# COMPARATIVE MARKET SYSTEM ANALYSIS: LIMIT ORDER MARKET AND DEALER MARKET*

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Abstract. In this paper, we consider the relationship between the equilibrium spreads and the liquidity supplier's risk aversion coefficient, which represents the degree of risk aversion in a limit order market and a dealer market. Thus, we analyze which market is more liquid, the limit order market system or the dealer market system. It is concluded that the spread in a limit order market is not necessarily narrower than that in a dealer market, i.e., market liquidity depends on a liquidity supplier's attitude to the risky asset.

## 1 Introduction

Markets are divided into two types of systems. One is the dealer market, where investors buy at the dealer's ask price and sell at the dealer's bid price. The other is the limit order market, where investors buy (sell) at the limit sell (buy) price that has been previously placed. The former system is in place in the NASDAQ Stock Market and the London Stock Exchange. The latter system is evidenced by the New York Stock Exchange (NYSE) [1], the Paris Bourse and the Tokyo Stock Exchange.

It is commonly noted that the dealer market system has wider spread than the limit order market system because the dealer, who is the liquidity supplier in the dealer market, is more responsible for providing immediate execution than the limit order trader, who is the liquidity supplier in the limit order market. Thus, the supplier in the former requires larger payments, which tend to widen the spread. Though academic research has discussed which system is better, there is no definitive answer.

For example, in an empirical study comparing the limit order system with the dealer market system, Huang and Stoll (1996) conclude that the execution cost in NASDAQ (a dealer system) is twice as large as that in the NYSE (a limit order system). It follows that the dealer market system causes wider spreads than the limit order market system in order to manage the market. On the other hand, Suzuki and Yasuda (2005) examined the securities that switched from the limit order market system to the dealer system and found that they reduced both relative and effective bid–ask spreads in the JASDAQ market. This market, which is for venture, combined the limit order market system with the dealer market system by March 2008.

In a theoretical study of the limit order market system and the dealer market system, Handa et al. (2003) extend Foucault (1999) by incorporating the asymmetric information to observe the relationship between the spread and the proportion of buyers in the market and shows that the spread in the limit order market is wider than the spread in the dealer system over almost all values of the proportion of buyers in the market.

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[1] In NYSE, investors trade under specialist market system, which is combined the dealer market system and the limit order market system. O'Hara (1995) reports that only 19.4 percent of all dealings on average are traded under market order system. Thus we think that NYSE is the limit order system in this paper.

The purpose of this paper is to consider which market system is more liquid, the limit order market or the dealer market, when the liquidity supplier's attitude to risky asset changes. This paper extends Handa et al. (2003) by assuming that the liquidity supplier has risk-averse utility to compare the liquidity in the limit order market with the liquidity in the dealer market.

The paper is organized as follows. In the next section, the market mechanism and the strategies of traders are described. In section 3, we derive the equilibrium prices of the bid and ask, and their spread in the dealer markets. In section 4, we numerically examine the relationship between the liquidity supplier's attitude to risk and the equilibrium bid–ask spread in the limit order and the dealer market systems. The conclusions are presented in section 5.

**2 Market mechanism** In the market, there are four types of traders, while the market participants are divided into two groups: a buyer group and a seller group. The buyer (seller) group attaches high (low) value $V_h$ ($V_l$) to the risky asset. Thus, $V_h$ ($V_l$) is interpreted as the reservation price or the present value of the future cash flows on the risky asset for the buyer (seller) group. Additionally, the traders in the buyer (seller) group are divided into two trader types: informed and uninformed traders. The proportions of the buyer group and the seller group in the market are defined as $k$ and $1-k$, respectively. Similarly, we define the proportion of the informed trader in each group as $\delta$. The trader trades one risky asset in the following manner.

The buyer (seller) arrives in the market and trades one risky asset by either a market order or a limit order at time 0. If the buyer (seller) decides to submit a market order, then the order is executed at time 0. Thus, the wealth that the buyer (seller) gains on the trading is given by:

$$\widetilde{W} = \begin{cases} V_h - A^M & \text{if the buyer submits a market order,} \\ B^M - V_l & \text{if the seller submits a market order,} \end{cases}$$

where $A^M$ and $B^M$ are the ask and the bid prices, respectively, at time 0. If the buyer (seller) submits a limit order, then the order is not executed at time 0, but would be executed at time 1. The wealth that the buyer (seller) gains at time 1 on trading is determined by the next trader who arrives to trade the risky asset by either a market order or a limit order. If the next trader submits a market sell (buy) order, then the wealth of the buyer (seller) who submits the limit order at time 0, is given at time 1 by:

$$\widetilde{W} = \begin{cases} V_h + \tilde{\epsilon} - P_B^L & \text{if the buyer submits a limit order at time 0,} \\ P_S^L - V_l - \tilde{\epsilon} & \text{if the seller submits a limit order at time 0,} \end{cases}$$

where $P_B^L$ ($P_S^L$) is the specified price at which the limit buy (sell) order is submitted at time 0. The random variable $\tilde{\epsilon}$ represents the private information, which takes a value of $+H$ or $-H$ with equal probability. Only an informed trader can observe the private information ex ante. The risky asset has an expected liquidation value equal to $\bar{V}$ at time 0, where $\bar{V} = kV_h + (1-k)V_l$.

For simplicity, it is assumed that, if the next trader submits a limit order, the limit order that the trader submits at time 0 expires; i.e., if the limit order holds for only one period then the wealth for the trader who submits the limit order is 0. It is assumed that all parameters ($k$, $\delta$, $V_h$, $V_l$ and $H$) are known to all traders at time 0. The strategies of informed and uninformed traders are as follows.

