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ON INTUITIONISTIC Q-FUZZY IDEALS OF SEMIGROUPS

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Abstract. In this paper, for a set Q, the notion of intuitionistic Q-fuzzy ideals of semigroups is introduced, and some properties of such ideals are investigated.

1 Introduction Zadeh, in his classic paper [10], introduced the notion of fuzzy sets and fuzzy set operations. Since then the fuzzy set theory developed by Zadeh and others has evoked great interest among researchers working in different branches of mathematics. Kuroki [4, 5] have studied sever properties of fuzzy left (right) ideals, fuzzy bi-ideal and fuzzy interior ideals in semigroups. Yuan and Shen [8] introduced a new fuzzy subgroup, called a S-fuzzy subgroup, of a group over a set S and they showed that a fuzzy subgroup of a group can be seen as a S-fuzzy subgroup. The idea of "intuitionistic Q-fuzzy set" was first published by Atanassov [1, 2], as a generalization of the notion of fuzzy set. In this paper, using the Atanassov's idea, we establish the intuitionistic Q-fuzzification of the concept of ideals in semigroups. For a set Q, the notions of intuitionistic Q-fuzzy subsemigroup, intuitionistic Q-fuzzy left(right) ideal and intuitionistic Q-fuzzy interior ideal of a semigroup X are given, and some properties of such ideals are investigated.

2 Preliminaries Let X be a semigroup. By a *subsemigroup* of X we mean a non-empty subset A of X such that $A^2 \subseteq A$, and by a *left* (resp. *right*) *ideal* of X we mean a non-empty subset A of G such that $GA \subseteq A$ (resp. $AX \subseteq A$). By *two-sided ideal* or simply *ideal,* we mean a non-empty subset of X which is both a left and a right ideal of X.

A *fuzzy set* in X is a function μ from X into the unit interval $[0, 1]$. A fuzzy set μ in X is called a *fuzzy subsemigroup* of X if it satisfies

$$(\forall x, y \in X)\ (\mu(xy) \geq \min\{\mu(x), \mu(y)\}),$$

and is called a *fuzzy left* (resp. *right*) *ideal* of X if

$$(\forall x, y \in X)\ (\mu(xy) \geq \mu(y)\ \ (\text{resp.}\mu(xy) \geq \mu(x))).$$

If μ is both a fuzzy left and a fuzzy right ideal of X, we say that μ is a *fuzzy ideal* of X.

In what follows let X and Q denote a semigroup and a non-empty set, respectively, unless otherwise specified. A mapping $f : X \times Q \to [0, 1]$ is called a *Q-fuzzy set* in X.

A Q-fuzzy set $f : X \times Q \to [0, 1]$ is called a *fuzzy subsemigroup* of X over Q (briefly, *Q-fuzzy subsemigroup of* X) if $f(xy, q) \geq \min\{f(x, q), f(y, q)\}$ for all $x, y \in X$ and $q \in Q$.

Example 2.1. Let $X = \{a, b, c, d\}$ be a semigroup with the following Cayley table:

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·	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	c	d
d	d	c	d	c

Let $f : X \times Q \to [0,1]$ be an Q-fuzzy set in X defined by $f(a,q) = 0.9, f(b,q) = 0.4, f(c,q) = 0.7, f(d,q) = 0.5$, for all $q \in Q$. By routine calculations one show that f is an Q-fuzzy subsemigroup of X.

Definition 2.2. An Q-fuzzy set f in X is called an *Q-fuzzy interior ideal* of X if it satisfies
(i) f is an Q- fuzzy subsemigroup of X,
(ii) $f(xwy, q) \geq f(w, q)$ for all $w, x, y \in X$ and $q \in Q$.

