order 5 which are not counted in [6].

### Theorem 4.7
Let $H_i = \{0, 1, 2, \ldots, i - 1\}, 1 \leq i \leq 6$. Then there are

1. 1 non-isomorphic quasi union hyper K-algebra on $H_1$,
2. 3 non-isomorphic quasi union hyper K-algebras on $H_2$,
3. 9 non-isomorphic quasi union hyper K-algebras on $H_3$,
4. 30 non-isomorphic quasi union hyper K-algebras on $H_4$,
5. 118 non-isomorphic quasi union hyper K-algebras on $H_5$,

**Proof.** To prove (i)-(iv) see Theorem 4.1[6]. To prove (v) we refer to Theorem 4.1(v)[6], but there are non isomorphic quasi union hyper K-algebras of order 5 which were not counted in [6], and they are as follows:

#### Type (1,2,2,2,3):

- $1 - \{\{0\}, \{0,1\}, \{0,1\}, \{0,3\}, \{0,3,4\}\}$

#### Type (1,2,2,3,3):

- $2 - \{\{0\}, \{0,1\}, \{0,1\}, \{0,3,4\}, \{0,3,4\}\}$
- $3 - \{\{0\}, \{0,1\}, \{0,2\}, \{0,1,2\}, \{0,1,4\}\}$
- $4 - \{\{0\}, \{0,1\}, \{0,2\}, \{0,1,3\}, \{0,2,4\}\}$
- $5 - \{\{0\}, \{0,1\}, \{0,2\}, \{0,3,4\}, \{0,3,4\}\}$

<table>
<thead>
<tr>
<th>$A(H_g)$</th>
<th>0</th>
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<th>2</th>
<th>3</th>
<th>4</th>
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</tr>
</thead>
<tbody>
<tr>
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<td>0</td>
<td>0</td>
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<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1 \circ 1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>2 \circ 2</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>3 \circ 3</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>4 \circ 4</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>4</td>
</tr>
</tbody>
</table>
Type \((1,2,2,3,4)\):

\[
6 = \{\{0\}, \{0, 1\}, \{0, 2\}, \{0, 1, 2\}, \{0, 1, 2, 4\}\},
7 = \{\{0\}, \{0, 1\}, \{0, 2\}, \{0, 1, 3\}, \{0, 1, 3, 4\}\}.
\]

Type \((1,2,2,4,4)\):

\[
8 = \{\{0\}, \{0, 1\}, \{0, 2\}, \{0, 1, 2, 3\}, \{0, 1, 2, 4\}\}.
\]

Type \((1,2,3,3,3)\):

\[
9 = \{\{0\}, \{0, 1\}, \{0, 1, 2\}, \{0, 3, 4\}, \{0, 3, 4\}\}.
\]

Type \((1,2,3,3,4)\):

\[
10 = \{\{0\}, \{0, 1\}, \{0, 1, 2\}, \{0, 1, 2, 4\}\},
11 = \{\{0\}, \{0, 1\}, \{0, 2, 3\}, \{0, 2, 3\}, \{0, 1, 2, 3\}\}.
\]

Type \((1,2,3,4,4)\):

\[
12 = \{\{0\}, \{0, 1\}, \{0, 1, 2\}, \{0, 1, 3, 4\}, \{0, 1, 3, 4\}\}.
\]

Also \(\{\{0\}, \{0, 1, 2, 3\}, \{0, 1, 2, 3\}, \{0, 1, 2, 3\}, \{0, 1, 2, 4\}\}\) of type \((1,4,4,4,4)\) is not a quasi union hyper K-algebra, and was more counted in [6]. So \(|\mathcal{H}_6^{12}| = 56 + 11 = 67\) and \(|\mathcal{H}_6| = 118\).

Proof of vi) To obtain all non isomorphic quasi union hyper K-algebras of order 6 and type \((l_0, l_1, \ldots, l_5)\) such that \(l_0 = 1, l_1 \geq 2\), we wrote a computer program and by using it we obtained all quasi union hyper K-algebras of order 6. Then by Theorem 4.5, we classified them. Finally we obtained 346 non isomorphic quasi union hyper K-algebras of order 6 and type \((l_0, l_1, \ldots, l_5)\) where \(l_0 = 1\) and \(l_1 \geq 2\). By Theorem 3.2(Going up), we can obtain all elements of \(\mathcal{H}_6^{11}\) and \(\mathcal{H}_6^{22}\) which we don’t bring them.

Elements of \(\mathcal{H}_6^{12}\) are as follows:

Type \((1,2,2,2,2,2)\):

\[
1 = \{\{0\}, \{0, 1\}, \{0, 1\}, \{0, 1\}, \{0, 1\}, \{0, 1\}\},
2 = \{\{0\}, \{0, 1\}, \{0, 1\}, \{0, 1\}, \{0, 4\}, \{0, 1\}\},
3 = \{\{0\}, \{0, 1\}, \{0, 1\}, \{0, 1\}, \{0, 4\}, \{0, 4\}\},
4 = \{\{0\}, \{0, 1\}, \{0, 1\}, \{0, 1\}, \{0, 4\}, \{0, 5\}\},
5 = \{\{0\}, \{0, 1\}, \{0, 1\}, \{0, 3\}, \{0, 3\}, \{0, 5\}\},
6 = \{\{0\}, \{0, 1\}, \{0, 1\}, \{0, 3\}, \{0, 4\}, \{0, 5\}\},
7 = \{\{0\}, \{0, 1\}, \{0, 2\}, \{0, 3\}, \{0, 4\}, \{0, 5\}\}.
\]