An informed buyer who realizes the value of the risky asset as $V_h + H$ ex ante submits a market buy order if and only if $V_h + H > A^M$ where $A^M$ is the ask price. Similarly, an informed seller who realizes the value of the risky asset as $V_l - H$ ex ante submits a market sell order if and only if $V_l - H < B^M$ where $B^M$ is the bid price. If informed buyers submit the limit order, their private information would be revealed because they post the limit order at the specified price, giving the private information. Thus, an informed trader submits the market order only to protect private information.

It is also assumed that the magnitude of the private signal $\tilde{\epsilon}$ satisfies $V_h - H < A^M$ and $V_l + H > B^M$, and that the informed seller (buyer) switches to buyer (seller) with probability $\gamma$. The probability $\gamma$ is 1 when the informed seller and buyer identify the magnitude of the private signal that satisfies $A^M < V_l + H$ and $V_h - H < B^M$, respectively.

On the other hand, uninformed buyers (sellers) expect the value of the risky asset to be $V_h$ ($V_l$); i.e., $E[V_h + \tilde{\epsilon}] = V_h$ ($E[V_l + \tilde{\epsilon}] = V_l$) since they do not know whether the private information $\tilde{\epsilon}$ takes $+H$ or $-H$ ex ante. If an uninformed buyer submits a market order, the order is instantaneously executed at the ask price $A^M$ independently of the private information. The uninformed buyer's wealth is then given by $\widetilde{W} = V_h - A^M$. However, the ask price may be an undesirable price. The limit order is used to execute the order at a desirable price. The limit buy order, which the uninformed trader submits at time 0, could be executed at time 1 if the incoming trader is (i) an uninformed seller who submits a market order, (ii) an informed seller who observes the value of the risky asset as $V_l - H < P_B^L$ ex ante, and (iii) an informed buyer who switches to a seller by observing the value as $V_h - H < P_B^L$ ex ante. The uninformed buyer's wealth is then given by:

$$\widetilde{W} = \begin{cases} V_h + \tilde{\epsilon} - P_B^L & \text{if case (i)} \\ \\ V_h - H - P_B^L & \text{if case (ii) or case (iii),} \end{cases}$$

where $P_B^L$ is the specified price at which the limit buy order is submitted at time 0. However, the limit buy order expires at time 1 if the incoming trader is (iv) an uninformed seller who submits a limit order, (v) an uninformed buyer, or (vi) an informed buyer who does not switch to a seller. For simplicity, if the limit order expires, wealth is normalized to zero. The uninformed buyer's wealth is thus given by $\widetilde{W} = 0$.

Similarly, an uninformed seller faces a situation similar to that of an uninformed buyer. If uninformed sellers submit a market sell order, then their wealth is given by $\widetilde{W} = B^M - V_l$. If they submit a limit sell order at the specified price $P_S^L$, they gain the following wealth:

$$\widetilde{W} = \begin{cases} 0 & \text{the limit order expires,} \\ \\ P_S^L - V_l - H & \text{the limit order executes } (\tilde{\epsilon} = +H), \\ \\ P_S^L - V_l + H & \text{the limit order executes } (\tilde{\epsilon} = -H). \end{cases}$$

The decision tree for an uninformed trader is shown in Figure 1 and Figure 2.

In the limit order market, an uninformed trader is the liquidity supplier because only the limit order posted by an uninformed trader is taken up by the market order that the liquidity demander submits. In order to introduce the risk of trading into the liquidity supplier's utility, an uninformed trader is also assumed to have negative exponential utility with risk aversion coefficient $R$, which is known to all traders:

$$U(\widetilde{W}) = -\exp(-R\widetilde{W}). \tag{1}$$

If a buyer submits a market order, the utility for the buyer is $U(V_h - A^M)$ where $A^M$ is the ask price. If a buyer submits the limit order at the specified price $P_B^L$, the expected utility for the buyer can be written as follows:

$$E[U] = \phi_B^{(1)}U(0) + \phi_B^{(2)}U(V_h + H - P_B^L) + \phi_B^{(3)}U(V_h - H - P_B^L),$$

where $\phi_B^{(n)}$ $(n = 1, 2, 3)$ is the probability that the limit buy order is executed. The execution probability is given by:

$$\begin{aligned}
\phi_B^{(1)} &= \frac{1}{2}\delta(1-k) + k\left(1 - \frac{1}{2}\delta\gamma\right), \\
\phi_B^{(2)} &= \frac{1}{2}(1-\delta)(1-k), \\
\phi_B^{(3)} &= \frac{1}{2}(1-k) + \frac{1}{2}k\delta\gamma.
\end{aligned}$$

Similarly, the utility of the seller who submits a market order is $U(B^M - V_l)$ where $B^M$ represents the bid price. The expected utility of the seller who submits a limit order at the specified price $P_S^L$ can be written as follows:

$$E[U] = \phi_S^{(1)}U(0) + \phi_S^{(2)}U(P_S^L - V_l + H) + \phi_S^{(3)}U(P_S^L - V_l - H),$$

where $\phi_S^{(n)}$ $(n = 1, 2, 3)$ is the probability that the limit sell order is executed. The execution probability is given by:

$$\begin{aligned}
\phi_S^{(1)} &= \frac{1}{2}\delta k + (1-k)\left(1 - \frac{1}{2}\delta\gamma\right), \\
\phi_S^{(2)} &= \frac{1}{2}(1-\delta)k, \\
\phi_S^{(3)} &= \frac{1}{2}k + \frac{1}{2}(1-k)\delta\gamma.
\end{aligned}$$

In the next section, we derive the equilibrium bid and ask prices and their spread in the dealer market.