Example 2.3. Let $S = \{0, e, f, a, b\}$ be a set with the following Cayley table:

·	0	e	f	a	b
0	0	0	0	0	0
e	0	e	0	a	0
f	0	0	f	0	b
a	0	a	0	0	e
b	0	0	b	f	0

Then X is a semigroup. Let $h : X \times Q \to [0,1]$ be an Q-fuzzy set in X defined by $h(0,q) = h(e,q) = h(f,q) = 1$, $h(a,q) = h(b,q) = 0$ for all $q \in Q$. By routine calculations one show that h is an Q-fuzzy interior ideal of X.

3 Intuitionistic Q-fuzzy ideals

Definition 3.1. An *intuitionistic Q-fuzzy set* (IQFS for short) A is an object having the form

$$A = \{(x, \mu_A(x,q), \gamma_A(x,q)) : x \in X, q \in Q\}$$

where the functions $\mu_A : X \times Q \to [0,1]$ and $\gamma_A : X \times Q \to [0,1]$ denote the degree of membership (namely $\mu_A(x,q)$) and the degree of nonmembership (namely $\gamma_A(x,q)$) of each element $(x,q) \in X \times Q$ to the set A, respectively, and $0 \leq \mu_A(x,q) + \gamma_A(x,q) \leq 1$ for all $x \in X$ and $q \in Q$.

For the sake of simplicity, we shall use the symbol $A = (\mu_A, \gamma_A)$ for the IQFS $A = \{(x, \mu_A(x,q), \gamma_A(x,q)) : x \in X, q \in Q\}$.

Definition 3.2. An IQFS $A = (\mu_A, \gamma_A)$ in X is called an *intuitionistic Q-fuzzy subsemigroup* of X if
(IQFS 1) $\mu_A(xy,q) \geq \min\{\mu_A(x,q), \mu_A(y,q)\}$
(IQFS 2) $\gamma_A(xy,q) \leq \max\{\gamma_A(x,q), \gamma_A(y,q)\}$
for all $x, y \in X$ and $q \in Q$.

Example 3.3. Let $X = \{a, b, c, d\}$ be a semigroup with the following Cayley table:

·	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	c	d
d	d	c	d	c

Let $A = (\mu_A, \gamma_A)$ be an IQFS in X defined by

$$\mu_A(a,q) = 0.9, \mu_A(b,q) = 0.4, \mu_A(c,q) = 0.5, \mu_A(d,q) = 0.7$$

$$\gamma_A(a,q) = 0.8, \gamma_A(b,q) = 0.3, \gamma_A(c,q) = 0.5, \gamma_A(d,q) = 0.5$$

for all $q \in Q$. Then $A = (\mu_A, \gamma_A)$ is an intuitionistic Q-fuzzy subsemigroup of X.

Proposition 3.4. If IQFS $A = (\mu_A, \gamma_A)$ is an intuitionistic Q-fuzzy subsemigroup of X, then the set

$$X_A = \{x \in X \mid \mu_A(x,q) = \mu_A(0,q), \gamma_A(x,q) = \gamma_A(0,q), q \in Q\}$$

is a subsemigroup of X.

Proof. Let $x, y \in X$ and $q \in Q$. Then $\mu_A(x,q) = \mu_A(y,q) = \mu_A(0,q)$ and $\gamma_A(x,q) = \gamma_A(y,q) = \gamma_A(0,q)$. Since $A = (\mu_A, \gamma_A)$ is an intuitionistic Q-fuzzy subsemigroup of X, it follows that

$$\mu_A(xy,q) \geq \min\{\mu_A(x,q), \mu_A(y,q)\} = \mu_A(0,q),$$

$$\gamma_A(xy,q) \leq \max\{\gamma_A(x,q), \gamma_A(y,q)\} = \gamma_A(0,q)$$

so that $\mu_A(xy,q) = \mu_A(0,q)$ and $\gamma_A(xy,q) = \gamma_A(0,q)$. Thus $xy \in X_A$, and consequently X_A is a subsemigroup of X. □