Type \((1,2,2,2,2,3)\):

\[
8 = \{\{0\}, \{0, 1\}, \{0, 1\}, \{0, 1\}, \{0, 1\}, \{0, 1, 5\}\},
9 = \{\{0\}, \{0, 1\}, \{0, 1\}, \{0, 1\}, \{0, 4\}, \{0, 1, 5\}\},
10 = \{\{0\}, \{0, 1\}, \{0, 1\}, \{0, 3\}, \{0, 3\}, \{0, 1, 3\}\},
11 = \{\{0\}, \{0, 1\}, \{0, 1\}, \{0, 4\}, \{0, 1, 5\}\},
12 = \{\{0\}, \{0, 1\}, \{0, 2\}, \{0, 3\}, \{0, 2\}, \{0, 1, 2\}\},
13 = \{\{0\}, \{0, 1\}, \{0, 1\}, \{0, 3\}, \{0, 4\}, \{0, 4, 5\}\},
14 = \{\{0\}, \{0, 1\}, \{0, 2\}, \{0, 3\}, \{0, 2\}, \{0, 1, 3\}\},
15 = \{\{0\}, \{0, 1\}, \{0, 1\}, \{0, 3\}, \{0, 3\}, \{0, 1, 5\}\},
16 = \{\{0\}, \{0, 1\}, \{0, 1\}, \{0, 3\}, \{0, 4\}, \{0, 1, 5\}\},
17 = \{\{0\}, \{0, 1\}, \{0, 1\}, \{0, 3\}, \{0, 4\}, \{0, 1, 5\}\},
18 = \{\{0\}, \{0, 1\}, \{0, 2\}, \{0, 3\}, \{0, 4\}, \{0, 1, 2\}\},
19 = \{\{0\}, \{0, 1\}, \{0, 2\}, \{0, 3\}, \{0, 4\}, \{0, 1, 5\}\}.
\]
Type (1,2,2,2,4):
20 – \{\{0\}, \{0,1\}, \{0,1\}, \{0,1\}, \{0,4\}, \{0,1,4,5\}\},
21 – \{\{0\}, \{0,1\}, \{0,1\}, \{0,3\}, \{0,4\}, \{0,1,3,4\}\},
22 – \{\{0\}, \{0,1\}, \{0,1\}, \{0,3\}, \{0,4\}, \{0,1,3,5\}\},
23 – \{\{0\}, \{0,1\}, \{0,1\}, \{0,3\}, \{0,4\}, \{0,3,4,5\}\},
24 – \{\{0\}, \{0,1\}, \{0,2\}, \{0,3\}, \{0,4\}, \{0,1,2,3\}\},
25 – \{\{0\}, \{0,1\}, \{0,2\}, \{0,3\}, \{0,4\}, \{0,1,2,5\}\}.

Type (1,2,2,2,5):
26 – \{\{0\}, \{0,1\}, \{0,1\}, \{0,3\}, \{0,4\}, \{0,1,3,4,5\}\},
27 – \{\{0\}, \{0,1\}, \{0,2\}, \{0,3\}, \{0,4\}, \{0,1,2,3,4\}\},
28 – \{\{0\}, \{0,1\}, \{0,2\}, \{0,3\}, \{0,4\}, \{0,1,2,3,5\}\}.

Type (1,2,2,2,6):
29 – \{\{0\}, \{0,1\}, \{0,2\}, \{0,3\}, \{0,4\}, \{0,1,2,3,4,5\}\}.

Type (1,2,2,3,3):
30 – \{\{0\}, \{0,1\}, \{0,1\}, \{0,1\}, \{0,1,4\}, \{0,1,4\}\},
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32 – \{\{0\}, \{0,1\}, \{0,1\}, \{0,1\}, \{0,1,4\}, \{0,1,5\}\},
33 – \{\{0\}, \{0,1\}, \{0,2\}, \{0,3\}, \{0,2,3\}, \{0,1,2\}\},
34 – \{\{0\}, \{0,1\}, \{0,1\}, \{0,1\}, \{0,4,5\}, \{0,4,5\}\},
35 – \{\{0\}, \{0,1\}, \{0,1\}, \{0,3\}, \{0,1,3\}, \{0,1,5\}\},
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41 – \{\{0\}, \{0,1\}, \{0,2\}, \{0,3\}, \{0,2,3\}, \{0,2,5\}\},
42 – \{\{0\}, \{0,1\}, \{0,1\}, \{0,3\}, \{0,1,4\}, \{0,3,5\}\},
43 – \{\{0\}, \{0,1\}, \{0,2\}, \{0,3\}, \{0,3,4\}, \{0,1,2\}\},
44 – \{\{0\}, \{0,1\}, \{0,1\}, \{0,3\}, \{0,3,4\}, \{0,3,5\}\},
45 – \{\{0\}, \{0,1\}, \{0,2\}, \{0,3\}, \{0,3,4\}, \{0,3,4\}\},
46 – \{\{0\}, \{0,1\}, \{0,1\}, \{0,3\}, \{0,4,5\}, \{0,4,5\}\},
47 – \{\{0\}, \{0,1\}, \{0,2\}, \{0,3\}, \{0,1,4\}, \{0,1,5\}\},
48 – \{\{0\}, \{0,1\}, \{0,2\}, \{0,3\}, \{0,4,5\}, \{0,4,5\}\}.

Type (1,2,2,3,4):
49 – \{\{0\}, \{0,1\}, \{0,1\}, \{0,1\}, \{0,1,4\}, \{0,1,4,5\}\},
50 – \{\{0\}, \{0,1\}, \{0,1\}, \{0,3\}, \{0,1,3\}, \{0,1,3,5\}\},
51 – \{\{0\}, \{0,1\}, \{0,1\}, \{0,3\}, \{0,1,4\}, \{0,1,3,4\}\},
52 – \{\{0\}, \{0,1\}, \{0,1\}, \{0,3\}, \{0,3,4\}, \{0,1,3,4\}\},
53 – \{\{0\}, \{0,1\}, \{0,2\}, \{0,3\}, \{0,1,2\}, \{0,1,2,3\}\},
54 – \{\{0\}, \{0,1\}, \{0,1\}, \{0,3\}, \{0,1,4\}, \{0,1,3,5\}\},
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62 − \{\{0\}, \{0, 1\}, \{0, 2\}, \{0, 3\}, \{0, 1, 3, 5\}\},
63 − \{\{0\}, \{0, 1\}, \{0, 2\}, \{0, 3\}, \{0, 1, 4\}, \{0, 1, 2, 5\}\},
64 − \{\{0\}, \{0, 1\}, \{0, 2\}, \{0, 3\}, \{0, 1, 4\}, \{0, 1, 4, 5\}\},
65 − \{\{0\}, \{0, 1\}, \{0, 2\}, \{0, 3\}, \{0, 1, 4\}, \{0, 2, 3, 5\}\}.