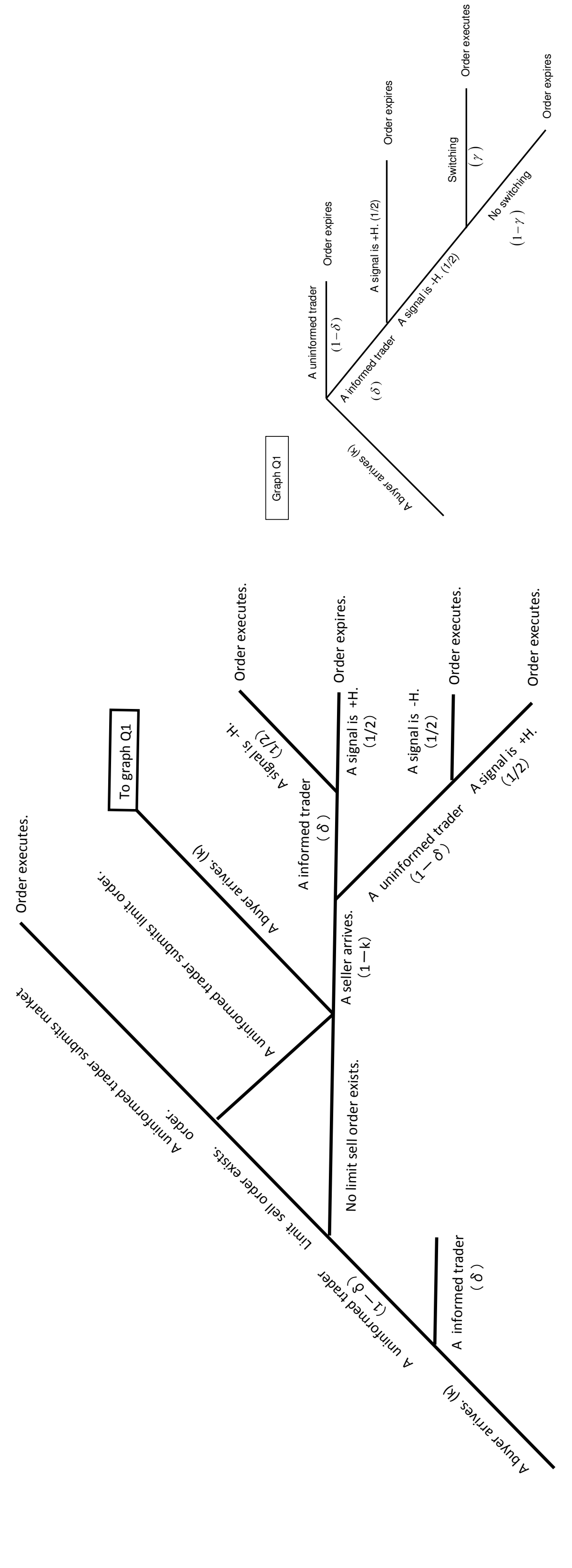


Figure 1: Decision tree for uninformed buyers.

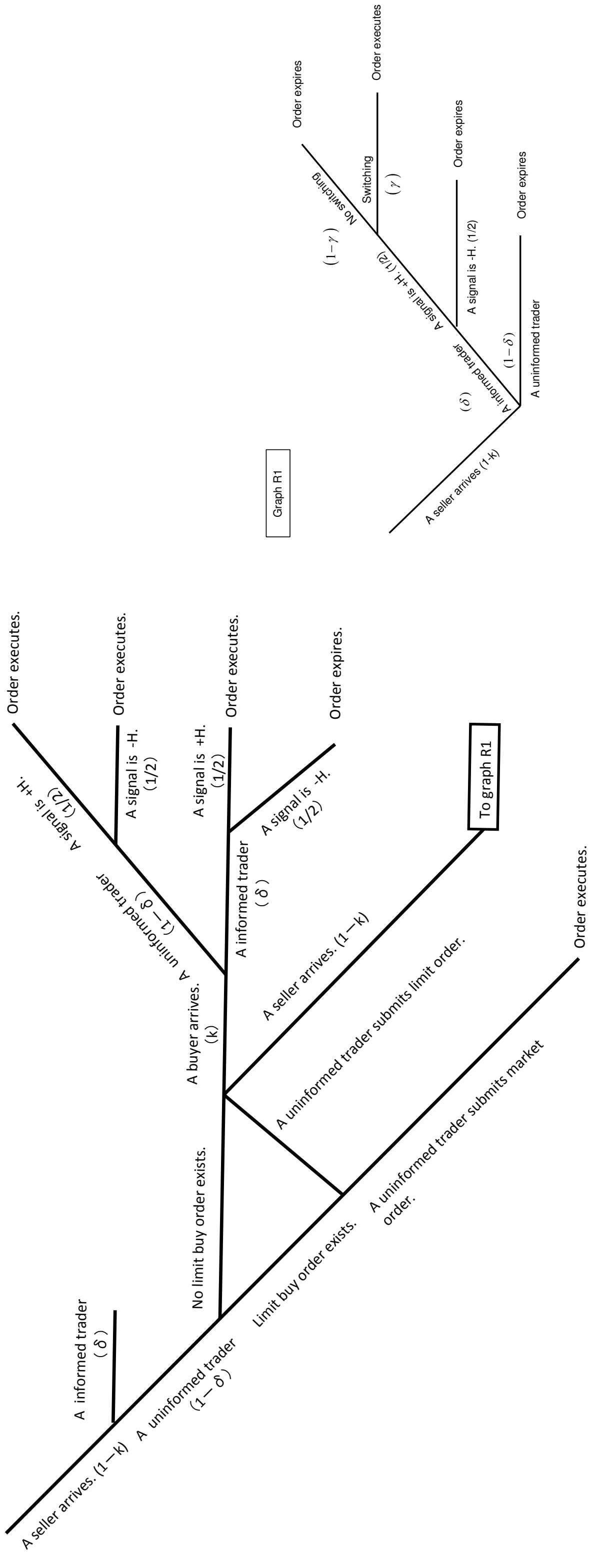


Figure 2: Decision tree for uninformed sellers.