Let $A = (\mu_A, \gamma_A)$ be an IQFS in a set X and let $\alpha + \beta \in [0,1]$ be such that $\alpha + \beta \leq 1$. Then we define the set

$$X_A^{(\alpha,\beta)} = \{x \in X \mid \mu_A(x,q) \geq \alpha, \gamma_A(x,q) \leq \beta, q \in Q\}.$$

Theorem 3.5. Let $A = (\mu_A, \gamma_A)$ be an intuitionistic Q-fuzzy subsemigroup of X. Then $X_A^{(\alpha,\beta)}$ is a subsemigroup of semigroup X for every $(\alpha, \beta) \in Im(\mu_A) \times Im(\gamma_A)$ with $\alpha + \beta \leq 1$.

Proof. Let $x, y \in X_A^{(\alpha,\beta)}$ and $q \in Q$. Then $\mu_A(x,q) \geq \alpha, \gamma_A(x,q) \leq \beta, \mu_A(y,q) \geq \alpha, \gamma_A(y,q) \leq \beta$ which imply that

$$\mu_A(xy,q) \geq \min\{\mu_A(x,q), \mu_A(y,q)\} \geq \alpha$$

$$\gamma_A(xy,q) \leq \max\{\gamma_A(x,q), \gamma_A(y,q)\} \leq \beta.$$

Thus $x - y \in X_A^{(\alpha,\beta)}$ Therefore $X_A^{(\alpha,\beta)}$ is a subsemigroup of semigroup X. □

A semigroup X is said to be a *monoid* if there exists an identity element $e \in X$ such that $xe = ex = x$ for all $x \in X$.

Definition 3.6. An intuitionistic Q-fuzzy subsemigroup IQFS $A = (\mu_A, \gamma_A)$ of a monoid M is said to be *intuitionistic Q-fuzzy submonoid* if

$$\mu_A(e,q) \geq \mu_A(x,q) \text{and} \gamma_A(e,q) \leq \gamma_A(x,q)$$

for all $x \in M$, where e is the identity element of M.

Let $f : M \to M'$ be a homomorphism of monoids M and M'. For any $IQFS = (\mu_A, \gamma_A)$ in Y, we define a new $IQFS^f = ({\mu_A}^f, {\gamma_A}^F)$ in X by

$${\mu_A}^f(x,q) = \mu_A(f(x),q), {\gamma_A}^f(x,q) = \gamma_A(f(x),q)$$

for all x and q.

Theorem 3.7. Let $f : M \to M'$ be a homomorphism of monoids M and M'. If an IQFS $A = (\mu_A, \gamma_A)$ is an intuitionistic Q-fuzzy submonoid of M', then $IQFS^f = ({\mu_A}^f, {\gamma_A}^f)$ is an intuitionistic Q-fuzzy submonoid in M.

Proof. Let $f : M \to M'$ be a homomorphism from M to M' and IQFS $A = (\mu_A, \gamma_A)$ be an intuitionistic Q-fuzzy submonoid of M'. Then the IQFS $A^f = ({\mu_A}^f, {\gamma_A}^f)$ of M satisfies

$$\begin{aligned} {\mu_A}^f(e, q) = \ & \mu_A(f(e), q) = \mu_A(e', q) \\ & \geq \mu_A(f(x), q) = {\mu_A}^f(x, q) \end{aligned}$$

and

$$\begin{aligned} {\gamma_A}^f(e, q) = \ & \gamma_A(f(e), q) = \gamma_A(e', q) \\ & \leq \gamma_A(f(x), q) = {\gamma_A}^f(x, q) \end{aligned}$$

where e, e' are identity elements of M and M'. Now we have

$$\begin{aligned} {\mu_A}^f(xy, q) = \ & \mu_A(f(xy), q) = \mu_A(f(x)f(y), q) \\ & \geq \min\{\mu_A(f(x), q), \mu_A(f(y), q)\} \\ & = \min\{{\mu_A}^f(x, q), {\mu_A}^f(y, q)\} \end{aligned}$$

and

$$\begin{aligned} {\gamma_A}^f(xy, q) = \ & \gamma_A(f(xy), q) = \gamma_A(f(x)f(y), q) \\ & \leq \max\{\gamma_A(f(x), q), \gamma_A(f(y), q)\} \\ & = \max\{{\gamma_A}^f(x, q), {\gamma_A}^f(y, q)\}. \end{aligned}$$

