A SIMPLE NUMBER THEORETIC PROBLEM III

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Abstract. In this note, we prove that positive integral solutions of $x_1^2+x_2^2+...+x_m^2 = y^2$ do not satisfy $x_1^n + x_2^n + ... + x_m^n = y^n$ by elementary way.

Let $a_1, a_2, ..., a_m, b$ be a positive integral solution of the equation

$$x_1^2 + x_2^2 + ... + x_m^2 = y^2.$$

Suppose that $a_1, a_2, ..., a_m, b$ is a solution of the equation

$$x_1^n + x_2^n + ... + x_m^n = y^n,$$

n is a natural number 3, 4, ...

$$b^n = b^{n-2}b^2 = b^{n-2}(a_1^2 + a_2^2 + ... + a_m^2)$$

$$> a_1^{n-2}a_1^2 + a_2^{n-2}a_2^2 + ... + a_m^{n-2}a_m^2 = b^n,$$

which is impossible.

Proposition. Any positive integral solution of

$$x_1^2 + x_2^2 + ... + x_m^2 = y^2$$

is not a solution of

$$x_1^n + x_2^n + ... + x_m^n = y^n (n = 3, 4, ...).$$

For example, in my notes [1],[2], there are identities

$$380^2 + 381^2 + ... + 9978^2 + 9979^2 = 575500^2$$

with 9600 terms,

$$307^2 + 309^2 + ... + 20303^2 + 20305^2 = 1181300^2$$

with 10000 terms.

These numbers do not satisfy the equation

$$x_1^n + x_2^n + ... + x_m^n = y^n,$$

where $m = 9600,\ 10000,\ n = 3, 4, ...$

For an identity in [3],

$$2^4 + 2^4 + 3^4 + 4^4 + 4^4 = 5^4$$

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we know that (2, 2, 3, 4, 4, 5) is not a solution of

$$x_1^n + x_2^n + x_3^n + x_4^n = y^n$$

for $n = 5, 6, ...$

All Pythagorean triples are not solutions of

$$x^n + y^n = z^n (n = 3, 4, ...)$$

By the same highschool math. technique, we can easily obtain more general proposition.

References

[1] K.Iseki and S.Lajos, Positive integral solutions of the equations $x^2 + (x+1)^2 + (x+2)^2 + ... + (x+n)^2 = y^2 (1 \leq x + n \leq 10000)$, Math. Japonica, 35(1990), 817-830.

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