**3 Equilibrium bid, ask prices and their spread** The bid and ask prices are important indexes for traders because they deal with the other traders at those prices. In the limit order market, the limit trader posts the prices. On the other hand, the dealer offers the prices in the dealer market. The spread, which involves the ask and bid prices, is one of the liquidity parameters in the security market. Therefore, the trader who offers the bid and ask prices is called the liquidity supplier.

In Hashimoto (2009), the equilibrium bid and ask prices and their spread are analyzed given the above market mechanism.

**Proposition 1.** *The equilibrium ask ($A^*$) and bid ($B^*$) prices and their spread ($S^*$) in the limit order market are given by:*

$$
\begin{aligned}
A^* &= \frac{1}{R}\log\left(\frac{(b_1-a_1)+a_0b_0\exp(R(V_h-V_l))+\sqrt{f(a_0,a_1,b_0,b_1,R,V_h,V_l)}}{2a_0\exp(-RV_l)}\right),\\
B^* &= \frac{1}{R}\log\left(\frac{2b_0\exp(RV_h)}{(a_1-b_1)+a_0b_0\exp(R(V_h-V_l))+\sqrt{f(a_0,a_1,b_0,b_1,R,V_h,V_l)}}\right),\\
S^* &= \frac{1}{R}\log\left(\frac{1}{2}\left[a_1+b_1+a_0b_0\exp(R(V_h-V_l)+\sqrt{f(a_0,a_1,b_0,b_1,R,V_h,V_l)}\right]\right),
\end{aligned}
$$

*where:*

$$
\begin{aligned}
b_0 &= -\phi_B^{(1)}U(0),\\
b_1 &= -\left[\phi_B^{(3)}U(-H)+\phi_B^{(2)}\{U(+H)+U(-H)\}\right],\\
a_0 &= -\phi_S^{(1)}U(0),\\
a_1 &= -\left[\phi_S^{(3)}U(-H)+\phi_S^{2}\{U(+H)+U(-H)\}\right],\\
f(a_0,a_1,b_0,b_1,R,V_h,V_l) &= (a_1-b_1)^2+2a_0b_0(a_1+b_1)\exp(R(V_h-V_l))\\
&\quad +a_0^2b_0^2\exp(2R(V_h-V_l)).
\end{aligned}
$$

*The parameters ($a_0$ , $a_1$ , $b_0$ , $b_1$ , $R$ , $V_h$ , $V_l$) also satisfy the following equations with the condition that $A^*>0$ , $B^*>0$ and $S^*>0$.*

$$
\begin{aligned}
a_1-b_1 &< -a_0\exp(-RV_l)+b_0\exp(RV_h),\\
a_1+b_1 &> 2-a_0b_0\exp(R(V_h-V_l))-\sqrt{f(a_0,a_1,b_0,b_1,R,V_h,V_l)}
\end{aligned}
$$

□

The equilibrium bid and ask prices and their spread in the dealer market are derived as follows, given the above market mechanism. The dealer is the liquidity supplier in the dealer market because the dealer offers bid and ask prices executing at least one share of the security immediately. In Glosten and Milgrom (1985), it is assumed that the bid and the ask prices are posted competitively among the dealers. Therefore, the dealer's expected wealth is zero at the optimal bid price $B^*$ and the optimal ask price $A^*$, which are offered conditional upon a sell order and a buy order, respectively.

The dealer posts the bid and ask prices based on $\bar{V}+\tilde{\epsilon}$, where $\bar{V}$ is the liquidation value of the risk asset at time 0, i.e., $\bar{V}=kV_h+(1-k)V_l$ and $\tilde{\epsilon}$ is the private information that the informed trader only observes ex ante. Informed buyers submit a buy order only when they observe the private information $\tilde{\epsilon}$ is $+H$ ex ante. While the uninformed buyer, who does not know whether the private information $\tilde{\epsilon}$ is $+H$ or $-H$, submits a buy order without regard to the private information. If a trader submits a buy order, the dealer's wealth is given by:

$$\widetilde{W} = \begin{cases} A-\bar{V}-H & \text{an informed trader submits } (\tilde{\epsilon}=+H), \\ 0 & \text{an informed trader submits } (\tilde{\epsilon}=-H), \\ A-\bar{V}\pm H & \text{an uninformed trader submits } (\tilde{\epsilon}=\pm H), \end{cases}$$

where $A$ is the ask price that the dealer offers.

Similarly, informed sellers submit a sell order only when they observe the private information $\tilde{\epsilon}$ is $-H$ ex ante. On the other hand, uninformed sellers necessarily submit a sell since they do not know the private information. If a trader submits a sell order, the wealth of the dealer who offers the bid price, $B$, is as follows:

$$\widetilde{W} = \begin{cases} 0 & \text{an informed trader submits } (\tilde{\epsilon}=+H), \\ \bar{V}-H-B & \text{an informed trader submits } (\tilde{\epsilon}=-H), \\ \bar{V}\pm H-B & \text{an uninformed trader submits } (\tilde{\epsilon}=\pm H), \end{cases}$$

where $B$ is the bid price that the dealer offers. The decision tree for the dealer trader is shown in Figure 3.