Hence $A^f = ({\mu_A}^f, {\gamma_A}^f)$ is an intuitionistic Q-fuzzy submonoid in M. □

Definition 3.8. An IQFS $A = (\mu_A, \gamma_A)$ is called an *intuitionistic Q-fuzzy left ideal* of X if
$\mu_A(xy, q) \geq \mu_A(y, q)$ and $\gamma_A(xy, q) \leq \gamma_A(y, q)$ for all $x, y \in X$ and $q \in Q$.

Definition 3.9. An IQFS $A = (\mu_A, \gamma_A)$ is called an *intuitionistic Q-fuzzy right ideal* of X if
$\mu_A(xy, q) \geq \mu_A(x, q)$ and $\gamma_A(xy, q) \leq \gamma_A(x, q)$ for all $x, y \in X$ and$q \in Q$.

An IQFS $A = (\mu_A, \gamma_A)$ in X is called an *intuitionistic Q-fuzzy ideal* of X if it is both an intuitionistic Q-fuzzy left ideal and intuitionistic Q-fuzzy right ideal of X.

Note that every intuitionistic Q-fuzzy left (resp. right) ideal of X is an intuitionistic Q-fuzzy subsemigroup of X. But the converse is not true. By Example 3.2, it is observed that $\mu_A(ca, q) = \mu_A(c, q) = 0.7 \leq \mu_A(a, q) = 0.9$ for all $q \in Q$.

Lemma 3.10. Let IQFS $A = (\mu_A, \gamma_A)$ be an intuitionistic Q-fuzzy subgroup of X such that $\mu_A(x, q) \geq \mu_A(y, q)$ (or $\mu_A(y, q) \geq \mu_A(x, q)$) and $\gamma_A(x, q) \leq \gamma_A(y, q)$(or $\gamma_A(y, q) \geq \gamma_A(x, q)$) for all $x, y, \in X$ and $q \in Q$. Then $A = (\mu_A, \gamma_A)$ is an intuitionistic Q-fuzzy left (or right) ideal of X.

Proof. Let $\mu_A(x, q) \geq \mu_A(y, q)$ and $\gamma_A(x, q) \leq \gamma_A(y, q)$ for all $x, y \in X$ and $q \in Q$. Then we have

$$\mu_A(xy, q) \geq \min\{\mu_A(x, q), \mu_A(y, q)\} = \mu_A(y, q)$$

$$\gamma_A(xy, q) \leq \max\{\gamma_A(x, q), \gamma_A(y, q)\} = \gamma_A(y, q).$$

Hence $A = (\mu_A, \gamma_A)$ is an intuitionistic Q-fuzzy left ideal of X. Similarly, if we let $\mu_A(y, q) \geq \mu_A(x, q)$ and $\gamma_A(y, q) \leq \gamma_A(x, q)$ for all $x, y \in X$ and $q \in Q$, then it is easy to prove that $A = (\mu_A, \gamma_A)$ is an intuitionistic Q-fuzzy right ideal of X. □

Definition 3.11. An intuitionistic Q-fuzzy subsemigroup $A = (\mu_A, \gamma_A)$ of X is called is called an *intuitionistic Q-fuzzy interior ideal* of X if

(IQFS 3) $\mu_A(xay, q) \geq \mu_A(a, q)$

(IQFS 4) $\gamma_A(xay, q) \leq \gamma_A(a, q)$

for all $x, y, a \in X$ and $q \in Q$.