**Type (1,2,2,2,3,5):**

66 − \{\{0\}, \{0, 1\}, \{0, 1\}, \{0, 1\}, \{0, 1, 3, 4, 5\}\},
67 − \{\{0\}, \{0, 1\}, \{0, 1\}, \{0, 1\}, \{0, 1, 3, 4, 5\}\},
68 − \{\{0\}, \{0, 1\}, \{0, 1\}, \{0, 1\}, \{0, 1, 3, 4, 5\}\},
69 − \{\{0\}, \{0, 1\}, \{0, 1\}, \{0, 1\}, \{0, 1, 2, 3, 5\}\},
70 − \{\{0\}, \{0, 1\}, \{0, 1\}, \{0, 1\}, \{0, 1, 2, 3, 5\}\},
71 − \{\{0\}, \{0, 1\}, \{0, 1\}, \{0, 1\}, \{0, 1, 2, 3, 5\}\}.

**Type (1,2,2,2,3,6):**

72 − \{\{0\}, \{0, 1\}, \{0, 2\}, \{0, 3\}, \{0, 1, 2, 3, 4, 5\}\}.

**Type (1,2,2,2,4,4):**

73 − \{\{0\}, \{0, 1\}, \{0, 1\}, \{0, 1\}, \{0, 1, 4, 5\}\},
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75 − \{\{0\}, \{0, 1\}, \{0, 1\}, \{0, 1\}, \{0, 1, 4, 5\}\},
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77 − \{\{0\}, \{0, 1\}, \{0, 1\}, \{0, 1\}, \{0, 1, 4, 5\}\},
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81 − \{\{0\}, \{0, 1\}, \{0, 1\}, \{0, 1\}, \{0, 1, 2, 3, 4\}\},
82 − \{\{0\}, \{0, 1\}, \{0, 1\}, \{0, 1\}, \{0, 1, 3, 5\}\},
83 − \{\{0\}, \{0, 1\}, \{0, 1\}, \{0, 1\}, \{0, 1, 4, 5\}\}.

**Type (1,2,2,2,4,5):**

84 − \{\{0\}, \{0, 1\}, \{0, 1\}, \{0, 1\}, \{0, 1, 2, 3, 4\}\},
85 − \{\{0\}, \{0, 1\}, \{0, 1\}, \{0, 1\}, \{0, 1, 2, 3, 5\}\},
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**Type (1,2,2,2,4,6):**

89 − \{\{0\}, \{0, 1\}, \{0, 1\}, \{0, 1\}, \{0, 1, 2, 3, 4\}\}.

**Type (1,2,2,2,5,5):**

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92 − \{\{0\}, \{0, 1\}, \{0, 1\}, \{0, 1\}, \{0, 1, 2, 3, 4\}\},
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**Type (1,2,2,2,5,6):**

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Type (1,2,2,2,6,6):
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Type (1,2,2,3,3,3):
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108 - \{\{0\}, \{0,1\}, \{0,2\}, \{0,1,2\}, \{0,4,5\}, \{0,4,5\}\},
109 - \{\{0\}, \{0,1\}, \{0,2\}, \{0,1,3\}, \{0,1,3\}, \{0,2,5\}\},
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Type (1,2,2,3,3,4):
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129 - \{\{0\}, \{0,1\}, \{0,2\}, \{0,1,3\}, \{0,2,4\}, \{0,1,2,3\}\},
130 - \{\{0\}, \{0,1\}, \{0,2\}, \{0,1,3\}, \{0,1,4\}, \{0,1,2,5\}\},
131 - \{\{0\}, \{0,1\}, \{0,2\}, \{0,1,3\}, \{0,1,4\}, \{0,1,3,5\}\},
132 - \{\{0\}, \{0,1\}, \{0,2\}, \{0,1,3\}, \{0,2,4\}, \{0,1,2,5\}\},
133 - \{\{0\}, \{0,1\}, \{0,2\}, \{0,1,3\}, \{0,2,4\}, \{0,1,3,5\}\},
134 - \{\{0\}, \{0,1\}, \{0,2\}, \{0,3,4\}, \{0,3,4\}, \{0,1,2,5\}\},
135 - \{\{0\}, \{0,1\}, \{0,2\}, \{0,3,4\}, \{0,3,4\}, \{0,3,4,5\}\}.

Type (1,2,2,3,3,5):
136 − \{\{0\}, \{0,1\}, \{0,1\}, \{0,1,3\}, \{0,1,4\}, \{0,1,3,4,5\}\},
137 − \{\{0\}, \{0,1\}, \{0,1\}, \{0,1,3\}, \{0,1,4\}, \{0,1,3,4,5\}\},
138 − \{\{0\}, \{0,1\}, \{0,2\}, \{0,1,2\}, \{0,1,4\}, \{0,1,2,4,5\}\},
139 − \{\{0\}, \{0,1\}, \{0,2\}, \{0,1,3\}, \{0,1,3\}, \{0,1,2,3,5\}\},
140 − \{\{0\}, \{0,1\}, \{0,2\}, \{0,1,3\}, \{0,1,4\}, \{0,1,2,3,4\}\},
141 − \{\{0\}, \{0,1\}, \{0,2\}, \{0,1,3\}, \{0,2,4\}, \{0,1,2,3,4\}\},
142 − \{\{0\}, \{0,1\}, \{0,2\}, \{0,3,4\}, \{0,3,4\}, \{0,1,2,3,4\}\},
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144 − \{\{0\}, \{0,1\}, \{0,2\}, \{0,1,3\}, \{0,1,4\}, \{0,1,3,4,5\}\},
145 − \{\{0\}, \{0,1\}, \{0,2\}, \{0,1,3\}, \{0,2,4\}, \{0,1,2,3,5\}\},
146 − \{\{0\}, \{0,1\}, \{0,2\}, \{0,3,4\}, \{0,3,4\}, \{0,1,3,4,5\}\}.