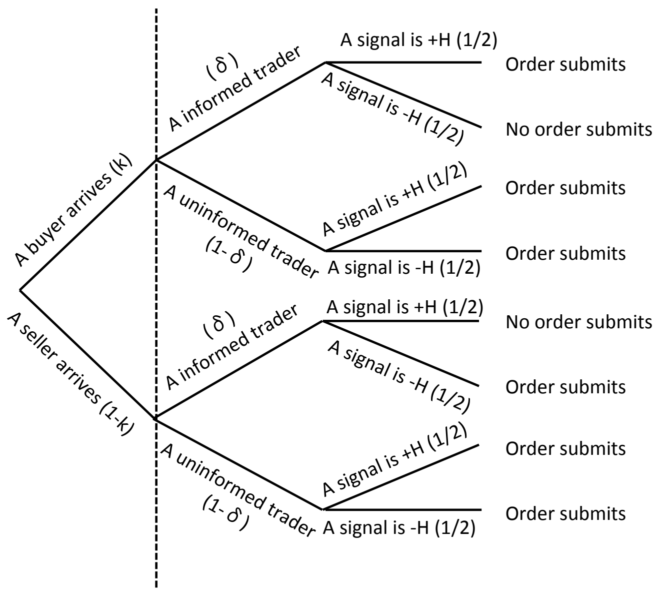


Figure 3: Decision tree for the dealer.

It is assumed that the dealer, who is the liquidity supplier in the dealer market, has negative exponential utility as with the limit order market [see Equation (1)]. The equilibrium bid and ask prices in the dealer market are characterized following Glosten and Milgrom (1985); i.e., the bid (ask) price is derived from the condition that the dealer's expected utility for a sell (buy) order equals the utility for zero profit. We can then express the expected utility of the dealer who offers the bid price $B$ as:

$$\begin{aligned} E\left[U_B\right] &= \frac{1}{2}(1-k)\delta U\left(0\right) + \frac{1}{2}(1-k)\delta U\left(\bar{V}-H-B\right) \\ &\quad + \frac{1}{2}(1-k)(1-\delta)U\left(\bar{V}+H-B\right) + \frac{1}{2}(1-k)(1-\delta)U\left(\bar{V}-H-B\right). \end{aligned}$$

In an equilibrium bid price, the dealer's expected utility on a sell order equals the utility of zero profit; i.e.,

$$U\left(0\right) = E\left[U_B\right]. \tag{2}$$

Similarly, the expected utility of the dealer who offers the ask price $A$ is expressed as:

$$\begin{aligned} E\left[U_A\right] &= \frac{1}{2}k\delta U\left(0\right) + \frac{1}{2}k\delta U\left(A-\bar{V}-H\right) + \frac{1}{2}k(1-\delta)U\left(A-\bar{V}-H\right) \\ &\quad + \frac{1}{2}k(1-\delta)U\left(A-\bar{V}+H\right). \end{aligned}$$

The equilibrium ask price is offered on the condition that the dealer's expected utility on a buy order equals the utility for zero profit; i.e.,

$$U\left(0\right) = E\left[U_A\right]. \tag{3}$$

The equilibrium ask price $A^*$, bid price $B^*$ and spread $S^*$ in Proposition 2 are derived from (2) and (3).

**Proposition 2.** *The equilibrium ask ($A^*$) and bid ($B^*$) prices and their spread ($S^*$) in the dealer market are given by:*

$$\begin{aligned} A^* &= \frac{1}{R}\log\left(\frac{k(1-\delta)\exp\left(R\left(\bar{V}+H\right)\right) + k\exp\left(R\left(\bar{V}-H\right)\right)}{2-k\delta}\right), \\ B^* &= \frac{1}{R}\log\left(\frac{2-(1-k)\delta}{(1-k)\exp\left(-R\left(\bar{V}-H\right)\right) + (1-k)(1-\delta)\exp\left(-R\left(\bar{V}+H\right)\right)}\right), \\ S^* &= \frac{1}{R}\log\left(\frac{k(1-k)\left\{(1-\delta)\exp\left(2RH\right) + (1-\delta)^2 + 1 + (1-\delta)\exp\left(-2RH\right)\right\}}{(2-k\delta)\left\{2-(1-k)\delta\right\}}\right). \end{aligned}$$

*The parameters ($k$ ,$\delta$ ,$H$ ,$R$ ,$V_h$ and $V_l$) also satisfy the following equations with the condition that $A^* > 0$ ,$B^* > 0$ and $S^* > 0$.*

$$\begin{aligned} &k(1-k)\left\{(1-\delta)\exp\left(2RH\right) + (1-\delta)^2 + 1 + (1-\delta)\exp\left(-2RH\right)\right\} \\ &\qquad > (2-k\delta)\left\{(1-k)\exp\left(-R\left(\bar{V}-H\right)\right) + (1-k)(1-\delta)\exp\left(-R\left(\bar{V}+H\right)\right)\right\} \end{aligned}$$

□

Because no useful result is obtained analytically at the equilibrium ask price $A^*$, bid price $B^*$ and their spread $S^*$ in the limit order and the dealer markets, we describe an explicit numerical method for characterizing these equilibriums in Section 4. In all the numerical analysis, we set the parameters as $V_h = 105,\ V_l = 95,\ H = 5,\ \delta = 0.375,\ R \geq 0.5$ and $\gamma = 0$. These parameters satisfy the conditions in Proposition 1 and the condition that $B^* < V_l + H < A^*$ and $B^* < V_h - H < A^*$ for $0 < k < 1$ where $k$ is the proportion of buyers in the market. These parameters also satisfy the conditions in Proposition 2.

In Section 4, we compare the equilibrium ask and bid prices, and their spread in the limit order market (Proposition 1) with them in the dealer market (Proposition 2).