Example 3.12. Let $X = \{0, e, f, a, b\}$ be a semigroup with the following Cayley table:

$\cdot$	0	e	f	a	b
0	0	0	0	0	0
e	0	e	0	0	0
f	0	0	f	0	b
a	0	a	0	0	e
b	0	0	b	f	0

Then X is a semigroup. Define an IQFS $A = (\mu_A, \gamma_A)$ in X by $\mu_A(0, q) = \mu_A(e, q) = \mu_A(f, q) = 1, \mu_A(a, q) = \mu_A(b, q) = 0, \gamma_A(0, q) = \gamma_A(e, q) = \gamma_A(f, q) = 0, \gamma_A(a, q) = \gamma_A(b, q) = 1$. By routine calculations we know that $A = (\mu_A, \gamma_A)$ is an intuitionistic Q-fuzzy subsemigroup of X and an intuitionistic Q-fuzzy interior ideal of X.

Theorem 3.13. If $\{A_i\}$ is a family of intuitionistic Q-fuzzy interior ideals of X, then $\cap A_i$ is an intuitionistic Q-fuzzy interior ideal of X, where $\cap A_i = (\vee \mu_{A_i}, \wedge \gamma_{A_i})$ and $\vee \mu_{A_i}$ and $\wedge \gamma_{A_i}$ are defined as follows:

$\vee \mu_{A_i} = \inf\{\mu_{A_i}(x, q) \mid i \in \Lambda, x \in X\}$ and $\wedge \gamma_{A_i} = \sup\{\gamma_{A_i}(x, q) \mid i \in \Lambda, x \in X\}$ for all $q \in Q$.

Proof. It is straightforward. □

Theorem 3.14. If an IQFS $A = (\mu_A, \gamma_A)$ in X is anintuitionistic Q-fuzzy interior ideal of X, then so is $\Box A$, where $\Box A = \{(x, \mu_A(x, q), 1 - \mu_A(x, q)) : x \in X, q \in Q\}$.

Proof. It is sufficient to show that $\overline{\mu_A}$ satisfies the conditions (IQFS 2) and (IQFS 4). For any $a, x, y \in X$, we have

$$\begin{aligned} \overline{\mu_A}(xy, q) = \quad & 1 - \mu_A(xy, q) \leq 1 - \min\{\mu_A(x, q), \mu_A(y, q)\} \\ & = \max\{1 - \mu_A(x, q), 1 - \mu_A(y, q)\} = \max\{\overline{\mu_A}(x, q), \overline{\mu_A}(y, q)\} \end{aligned}$$

and $\overline{\mu_A}(xay, q) = 1 - \mu_A(xay, q) \geq 1 - \mu_A(a, q) = \overline{\mu_A}(a, q)$. Therefore $\Box A$ is an intuitionistic Q-fuzzy interior ideal of X. □

Definition 3.15. Let $A = (\mu_A, \gamma_A)$ be an IQFS in X and let $\alpha \in [0, 1]$. Then the set $\mu^{\geq}_{A,\alpha} := \{x \in X : \mu_A(x, q) \geq \alpha$ for all $q \in Q\}$ is called a *μ-level α-cut* and $\gamma^{\leq}_{A,\alpha} := \{x \in X : \gamma_A(x, q) \leq \alpha$ for all $q \in Q\}$ is called a *γ-level α-cut* of A.

Theorem 3.16. If an IQFS $A = (\mu_A, \gamma_A)$ in X is an intuitionistic Q-fuzzy interior ideal of X, then the μ-level α-cut $\mu^{\geq}_{A,\alpha}$ and γ-level α-cut $\gamma^{\leq}_{A,\alpha}$ of A are interior ideals of X for every $\alpha \in \mathrm{Im}(\mu_A) \cap \mathrm{Im}(\gamma_A) \subseteq [0, 1]$.