**Type (1,2,2,3,3,6):**

147 − \{\{0\}, \{0,1\}, \{0,2\}, \{0,1,3\}, \{0,1,4\}, \{0,1,2,3,4,5\}\},
148 − \{\{0\}, \{0,1\}, \{0,2\}, \{0,1,3\}, \{0,2,4\}, \{0,1,2,3,4,5\}\},
149 − \{\{0\}, \{0,1\}, \{0,2\}, \{0,3,4\}, \{0,3,4\}, \{0,1,2,3,4,5\}\}.

**Type (1,2,2,3,4,4):**

150 − \{\{0\}, \{0,1\}, \{0,1\}, \{0,1,3\}, \{0,1,3,4\}, \{0,1,3,4\}\},
151 − \{\{0\}, \{0,1\}, \{0,2\}, \{0,1,2\}, \{0,1,2,4\}, \{0,1,2,4\}\},
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159 − \{\{0\}, \{0,1\}, \{0,2\}, \{0,1,3\}, \{0,1,2,4\}, \{0,1,2,3\}\},
160 − \{\{0\}, \{0,1\}, \{0,2\}, \{0,1,3\}, \{0,1,2,4\}, \{0,1,2,4\}\},
161 − \{\{0\}, \{0,1\}, \{0,2\}, \{0,1,3\}, \{0,1,3,4\}, \{0,1,2,3\}\},
162 − \{\{0\}, \{0,1\}, \{0,2\}, \{0,1,3\}, \{0,1,3,4\}, \{0,1,3,4\}\},
163 − \{\{0\}, \{0,1\}, \{0,2\}, \{0,1,3\}, \{0,1,2,4\}, \{0,1,2,5\}\},
164 − \{\{0\}, \{0,1\}, \{0,2\}, \{0,1,3\}, \{0,1,2,4\}, \{0,1,3,5\}\},
165 − \{\{0\}, \{0,1\}, \{0,2\}, \{0,1,3\}, \{0,1,3,4\}, \{0,1,3,5\}\},
166 − \{\{0\}, \{0,1\}, \{0,2\}, \{0,1,3\}, \{0,1,4,5\}, \{0,1,4,5\}\},
167 − \{\{0\}, \{0,1\}, \{0,2\}, \{0,1,3\}, \{0,2,4,5\}, \{0,2,4,5\}\}.

**Type (1,2,2,3,4,5):**

168 − \{\{0\}, \{0,1\}, \{0,1\}, \{0,1,3\}, \{0,1,3,4\}, \{0,1,3,4,5\}\},
169 − \{\{0\}, \{0,1\}, \{0,2\}, \{0,1,2\}, \{0,1,2,4\}, \{0,1,2,4,5\}\},
170 − \{\{0\}, \{0,1\}, \{0,2\}, \{0,1,3\}, \{0,1,2,3\}, \{0,1,2,3,5\}\},
171 − \{\{0\}, \{0,1\}, \{0,2\}, \{0,1,3\}, \{0,1,2,4\}, \{0,1,2,3,4\}\},
172 − \{\{0\}, \{0,1\}, \{0,2\}, \{0,1,3\}, \{0,1,3,4\}, \{0,1,2,3,4\}\},
173 − \{\{0\}, \{0,1\}, \{0,2\}, \{0,1,3\}, \{0,1,2,4\}, \{0,1,2,3,5\}\},
174 − \{\{0\}, \{0,1\}, \{0,2\}, \{0,1,3\}, \{0,1,2,4\}, \{0,1,2,4,5\}\},
175 − \{\{0\}, \{0,1\}, \{0,2\}, \{0,1,3\}, \{0,1,3,4\}, \{0,1,2,3,5\}\},
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**Type (1,2,2,3,4,6):**
177 − \{0\}, \{0, 1\}, \{0, 2\}, \{0, 1, 3\}, \{0, 1, 2, 3, 4, 5\}
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**Type (1,2,2,3,5,5):**

179 − \{0\}, \{0, 1\}, \{0, 1, 3\}, \{0, 1, 3, 4, 5\}, \{0, 1, 3, 4, 5\}
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**Type (1,2,2,3,5,6):**

185 − \{0\}, \{0, 1\}, \{0, 2\}, \{0, 1, 3\}, \{0, 1, 2, 3, 4, 5\}

**Type (1,2,2,3,6,6):**

186 − \{0\}, \{0, 1\}, \{0, 2\}, \{0, 1, 3\}, \{0, 1, 2, 3, 4, 5\}

**Type (1,2,2,4,4,4):**

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193 − \{0\}, \{0, 1\}, \{0, 2\}, \{0, 1, 2, 3\}, \{0, 1, 4, 5\}, \{0, 1, 4, 5\}
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**Type(1,2,2,4,4,5):**

195 − \{0\}, \{0, 1\}, \{0, 1, 3, 4\}, \{0, 1, 3, 4\}, \{0, 1, 3, 4\}, \{0, 1, 3, 4\}
196 − \{0\}, \{0, 1\}, \{0, 2\}, \{0, 1, 2, 3\}, \{0, 1, 2, 3\}, \{0, 1, 2, 3\}, \{0, 1, 2, 3\}
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198 − \{0\}, \{0, 1\}, \{0, 2\}, \{0, 1, 3, 4\}, \{0, 1, 3, 4\}, \{0, 1, 2, 3, 4\}
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200 − \{0\}, \{0, 1\}, \{0, 2\}, \{0, 1, 3, 4\}, \{0, 1, 3, 4\}, \{0, 1, 3, 4\}