**4 A numerical analysis** In the analysis of the market, the equilibrium spread is defined as:

$$Spread = A^* - B^*,$$

where $A^*$ and $B^*$ are the equilibrium ask and bid prices, respectively. This is an important measure characterizing the liquidity in the security market, because the trader who sells the securities after buying them incurs bigger losses when the spread is wider. Similarly the trader who buys back the securities after selling short also suffers bigger losses when the spread is wider. Therefore, the narrower and wider spreads mean more and less liquid markets, respectively.

**4.1 Comparison spread with risk aversion parameter** We analyze the relationship between the equilibrium spread and the risk aversion coefficient of the liquidity supplier's utility in each of the limit order and the dealer markets given the proportion of buyers in the market. In order to compare the limit order and the dealer markets on the liquidity supplier's risk-averse coefficient, see Figure 4. Figure 4 plots the spreads for the liquidity suppliers in each market on condition that suppliers from both market types have the same risk aversion parameter.

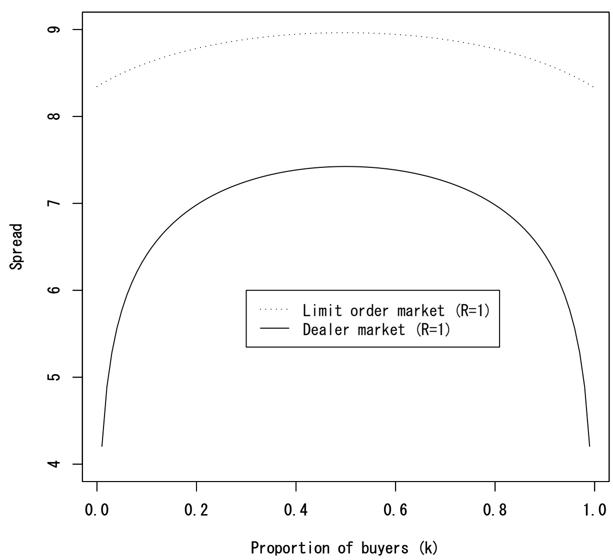


Figure 4: Spreads for limit order and dealer markets with same risk-averse coefficient.

The spread in the limit order market is wider than the spread in the dealer market for all values of the proportion of buyers in the market, i.e., the limit order market is less liquid than the dealer market. This conclusion is consistent with Suzuki and Yasuda (2005) showing that the spread in the limit order market is wider than the spread in the dealer market. Intuitively, the risk of whether the limit order is executed is specific to the liquidity supplier in the limit order market. This risk for the liquidity supplier in the dealer market does not exist. Therefore, the liquidity in the limit market is lower than that in the dealer market.

In order to analyze the relationship between the spread and the risk aversion coefficient $R$ of the liquidity supplier in each market, see Figure 5. Figure 5 plots the spreads for the case of the liquidity supplier who has risk aversion coefficient $R$ that is greater than 0.5 in each market. The proportions of buyers in the market $k$ are set as 0.01, 0.5 and 0.99, which mean that the liquidity supplier deals with buyers with low, equal and high probability, respectively.

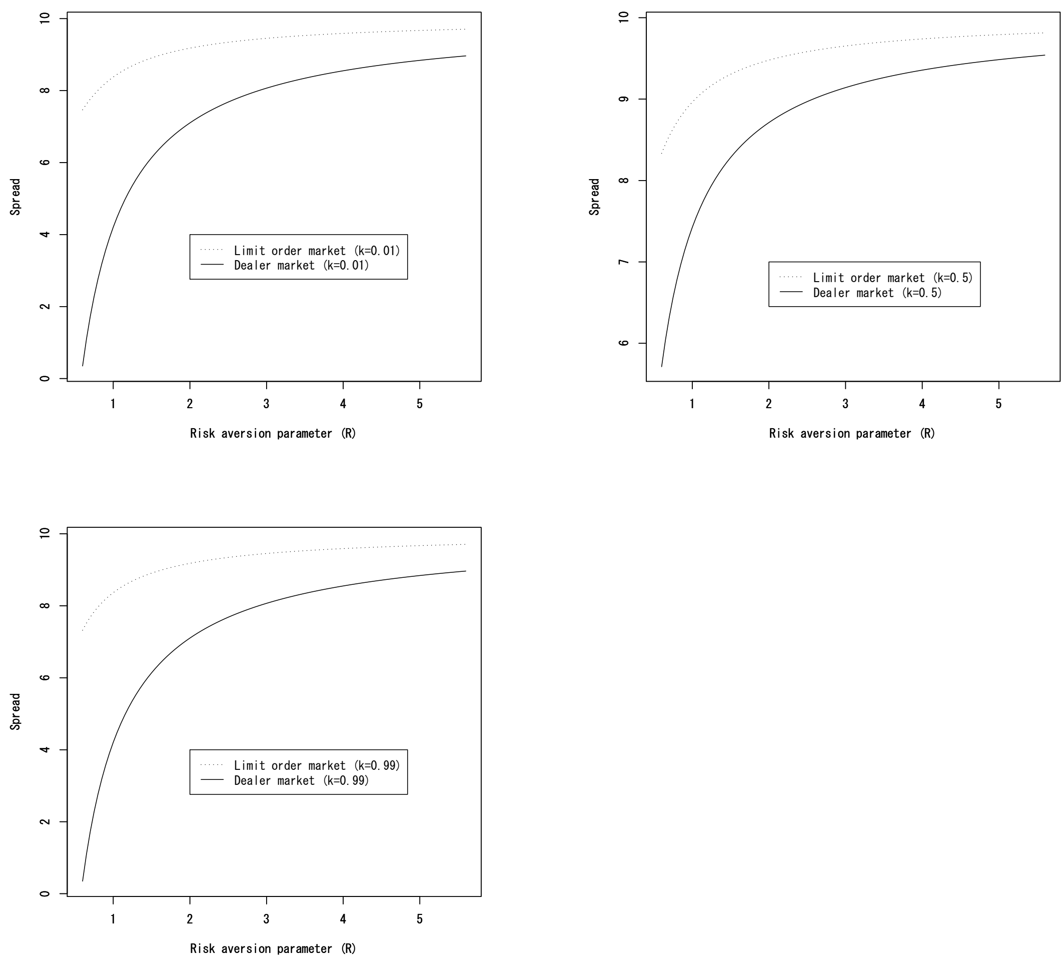


Figure 5: Spreads and risk-averse coefficient for different proportions of buyers.