Proof. Let $\alpha \in \mathrm{Im}(\mu_A) \cap \mathrm{Im}(\gamma_A) \subseteq [0, 1]$ and let $x, y \in \mu^{\geq}_{A,\alpha}$. Then $\mu_A(x, q) \geq \alpha$ and $\mu_A(y, q) \geq \alpha$ for all $q \in Q$. It follows from (IQFS 1) that

$$\mu_A(xy, q) \geq \min\{\mu_A(x, q), \mu_A(y, q)\} \geq \alpha \text{ so that } xy \in \mu^{\geq}_{A,\alpha}.$$

If $x, y \in \gamma^{\leq}_{A,\alpha}$, then $\gamma_A(x, q) \leq \alpha$ and $\gamma_A(y, q) \leq \alpha$, and so

$$\gamma_A(xy, q) \leq \max\{\gamma_A(x, q), \gamma_A(y, q)\} \leq \alpha, \text{ i.e., } xy \in \gamma^{\leq}_{A,\alpha}.$$

Hence $\mu^{\geq}_{A,\alpha}$ and $\gamma^{\leq}_{A,\alpha}$ are subsemigroups of X. Now let $x, y \in X$ and $a \in \mu^{\geq}_{A,\alpha}$. Then $\mu_A(xay, q) \geq \mu_A(a, q) \geq \alpha$ and so $xay \in \mu^{\geq}_{A,\alpha}$. If $a \in \gamma^{\leq}_{A,\alpha}$, then $\gamma_A(xay, q) \leq \gamma_A(a, q) \leq \alpha$ and thus $xay \in \gamma^{\leq}_{A,\alpha}$. Therefore $\mu^{\geq}_{A,\alpha}$ and $\gamma^{\leq}_{A,\alpha}$ are interior ideals of X. □ □

Theorem 3.17. Let $A = (\mu_A, \gamma_A)$ be an IQFS in X such that the non-empty sets $\mu^{\geq}_{A,\alpha}$ and $\gamma^{\leq}_{A,\alpha}$ are interior ideals of X for all $\alpha \in [0, 1]$. Then $A = (\mu_A, \gamma_A)$ is an intuitionistic Q-fuzzy interior ideal of X.

Proof. Let $\alpha \in [0, 1]$ and suppose that $\mu^{\geq}_{A,\alpha}(\neq \emptyset)$ and $\gamma^{\leq}_{A,\alpha}(\neq \emptyset)$ are interior ideals of X. We must show that $A = (\mu_A, \gamma_A)$ satisfies the conditions (IQFS 1)-(IFQS 4). If the condition (IQFS 1) is false, then there exist $x_0, y_0 \in X$ such that $\mu_A(x_0y_0, q) < \min\{\mu_A(x_0, q), \mu_A(y_0, q)\}$ for all $q \in Q$. Taking

$$\alpha_0 := \frac{1}{2}(\mu_A(x_0y_0, q) + \min\{\mu_A(x_0, q), \mu_A(y_0, q)\}),$$

we have $\mu_A(x_0y_0, q) < \alpha_0 < \min\{\mu_A(x_0, q), \mu_A(y_0, q)\}$. It follows that $x_0, y_0 \in \mu^{\geq}_{A,\alpha_0}$ and $x_0y_0 \notin \mu^{\geq}_{A,\alpha_0}$, which is a contradiction. Hence the condition (IQFS 1) is true. The proof of other conditions are similar to the case (IQFS1), we omit the proof. □

Theorem 3.18. Let M be an interior ideal of X and let $A = (\mu_A, \gamma_A)$ be an IQFS in X defined by

$$\mu_A(x, q) := \begin{cases} \alpha_0 & \text{if } \ x \in M, \\ \alpha_1 & \text{otherwise}, \end{cases} \qquad \gamma_A(x, q) := \begin{cases} \beta_0 & \text{if } \ x \in M, \\ \beta_1 & \text{otherwise}, \end{cases}$$

for all $x \in X, q \in Q$ and $\alpha_i, \beta_i \in [0, 1]$ such that $\alpha_0 > \alpha_1$, $\beta_0 < \beta_1$ and $\alpha_i + \beta_i \leq 1$ for $i = 0, 1$. Then $A = (\mu_A, \gamma_A)$ is an intuitionistic Q-fuzzy interior ideal of X and $\mu^{\geq}_{A,\alpha_0} = M = \gamma^{\leq}_{A,\beta_0}$.

