RALPH HENSTOCK AN OBITUARY

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Ralph Henstock was born in a mining village near to Nottingham in 1923, June 2, the son of a mining family. His father was determined that his son not follow in his footsteps. This determination was rewarded as young Raph was talented and obtained a scholarship that enabled him to attend the University of Cambridge, St. John's College, starting in 1941. His studies and early career were complicated by the war but he finally settled into an academic career that he liked, and not in the civil service that he did not, having experienced this kind of work several times during the war. He then spent the rest of his life in various provincial universities in Britain: Assistant lecturer in Bedford College, London, 1947–1948, Birkbeck College, London, 1948–1951; lecturer at Queen's University, Belfast 1951–1956, Bristol University 1956–1960; senior lecturer and reader at Queen's University 1960–1964; reader at Lancaster University 1964–1970 and finally appointed to the Chair of Pure Mathematics at the New University of Ulster in Coleraine, 1970–1988. After his retirement he was a Leverhulme fellow, 1988–1991.

He remained the modest devout son of his father all his life, a kind man who held strong opinions that however were his opinions and not to be forced on anyone else. A staunch Protestant he did not, when living in Northern Ireland, become a part of the fanatical group and only if asked would declare his strong support for the Ulster settlers' cause but in terms so reasonable that that it was difficult to argue with him. Throughout his life he showed a keen interest in religion and in Methodism in particular. He married in 1949, his wife pre-deceasing him after an illness that lasted many years and restricted his ability to accept opportunities to travel that became open to him when his renown spread. He had one child, a son John, who ironically is a civil servant,

Almost all of Henstock's mathematical work was in the area of integration and this interest resulted from advice given to him by his thesis supervisor Professor Dienes; Henstock had originally thought of a topic in divergent series, the subject of two of his early papers [**2, 4**];(references are to the bibliography of List of Publications of Ralph Henstock.) His thesis "Interval Functions and Their Integrals", the topic of his first published papers,[**1, 3**], extended ideas of J.C. Burkill and was a topic that re-appeared in various forms in his later work.

It was this work and in particular his deep studies of the Ward-Perron-Stieltjes integral that led him to introduce a new approach to integration, an approach that Professor Rogers in his eulogy at the conference dinner in Coleraine August 1988 characterized as " obtaining an impossible result that actually turned out to be possible". In the early papers on integration, in the fifties and early sixties of the last century, in which the new approach to integration was used there was not much emphasis on the generality of the process that Professor Henstock had discovered. So no-one noticed the extraordinary nature of his work. The same is true of the work by Professor Kurzweil who independently, and almost simultaneously, came up with the same idea. This changed with the publication of Henstock's first book,[**18**], and with the very full review that it received by Professor Hildebrandt, [MR0158047 (28 #1274)].

It was Professor Besicovitch who said something along the lines "a good mathematician is known by his bad proofs" and Professor Henstock gives an illustration of this remark. Consider the definition he gave of his integral as quoted in the Hildebrandt review:

"A set of right intervals R is said to be complete on a closed interval $[a,b]$ if for every x with $a \le x < b$, there exists a $\delta(x)$ such that, for $0 < d \le \delta(x)$, the interval $(x, x+d)$ belongs to R. A similar definition holds for a left complete set L on a closed interval $[a,b]$ or an elementary set E. A complete set A on E consists of a complete set R and a complete set L. .... The integrals are applied to interval functions. To each interval $[u,v]$ in E there corresponds a two-valued function $h(u,v)$: $h_r(u,v)$ if u is the associated point and $h_l(u,v)$ if v is the associated point. Then $h = (h_r, h_l)$ has a Riemann complete (RC) integral on E if for every $\varepsilon > 0$, there exists A_ε on E such that $|\sum_\sigma h_s(u,v) - (RC)\int_E dh| < \varepsilon$ for all subdivisions σ of E consisting of intervals in A_ε, s being r or l associated with the interval."

Compare this to the definition as now given of the same integral:

A function f has a generalized Riemann integral on I if $\exists\, A \in \mathbb{R}$ and $\forall\, \epsilon > 0\ \exists$ a function $\delta > 0$ such that for all partitions of I, $\{a_0, \dots, a_n; c_1, \dots, c_n\}$, with $a_{i-1} \le c_i \le a_i$ and $a_i - a_{i-1} < \delta(c_i)$, $1 \le i \le n$

$$\Big|\sum_{\varpi} f(c_i)(a_i - a_{i-1}) - A\Big| < \epsilon.$$

As is seen even the name of the integral has changed and in fact various other names have been given, Henstock integral, Kurzweil integral, Henstock-Kurzweil integral, Kurzweil-Henstock integral and finally the name that Henstock himself preferred in end, the gauge integral after the name given to the function δ used to obtain the partitions in the definition.

In any case bad proofs or not Henstock developed all the tools necessary for this approach to integration and showed that it was equivalent to the classical Denjoy-Perron integral. Further the above definition can be easily modified to apply to the integration of interval functions or even to point-interval functions. This means that there is a very easy extension to Stieltjes type integrals and a simple proof of integration by parts formulæ. In addition his approach was so easy and general that simple modifications gave rise to gauge integrals that were equivalent to most of the known integrals in use at the time of definition, some of these being given as exercises in his second book. Generally speaking if there is a derivative, no matter how general, Henstock's approach enables the definition of an integral that will integrate this derivative. This is not so easy to see for approximate derivatives although Henstock did point this out in his second book. It is even less obvious for symmetric derivatives and much work was done by others, in particular Brian Thomson, to enable the simple symmetric gauge integrals to be defined, integrals of great importance in the theory of trigonometric series.

Henstock extended his work very readily to higher dimensions where however non-absolute integrals are less easy to work with and less useful relative to derivatives. However others, especially Washek Pfeffer, considered the problem of general gradients defining a gauge integral that would handle such quantities leading to very general Green and Stokes theorems. Many elementary texts have been written with the gauge integral as the one used to introduce integration to students as it puts in their hands, with no more trouble than the usual Riemann approach, a tool that has Lebesgue power. In these many ways we see that Henstock's work had a great influence. Further evidence of the spread of and influence of Henstock's work is given by the steady rise of references to his books on integration. To date the Mathematical Reviews notes only 5 citations of his first book, [**18**], 8 citations of the third, [**38**], and 22 of his last, [**43**].

Henstock himself spent the last part of his life extending his idea to integrals in spaces with an infinite number of dimensions, in particular trying to give a proper definition of the Feynman integral using his methods. In this he was helped by P.Muldowney and in the end was successful in this audacious and difficult task. It is of interest to note that in this far reach of his theory there were problems with Cousin's lemma, called Siepinski's lemma in the earlier work, just as there appears to have been in the first book as pointed out by Hildebrandt in his review mentioned above, In the end with the help of Pat Muldowney and Valentin Skvortsov the difficulty was settled in his last publication, a joint work with these two colleagues, [**50**].

Henstock seems to have had few graduate students but those he had speak highly of him as a hard but kind taskmaster. In all a gentleman and a scholar of the first rank who will be much missed. Ralph Henstock died in 2007, January 7, after a short illness.

References

[**Biographical Sources**(The author is very indebted to these references)]

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