Figure 5 shows that the increment of risk aversion makes the spread wider, i.e., less liquid in both markets. The sensitivity to risk induces the liquidity suppliers to widen the spread.

This result is consistent with Subrahmanyam (1991), which concludes that the increment in the risk-averse parameter makes the market less liquid.

Next, we analyze the spreads in the limit order and dealer markets with the condition that the liquidity suppliers in each market have a different risk aversion coefficient, see Figure 6. Figure 6 shows that the spread in the limit order market is not always wider than the spread in the dealer market for all $k$ when the liquidity supplier's risk aversion coefficient in each market is different. The result is consistent with Huang and Stoll (1996), which concludes that the spread in NASDAQ (a dealer system) is larger than the spread in the NYSE (a limit order system). Thus, it indicates that the liquidity in the market depends on the liquidity supplier's attitude to the risky asset.

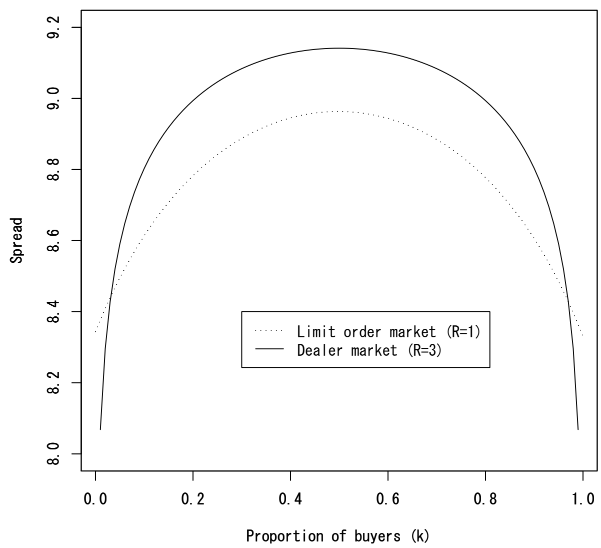


Figure 6: Spreads for limit order and dealer markets with different risk-averse coefficient.

In the following subsection, we consider bid and ask commissions for differnt risk aversion coefficient over the proportion of buyers in the market ($k$).

**4.2 Comparison bid, ask commissions with risk aversion parameter** We define the ask and bid commissions as in Handa et al. (2003), i.e.,

$$\begin{aligned} Ask\ Commission &= A^* - \bar{V}, \\ Bid\ Commission &= \bar{V} - B^*, \end{aligned}$$

where $\bar{V} = kV_h + (1-k)V_l$ is interpreted as the liquidation value of the risky asset. The smaller ask and bid commissions are profitable for the liquidity demanders who submit the market order.

The left and right panels of Figure 7 represent ask and bid commissions, respectively, in each market. This left panels of Figure 7 indicate that the ask commission in the limit order market is smaller than that in the dealer market when the proportion of buyers $k$ is closing 1, i.e., the buyer side is thicker. Since the probability of the limit sell order's expiration is lower, the limit sell trader, who is the liquidity supplier for buyer in the limit order market, requires low ask commission. On the other hand, the dealer, who is the liquidity supplier for buyer in the dealer market, posts high ask commission in order to face many buyers.

Similarly, the right panels of Figure 7 show that the bid commission in the limit order market is larger than that in the dealer market when $k$ is closing to 1. This means that the high probability of the limit buy order's expiration induces the limit buy trader, who is liquidity supplier for seller in the limit order market, to require high bid commission. Meanwhile, the dealer, who is the liquidity supplier for seller in the dealer market, posts the low bid commission in order to face a few sellers.