Proof. Let $x, y \in X$ and $q \in Q$. If any one of x and y does not belong to M, then

$$\mu_A(xy, q) \geq \alpha_1 = \min\{\mu_A(x, q), \mu_A(y, q)\}$$

and

$$\gamma_A(xy, q) \leq \beta_1 = \max\{\gamma_A(x, q), \gamma_A(y, q)\}.$$

Other cases are trivial, and we omit the proof. Hence $A = (\mu_A, \gamma_A)$ is an intuitionistic Q-fuzzy subsemigroup of X. Now let $x, y, a \in X$ and $q \in Q$. If $a \notin M$, then $\mu_A(xay, q) \geq \alpha_1 = \mu_A(a, q)$ and $\gamma_A(xay, q) \leq \beta_1 = \gamma_A(a, q)$. Assume that $a \in M$. Since M is an interior ideal of X, it follows that $xay \in M$. Hence $\mu_A(xay, q) = \alpha_0 = \mu_A(a, q)$ and $\gamma_A(xay, q) = \beta_0 = \gamma_A(a, q)$. Therefore $A = (\mu_A, \gamma_A)$ is an intuitionistic Q-fuzzy interior ideal of X. Obviously $\mu^{\geq}_{A,\alpha_0} = M = \gamma^{\leq}_{A,\beta_0}$. □

Corollary 3.19. Let χ_M be the characteristic function of an interior ideal M of X. Then the IQFS $\tilde{M} = (\chi_M, \overline{\chi}_M)$ is an intuitionistic Q-fuzzy interior ideal of X.

Theorem 3.20. An IQFS $A = (\mu_A, \gamma_A)$ is an intuitionistic Q-fuzzy interior ideal of X if and only if the fuzzy sets μ_A and $\overline{\gamma}_A$ are Q-fuzzy interior ideals of X.

Proof. Let $A = (\mu_A, \gamma_A)$ be an intuitionistic Q-fuzzy interior ideal of X. Then clearly μ_A is an Q-fuzzy interior ideal of X. Let $x, a, y \in X$ and $q \in Q$. Then

$$\begin{aligned} \overline{\gamma}_A(xy, q) &= 1 - \gamma_A(xy, q) \\ &\geq 1 - \max\{\gamma_A(x, q), \gamma_A(y, q)\} \\ &= \min\{1 - \gamma_A(x, q), 1 - \gamma_A(y, q)\} \\ &= \min\{\overline{\gamma}_A(x, q), \overline{\gamma}_A(y, q)\}, \text{ and} \end{aligned}$$

$$\overline{\gamma}_A(xay, q) = 1 - \gamma_A(xay, q) \geq 1 - \gamma_A(a, q) = \overline{\gamma}_A(a, q).$$

Hence $\overline{\gamma}_A$ is a Q-fuzzy interior ideal of X.