**Type (1,2,2,4,4,6):**

201 − \{0\}, \{0, 1\}, \{0, 2\}, \{0, 1, 2, 3\}, \{0, 1, 2, 4\}, \{0, 1, 2, 3, 4, 5\}
202 − \{0\}, \{0, 1\}, \{0, 2\}, \{0, 1, 3, 4\}, \{0, 1, 3, 4\}, \{0, 1, 2, 3, 4, 5\}

**Type (1,2,2,4,5,5):**

203 − \{0\}, \{0, 1\}, \{0, 2\}, \{0, 1, 2, 3\}, \{0, 1, 2, 3, 4\}, \{0, 1, 2, 3, 4\}
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**Type (1,2,2,4,5,6):**

206 − \{0\}, \{0, 1\}, \{0, 2\}, \{0, 1, 2, 3\}, \{0, 1, 2, 3, 4\}, \{0, 1, 2, 3, 4, 5\}

**Type (1,2,2,4,6,6):**

207 − \{0\}, \{0, 1\}, \{0, 2\}, \{0, 1, 2, 3\}, \{0, 1, 2, 3, 4, 5\}, \{0, 1, 2, 3, 4, 5\}.
CLASSIFICATION OF QUASI UNION HYPER K-ALGEBRAS OF ORDER 6

Type (1,2,2,5,5,5):

208 = \{\{0\}, \{0, 1\}, \{0, 1\}, \{0, 1, 3, 4, 5\}, \{0, 1, 3, 4, 5\}, \{0, 1, 3, 4, 5\}\},
209 = \{\{0\}, \{0, 1\}, \{0, 2\}, \{0, 1, 2, 3, 4\}, \{0, 1, 2, 3, 4\}, \{0, 1, 2, 3, 4\}\},
210 = \{\{0\}, \{0, 1\}, \{0, 2\}, \{0, 1, 3, 4, 5\}, \{0, 1, 3, 4, 5\}, \{0, 1, 3, 4, 5\}\}.

Type (1,2,2,5,5,6):

211 = \{\{0\}, \{0, 1\}, \{0, 2\}, \{0, 1, 2, 3, 4\}, \{0, 1, 2, 3, 4\}, \{0, 1, 2, 3, 4, 5\}\}.

Type (1,2,2,6,6,6):

212 = \{\{0\}, \{0, 1\}, \{0, 2\}, \{0, 1, 2, 3, 4, 5\}, \{0, 1, 2, 3, 4, 5\}, \{0, 1, 2, 3, 4, 5\}\}.

Type (1,2,3,3,3,3):

213 = \{\{0\}, \{0, 1\}, \{0, 1, 2\}, \{0, 1, 2\}, \{0, 1, 2\}\},
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215 = \{\{0\}, \{0, 1\}, \{0, 2, 3\}, \{0, 2, 3\}, \{0, 2, 3\}\},
216 = \{\{0\}, \{0, 1\}, \{0, 1, 2\}, \{0, 1, 2\}, \{0, 1, 4\}, \{0, 1, 5\}\},
217 = \{\{0\}, \{0, 1\}, \{0, 1, 2\}, \{0, 1, 2\}, \{0, 1, 4, 5\}, \{0, 1, 4, 5\}\},
218 = \{\{0\}, \{0, 1\}, \{0, 1, 2\}, \{0, 3, 4\}, \{0, 3, 4\}, \{0, 3, 4\}\},
219 = \{\{0\}, \{0, 1\}, \{0, 1, 2\}, \{0, 1, 3\}, \{0, 1, 4\}, \{0, 1, 5\}\},
220 = \{\{0\}, \{0, 1\}, \{0, 1, 2\}, \{0, 1, 3\}, \{0, 4, 5\}, \{0, 4, 5\}\},
221 = \{\{0\}, \{0, 1\}, \{0, 2, 3\}, \{0, 2, 3\}, \{0, 4, 5\}, \{0, 4, 5\}\}.

Type (1,2,3,3,3,4):

222 = \{\{0\}, \{0, 1\}, \{0, 1, 2\}, \{0, 1, 2\}, \{0, 1, 2, 5\}\},
223 = \{\{0\}, \{0, 1\}, \{0, 1, 2\}, \{0, 1, 2\}, \{0, 1, 4, 5\}\},
224 = \{\{0\}, \{0, 1\}, \{0, 2, 3\}, \{0, 2, 3\}, \{0, 1, 2, 3\}\},
225 = \{\{0\}, \{0, 1\}, \{0, 1, 2\}, \{0, 1, 2\}, \{0, 1, 4, 5\}\},
226 = \{\{0\}, \{0, 1\}, \{0, 1, 2\}, \{0, 1, 3\}, \{0, 1, 4\}, \{0, 1, 2, 3\}\},
227 = \{\{0\}, \{0, 1\}, \{0, 1, 2\}, \{0, 1, 3\}, \{0, 3, 4\}, \{0, 1, 3, 4\}\},
228 = \{\{0\}, \{0, 1\}, \{0, 2, 3\}, \{0, 2, 3\}, \{0, 2, 3, 5\}\},
229 = \{\{0\}, \{0, 1\}, \{0, 1, 2\}, \{0, 1, 3\}, \{0, 1, 4\}, \{0, 1, 2, 5\}\},
230 = \{\{0\}, \{0, 1\}, \{0, 1, 2\}, \{0, 3, 4\}, \{0, 3, 4\}, \{0, 1, 2, 5\}\},
231 = \{\{0\}, \{0, 1\}, \{0, 1, 2\}, \{0, 3, 4\}, \{0, 3, 4\}, \{0, 3, 4, 5\}\}.