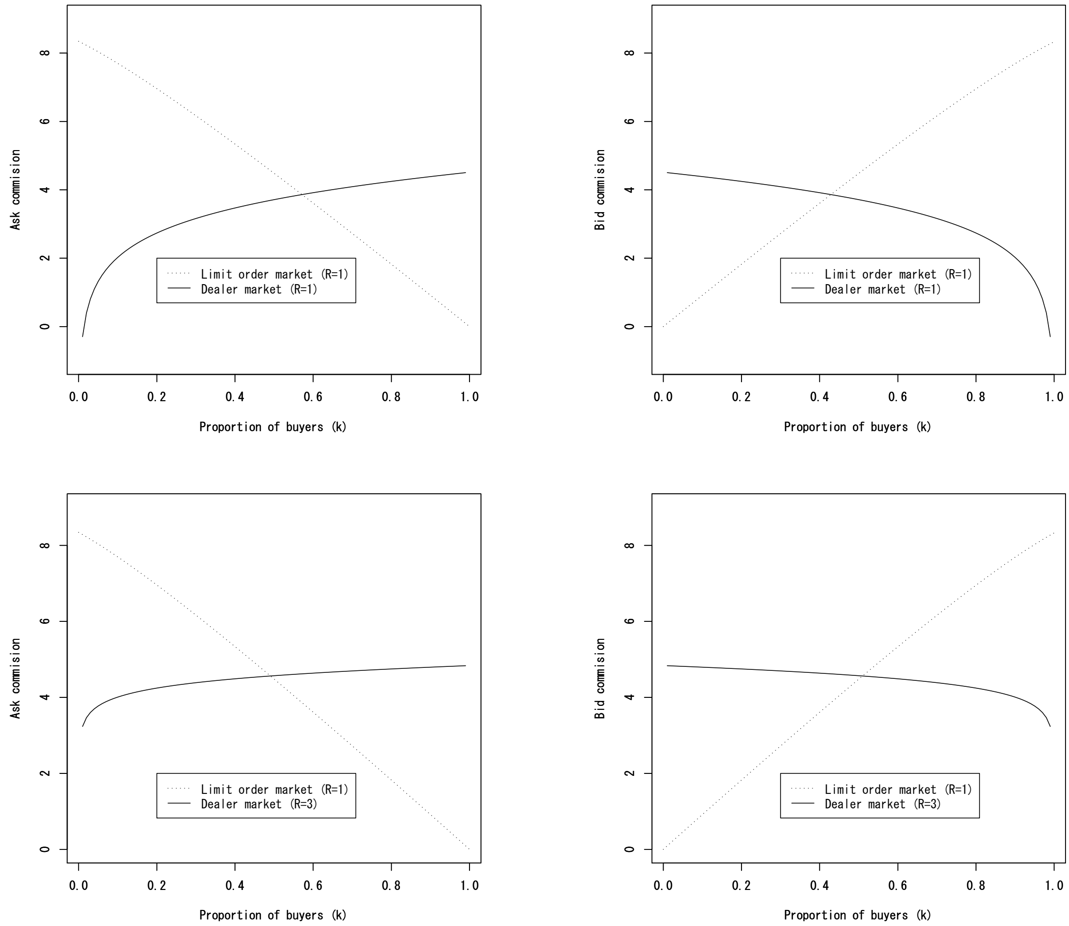


Figure 7: Ask and bid commissions for different risk-averse coefficient.

In next subsection, the relationship between the spreads and the unconditional variance of the risky asset is analyzed.

**4.3 Comparison spread with the variance of the risky asset** The unconditional variance of the risky asset in this model, $\sigma^2_{V\epsilon}$, is given by:

$$\sigma^2_{V\epsilon} = k(1-k)(V_h - V_l)^2 + H^2, \tag{4}$$

where $V_h(V_l)$, $k$ and $H$ are the buyer (seller) belief for the present value of the future cash flows on the risky asset, the proportion of buyers in the market and the value of private information, respectively [see Proposition 4 in the Handa et al. (2002)]. The Equation (4) is substituted into $S^*$ in Proposition 1 and $S^*$ in Proposition 2 in order to consider the relationship the spreads and the risk of the asset. The following figure is derived when the liquidity suppliers' risk aversion coefficient and the proportion of the buyers in the market are 1 and 0.5, respectively.

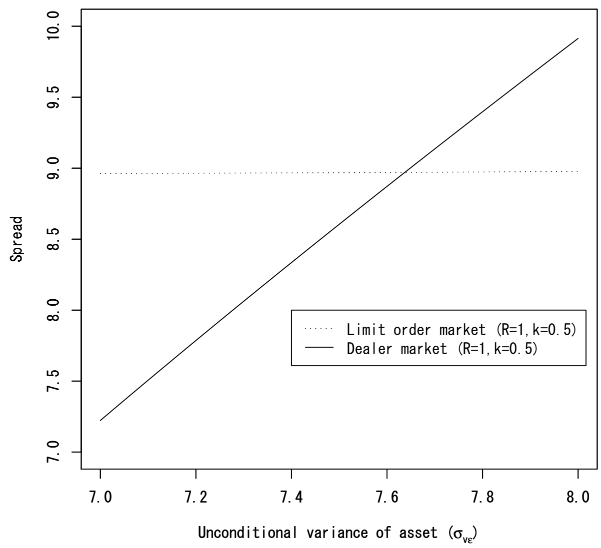


Figure 8: Spreads over the unconditional variance of the risky asset.

Figure 8 indicates that the spread in the limit order market is wider (narrower) than the spread in the dealer market if the unconditional variance of the risky asset is small (large). Thus, the limit order system is more liquid than the dealer market system if the risk of the asset is high. On the other hand, the dealer market system is more liquid than the limit order system if the risk of the asset is low.

**5 Conclusion** This paper compares the spread in the limit order market system with that in the dealer market system, allowing for the liquidity suppliers in each market who have risk-averse utility in the Arrow–Pratt sense.

This paper shows that the spread in the limit order market is wider than that in the dealer market when the liquidity suppliers in each market have the same risk aversion coefficient because the limit order in a limit order market is not always executed.

However, the spread in the limit order market is not necessarily wider than the spread in the dealer market when the liquidity suppliers in each market have different risk aversion coefficients.

We also show that the spreads in the limit order and the dealer markets are increasing in the liquidity supplier's risk aversion coefficient because they are sensitive to the risky asset. This result is consistent with Subrahmanyam (1991).

Another implication of this paper is that the ask commission in the limit order market is lower than that in the dealer market when the buyer side is thicker. Similarly, the bid commission in the limit order market is higher than that in the dealer market when the buyer side is thicker. This result suggests that the ask (bid) commission in the limit order market is narrower (wider) than that in the dealer market when the proportion of the buyers in the market is larger.

Finally, we note that the spread in the dealer market is wider than the spread in the limit order market when the unconditional variance of the risky asset is increasing. Thus, it is concluded that the limit order market system is more liquid than the dealer market system if the security market is uninformative.

This paper clearly indicates that the market's liquidity depends on the liquidity supplier's attitude to risk and the variance of the risky asset. Therefore, it is important to analyze the liquidity supplier's attitude to the risky asset from tick or daily transaction data in the security market. The research has a key role in deciding which market system is liquid for the existing markets. This remains a fruitful subject for further research.

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