Conversely suppose that μ_A and $\overline{\gamma}_A$ are Q-fuzzy interior ideals of X. Let $a, x, y \in X$ and $q \in Q$. Then

$$\begin{aligned} 1 - \gamma_A(xy, q) &= \overline{\gamma}_A(xy, q) \geq \min\{\overline{\gamma}_A(x, q), \overline{\gamma}_A(y, q)\} \\ &= \min\{1 - \gamma_A(x, q), 1 - \gamma_A(y, q)\} \\ &= 1 - \max\{\gamma_A(x, q), \gamma_A(y, q)\}, \text{ and} \end{aligned}$$

$$1 - \gamma_A(xay, q) = \overline{\gamma}_A(xay, q) \geq \overline{\gamma}_A(a, q) = 1 - \gamma_A(a, q),$$

which imply that $\gamma_A(xy, q) \leq \max\{\gamma_A(x, q), \gamma_A(y, q)\}$ and $\gamma_A(xay, q) \leq \gamma_A(a, q)$. This completes the proof. □

Corollary 3.21. An IQFS $A = (\mu_A, \gamma_A)$ is an intuitionistic Q-fuzzy interior ideal of X if and only if $\Box A = (\mu_A, \overline{\mu}_A)$ and $\Diamond A = (\overline{\gamma}_A, \gamma_A)$ are intuitionistic Q-fuzzy interior ideals of X.

Proof. It is straightforward by Theorem 3.20. □

Let f be a map from a set X to a set Y. If $A = (\mu_A, \gamma_A)$ and $B = (\mu_B, \gamma_B)$ are IQFSs in X and Y respectively, then the *preimage* of B under f, denoted by $f^{-1}(B)$, is an IQFS in X defined by

$$f^{-1}(B) = (f^{-1}(\mu_B), f^{-1}(\gamma_B)), \text{ where } f^{-1}(\mu_B) = \mu_B(f).$$

Theorem 3.22. Let $f : X \to Y$ be a homomorphism of semigroups. If $B = (\mu_B, \gamma_B)$ is an intuitionistic Q-fuzzy interior ideal of Y, then the preimage $f^{-1}(B) = (f^{-1}(\mu_B), f^{-1}(\gamma_B))$ of B under f is an intuitionistic Q-fuzzy interior ideal of X.

Proof. Assume that $B = (\mu_B, \gamma_B)$ is an intuitionistic Q-fuzzy interior ideal of Y and let $x, y \in X$ and $q \in Q$. Then

$$\begin{aligned} f^{-1}(\mu_B)(xy, q) &= \mu_B(f(xy), q) \\ &= \mu_B(f(x)f(y), q) \\ &\geq \min\{\mu_B(f(x), q), \mu_B(f(y), q)\} \\ &= \min\{f^{-1}(\mu_B(x, q)), f^{-1}(\mu_B(y, q))\}, \text{ and} \end{aligned}$$

$$\begin{aligned} f^{-1}(\gamma_B)(xy, q) &= \gamma_B(f(xy), q) \\ &= \gamma_B(f(x)f(y), q) \\ &\leq \max\{\gamma_B(f(x), q), \gamma_B(f(y), q)\} \\ &= \max\{f^{-1}(\gamma_B(x, q)), f^{-1}(\gamma_B(y, q))\}. \end{aligned}$$

Hence $f^{-1}(B) = (f^{-1}(\mu_B), f^{-1}(\gamma_B))$ is an intuitionistic Q-fuzzy subsemigroup of X. For any $a, x, y \in X$ and $q \in Q$, we have

$$\begin{aligned} f^{-1}(\mu_B)(xay, q) &= \mu_B(f(xay), q)) \\ &= \mu_B(f(x)f(a)f(y), q) \\ &\geq \mu_B(f(a), q) \\ &= f^{-1}(\mu_B(a, q)), \text{ and} \end{aligned}$$

$$\begin{aligned} f^{-1}(\gamma_B)(xay, q) &= \gamma_B(f(xay), q) \\ &= \gamma_B(f(x)f(a)f(y), q) \\ &\leq \gamma_B(f(a), q) \\ &= f^{-1}(\gamma_B(a, q)). \end{aligned}$$

Therefore $f^{-1}(B) = (f^{-1}(\mu_B), f^{-1}(\gamma_B))$ is an intuitionistic Q-fuzzy interior ideal of X. □

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