Type (1,2,3,3,3,5):

232 = \{\{0\}, \{0, 1\}, \{0, 1, 2\}, \{0, 1, 2\}, \{0, 1, 4, 5\}\},
233 = \{\{0\}, \{0, 1\}, \{0, 1, 2\}, \{0, 1, 3\}, \{0, 1, 4\}, \{0, 1, 2, 3, 4\}\},
234 = \{\{0\}, \{0, 1\}, \{0, 1, 2\}, \{0, 3, 4\}, \{0, 3, 4\}, \{0, 1, 2, 3, 4\}\},
235 = \{\{0\}, \{0, 1\}, \{0, 2, 3\}, \{0, 2, 3\}, \{0, 1, 2, 3, 5\}\},
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Type (1,2,3,3,3,6):

238 = \{\{0\}, \{0, 1\}, \{0, 1, 2\}, \{0, 1, 3\}, \{0, 1, 4\}, \{0, 1, 2, 3, 4, 5\}\},
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Type (1,2,3,3,4,4):
240 – \{\{0\}, \{0, 1\}, \{0, 1, 2\}, \{0, 1, 2, 4\}, \{0, 1, 2, 3\}\}.
241 – \{\{0\}, \{0, 1\}, \{0, 1, 2\}, \{0, 1, 3\}, \{0, 1, 2, 3\}\}.
242 – \{\{0\}, \{0, 1\}, \{0, 2, 3\}, \{0, 2, 3\}, \{0, 1, 2, 3\}\}.
243 – \{\{0\}, \{0, 1\}, \{0, 1, 2\}, \{0, 1, 2, 4\}, \{0, 1, 2, 5\}\}.
244 – \{\{0\}, \{0, 1\}, \{0, 1, 2\}, \{0, 1, 4, 5\}, \{0, 1, 4, 5\}\}.
245 – \{\{0\}, \{0, 1\}, \{0, 1, 2\}, \{0, 1, 3\}, \{0, 1, 2, 3\}\}.
246 – \{\{0\}, \{0, 1\}, \{0, 1, 2\}, \{0, 1, 3\}, \{0, 1, 2, 4\}\}.
247 – \{\{0\}, \{0, 1\}, \{0, 1, 2\}, \{0, 1, 2, 3\}, \{0, 2, 3, 5\}\}.
248 – \{\{0\}, \{0, 1\}, \{0, 2, 3\}, \{0, 2, 3\}, \{0, 2, 3, 4\}\}.
249 – \{\{0\}, \{0, 1\}, \{0, 1, 2\}, \{0, 1, 3\}, \{0, 1, 2, 4\}\}.
250 – \{\{0\}, \{0, 1\}, \{0, 1, 2\}, \{0, 1, 3\}, \{0, 1, 4, 5\}\}.
251 – \{\{0\}, \{0, 1\}, \{0, 2, 3\}, \{0, 2, 3\}, \{0, 1, 4, 5\}\}.
252 – \{\{0\}, \{0, 1\}, \{0, 2, 3\}, \{0, 2, 3\}, \{0, 2, 3, 4\}\}.

**Type (1,2,3,4,5):**

253 – \{\{0\}, \{0, 1\}, \{0, 1, 2\}, \{0, 1, 2\}, \{0, 1, 2, 4\}, \{0, 1, 2, 4, 5\}\}.
254 – \{\{0\}, \{0, 1\}, \{0, 1, 2\}, \{0, 1, 3\}, \{0, 1, 2, 3\}\}.
255 – \{\{0\}, \{0, 1\}, \{0, 1, 2\}, \{0, 1, 3\}, \{0, 1, 2, 4\}, \{0, 1, 2, 3, 5\}\}.
256 – \{\{0\}, \{0, 1\}, \{0, 2, 3\}, \{0, 2, 3\}, \{0, 1, 2, 4\}\}.
257 – \{\{0\}, \{0, 1\}, \{0, 2, 3\}, \{0, 2, 3\}, \{0, 1, 2, 3, 5\}\}.
258 – \{\{0\}, \{0, 1\}, \{0, 1, 2\}, \{0, 1, 2\}, \{0, 1, 2, 4\}\}.
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260 – \{\{0\}, \{0, 1\}, \{0, 2, 3\}, \{0, 2, 3\}, \{0, 2, 3, 4\}\}.
261 – \{\{0\}, \{0, 1\}, \{0, 2, 3\}, \{0, 2, 3\}, \{0, 2, 3, 4\}\}.

**Type (1,2,3,4,6):**

262 – \{\{0\}, \{0, 1\}, \{0, 1, 2\}, \{0, 1, 3\}, \{0, 1, 2, 4\}, \{0, 1, 2, 3, 4, 5\}\}.
263 – \{\{0\}, \{0, 1\}, \{0, 2, 3\}, \{0, 2, 3\}, \{0, 1, 2, 3, 4, 5\}\}.

**Type (1,2,3,5,5):**

264 – \{\{0\}, \{0, 1\}, \{0, 1, 2\}, \{0, 1, 2\}, \{0, 1, 2, 4, 5\}\}.
265 – \{\{0\}, \{0, 1\}, \{0, 1, 2\}, \{0, 1, 3\}, \{0, 1, 2, 3, 4\}\}.
266 – \{\{0\}, \{0, 1\}, \{0, 2, 3\}, \{0, 2, 3\}, \{0, 1, 2, 3, 4\}\}.
267 – \{\{0\}, \{0, 1\}, \{0, 1, 2\}, \{0, 1, 3\}, \{0, 1, 2, 3, 4\}\}.
268 – \{\{0\}, \{0, 1\}, \{0, 1, 2\}, \{0, 1, 3\}, \{0, 1, 2, 4, 5\}\}.
269 – \{\{0\}, \{0, 1\}, \{0, 2, 3\}, \{0, 2, 3\}, \{0, 1, 2, 3, 5\}\}.
270 – \{\{0\}, \{0, 1\}, \{0, 2, 3\}, \{0, 2, 3\}, \{0, 2, 3, 4, 5\}\}.

**Type (1,2,3,5,6):**

271 – \{\{0\}, \{0, 1\}, \{0, 1, 2\}, \{0, 1, 3\}, \{0, 1, 2, 3, 4, 5\}\}.
272 – \{\{0\}, \{0, 1\}, \{0, 2, 3\}, \{0, 2, 3\}, \{0, 1, 2, 3, 4, 5\}\}.

**Type (1,2,3,6,6):**

273 – \{\{0\}, \{0, 1\}, \{0, 1, 2\}, \{0, 1, 3\}, \{0, 1, 2, 3, 4, 5\}\}.
274 – \{\{0\}, \{0, 1\}, \{0, 2, 3\}, \{0, 2, 3\}, \{0, 1, 2, 3, 4, 5\}\}.

**Type (1,2,3,4,4):**
275 – \{\{0\}, \{0,1\}, \{0,1,2\}, \{0,1,2,3\}, \{0,1,2,3\}\},
276 – \{\{0\}, \{0,1\}, \{0,1,2\}, \{0,1,2,3\}, \{0,1,2,3,4\}\},
277 – \{\{0\}, \{0,1\}, \{0,1,2\}, \{0,1,2,3\}, \{0,1,2,3,4\}\},
278 – \{\{0\}, \{0,1\}, \{0,1,2\}, \{0,1,2,3\}, \{0,1,2,4\}\},
279 – \{\{0\}, \{0,1\}, \{0,1,2\}, \{0,1,2,3\}, \{0,1,4,5\}\},
280 – \{\{0\}, \{0,1\}, \{0,1,2\}, \{0,3,4,5\}\}.

**Type (1,2,3,4,4,5):**

281 – \{\{0\}, \{0,1\}, \{0,1,2\}, \{0,1,2,3\}, \{0,1,2,3,4\}\},
282 – \{\{0\}, \{0,1\}, \{0,1,2\}, \{0,1,2,3\}, \{0,1,2,3,4\}\},
283 – \{\{0\}, \{0,1\}, \{0,1,2\}, \{0,1,2,3\}, \{0,1,3,4\}\},
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**Type (1,2,3,4,4,6):**

286 – \{\{0\}, \{0,1\}, \{0,1,2\}, \{0,1,2,3\}, \{0,1,2,3,4\}\},
287 – \{\{0\}, \{0,1\}, \{0,1,2\}, \{0,1,3,4\}\}.

**Type (1,2,3,4,5,5):**

288 – \{\{0\}, \{0,1\}, \{0,1,2\}, \{0,1,2,3\}, \{0,1,2,4,5\}\},
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**Type (1,2,3,4,5,6):**

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**Type (1,2,3,4,6,6):**

293 – \{\{0\}, \{0,1\}, \{0,1,2\}, \{0,1,2,3,4\}\},
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**Type (1,2,3,5,5,5):**

295 – \{\{0\}, \{0,1\}, \{0,1,2\}, \{0,1,2,3,4\}\},
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**Type (1,2,3,6,6,6):**

297 – \{\{0\}, \{0,1\}, \{0,1,2\}, \{0,1,2,3\}, \{0,1,2,3,5\}\},
298 – \{\{0\}, \{0,1\}, \{0,1,2,3\}, \{0,1,2,3,4\}\},
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**Type (1,2,4,4,4,5):**

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301 – \{\{0\}, \{0,1\}, \{0,2,3,4\}\},
302 – \{\{0\}, \{0,1\}, \{0,2,3,4\}\}.

**Type (1,2,4,4,4,6):**
303 – \{\{0\}, \{0, 1\}, \{0, 2, 3, 4\}, \{0, 2, 3, 4\}, \{0, 2, 3, 4, 5\}\}.

**Type (1,2,4,4,5,5):**

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305 – \{\{0\}, \{0, 1\}, \{0, 1, 2, 3\}, \{0, 1, 2, 3\}, \{0, 1, 2, 3, 4\}\}.

**Type (1,2,4,4,5,6):**

306 – \{\{0\}, \{0, 1\}, \{0, 1, 2, 3\}, \{0, 1, 2, 3\}, \{0, 1, 2, 3, 4, 5\}\}.

**Type (1,2,4,4,6,6):**

307 – \{\{0\}, \{0, 1\}, \{0, 1, 2, 3\}, \{0, 1, 2, 3, 4\}, \{0, 1, 2, 3, 4, 5\}\}.

**Type (1,2,5,5,5,5):**

308 – \{\{0\}, \{0, 1\}, \{0, 1, 2, 3, 4\}, \{0, 1, 2, 3, 4\}, \{0, 1, 2, 3, 4\}\},

309 – \{\{0\}, \{0, 1\}, \{0, 2, 3, 4\}, \{0, 2, 3, 4\}, \{0, 2, 3, 4, 5\}\}.

**Type (1,2,5,5,5,6):**

310 – \{\{0\}, \{0, 1\}, \{0, 1, 2, 3, 4\}, \{0, 1, 2, 3, 4\}, \{0, 1, 2, 3, 4, 5\}\}.

**Type (1,2,6,6,6,6):**

311 – \{\{0\}, \{0, 1\}, \{0, 1, 2, 3, 4\}, \{0, 1, 2, 3, 4\}, \{0, 1, 2, 3, 4, 5\}\}.

**Type (1,3,3,3,3,3):**

312 – \{\{0\}, \{0, 1\}, \{0, 1, 2\}, \{0, 1, 2\}, \{0, 1, 2\}\},

313 – \{\{0\}, \{0, 1\}, \{0, 1, 2\}, \{0, 1, 2\}, \{0, 4, 5\}\}.

**Type (1,3,3,3,3,4):**

314 – \{\{0\}, \{0, 1\}, \{0, 1, 2\}, \{0, 1, 2\}, \{0, 1, 2, 5\}\},

315 – \{\{0\}, \{0, 1\}, \{0, 1, 2\}, \{0, 3, 4\}, \{0, 3, 4\}, \{0, 1, 2, 5\}\}.

**Type (1,3,3,3,3,5):**

316 – \{\{0\}, \{0, 1\}, \{0, 1, 2\}, \{0, 3, 4\}, \{0, 3, 4\}, \{0, 1, 2, 3, 4\}\}.

**Type (1,3,3,3,3,6):**

317 – \{\{0\}, \{0, 1\}, \{0, 1, 2\}, \{0, 3, 4\}, \{0, 3, 4\}, \{0, 1, 2, 3, 4, 5\}\}.

**Type (1,3,3,3,4,4):**

318 – \{\{0\}, \{0, 1\}, \{0, 1, 2\}, \{0, 1, 2\}, \{0, 1, 2, 4\}\},

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**Type (1,3,3,3,4,5):**

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**Type (1,3,3,3,5,5):**

321 – \{\{0\}, \{0, 1\}, \{0, 1\}, \{0, 1, 2\}, \{0, 1, 2, 4, 5\}\}.

**Type (1,3,3,4,4,4):**
CLASSIFICATION OF QUASI UNION HYPER K-ALGEBRAS OF ORDER 6 181

322 − \{0\}, \{0, 1, 2\}, \{0, 1, 2\}, \{0, 1, 2, 3\}, \{0, 1, 2, 3\}, \{0, 1, 2, 3\}.
323 − \{0\}, \{0, 1, 2\}, \{0, 1, 2\}, \{0, 1, 2, 3\}, \{0, 1, 2, 3\}, \{0, 1, 2, 3\}.
324 − \{0\}, \{0, 1, 2\}, \{0, 1, 2\}, \{0, 1, 2, 3\}, \{0, 1, 2, 4\}, \{0, 1, 2, 5\}.
325 − \{0\}, \{0, 1, 2\}, \{0, 1, 2\}, \{0, 3, 4, 5\}, \{0, 3, 4, 5\}, \{0, 3, 4, 5\}.

Type (1,3,3,4,4,5):
326 − \{0\}, \{0, 1, 2\}, \{0, 1, 2\}, \{0, 1, 2, 3\}, \{0, 1, 2, 3\}, \{0, 1, 2, 3, 5\}.
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328 − \{0\}, \{0, 1, 2\}, \{0, 1, 2\}, \{0, 1, 2, 3\}, \{0, 1, 2, 4\}, \{0, 1, 2, 3, 5\}.

Type (1,3,3,4,4,6):
329 − \{0\}, \{0, 1, 2\}, \{0, 1, 2\}, \{0, 1, 2, 3\}, \{0, 1, 2, 4\}, \{0, 1, 2, 3, 4, 5\}.

Type (1,3,3,4,5,5):
330 − \{0\}, \{0, 1, 2\}, \{0, 1, 2\}, \{0, 1, 2, 3\}, \{0, 1, 2, 3, 4\}, \{0, 1, 2, 3, 5\}.
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332 − \{0\}, \{0, 1, 2\}, \{0, 1, 2\}, \{0, 1, 2, 3\}, \{0, 1, 2, 4, 5\}, \{0, 1, 2, 4\}.

Type (1,3,3,4,5,6):
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Type (1,3,3,4,6,6):
334 − \{0\}, \{0, 1, 2\}, \{0, 1, 2\}, \{0, 1, 2, 3\}, \{0, 1, 2, 3, 4, 5\}, \{0, 1, 2, 3, 4\}.

Type (1,3,3,5,5,5):
335 − \{0\}, \{0, 1, 2\}, \{0, 1, 2\}, \{0, 1, 2, 3, 4\}, \{0, 1, 2, 3, 4\}, \{0, 1, 2, 3, 4\}.

Type (1,3,3,5,5,6):
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Type (1,3,3,6,6,6):
337 − \{0\}, \{0, 1, 2\}, \{0, 1, 2\}, \{0, 1, 2, 3, 4, 5\}, \{0, 1, 2, 3, 4, 5\}, \{0, 1, 2, 3, 4\}.

Type (1,4,4,4,4,4):
338 − \{0\}, \{0, 1, 2, 3\}, \{0, 1, 2, 3\}, \{0, 1, 2, 3\}, \{0, 1, 2, 3\}, \{0, 1, 2, 3\}.

Type (1,4,4,4,4,5):
339 − \{0\}, \{0, 1, 2, 3\}, \{0, 1, 2, 3\}, \{0, 1, 2, 3\}, \{0, 1, 2, 3, 5\}.

Type (1,4,4,4,5,5):
340 − \{0\}, \{0, 1, 2, 3\}, \{0, 1, 2, 3\}, \{0, 1, 2, 3, 4\}, \{0, 1, 2, 3, 4, 5\}.
341 − \{0\}, \{0, 1, 2, 3\}, \{0, 1, 2, 3\}, \{0, 1, 2, 3, 4\}, \{0, 1, 2, 3, 5\}.

Type (1,4,4,4,5,6):
342 − \{0\}, \{0, 1, 2, 3\}, \{0, 1, 2, 3\}, \{0, 1, 2, 3, 4\}, \{0, 1, 2, 3, 4, 5\}.

Type (1,4,4,4,6,6):
343 − \{0\}, \{0, 1, 2, 3\}, \{0, 1, 2, 3\}, \{0, 1, 2, 3, 4, 5\}, \{0, 1, 2, 3, 4, 5\}.

Type (1,5,5,5,5,5):
Moreover by Theorem 3.5, we have

\[ |H_{11}^6| + |H_{22}^6| = 2|H_5| - |H_4| = 2 \times 118 - 30 = 206, \]

so

\[ |H_6| = 206 + |H_{12}^6| = 206 + 346 = 552, \]

and these complete the proof of (vi).

References


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