SAUNDERS MAC LANE 1909-2005
MEETING A GRAND LEADER

Walter Tholen

Received December 15, 2005

Preamble

It was in August of 2000 at the Category Theory meeting in Como, Italy, when I saw Saunders Mac Lane for the last time. The ambulance turned in front of the conference site, the stately Villa Olmo at the lake shore, and Saunders, who had just collapsed and looked very pale indeed, waved graciously from inside the vehicle to the worried bystanders. Immediately it almost felt to me like the royal hand wave by a leader par excellence, who was saying "Good Bye" to his people; to the followers of the field that he had helped to create almost sixty years earlier, that is. Although it turned out that Saunders had only suffered a temporary weakness, probably caused by the stress of overseas travel, he in fact did not attend any of the major Category Theory conferences afterwards. When I spoke to him a few times on the telephone in 2002 to invite him to be the guest of honour at a meeting at the Fields Institute, he was obviously tempted to accept but eventually concluded that travel at the age of 93 would just be too hard on him. Saunders died almost three years later, on April 14, 2005. An outspoken mathematician of extraordinary vision, determination, and uncompromising principles left the stage.

It was in the summer of 1972 when I first met Saunders Mac Lane; more precisely, I just saw him. As a graduate student two years prior to completion of my Ph.D. thesis, I stayed down in the village of Oberwolfach and just sat in on the lectures given up the hill at the Category Theory conference in the Mathematical Institute. A year later I finally had the courage to introduce myself to him and was very impressed that he gave me all his attention. The leader, who could be equally strong in his encouragement and criticism of any mathematical endeavours, was indeed always willing to listen and learn, no matter from whom, and give clear direction and advice. Like many of his students, I felt these qualities even in my first brief discussion with him, and I quickly discovered how kind and down to earth Saunders was, despite his celebrated status as one of the two great old men of category theory. Of course, his coworker since the 1940s, Samuel Eilenberg, also attended these Oberwolfach meetings of the 1970s, organized by Horst Schubert and John Gray, but normally kept a much lower profile than Saunders. Sammy's comments, however, made from the back rather than the front row, and often wrapped in subtle humour, could be equally sharp as Saunders' more direct reactions. Still, even if, in the heat of the often very emotional discussions on algebraic theories, monads, and topos and sheaf theory, with people like Bill Lawvere, Peter Freyd, Michael Barr, Max Kelly, John Isbell, André Joyal and Jean Bénabou impressing the audience with their often extreme views, one may have forgotten who the true leader was, one would certainly be reminded at the hike on Wednesday afternoon. I will never forget the scene when, very shortly after leaving the Institute, at the fork of the trail going further up the hill, John Gray was heading in one direction and Saunders was pointing his cane in the other, while vehemently disputing John's claims about the right direction. Of course, neither of the two men retreated, leaving everybody else with the difficult decision of whether to follow what would most likely be the better direction, or to simply follow the boss. With most people choosing the latter option, only few arrived back in time for dinner at the Institute.

Throughout the years I had my best encounters with Saunders on those conference hikes, wherever in the world they took place, since we both seemed to like the challenge of a mountain hike. In his wonderful essay about "Concepts and Categories in Perspective", he indeed uses the image of hiking to explain the very nature of research in mathematics:

> *The progress of mathematics is like the difficult exploration of possible trails up a massive infinitely high mountain, shrouded in a heavy mist which will occasionally lift a little to afford new and charming perspectives. This or that route is explored a bit more, and we hope that some will lead on higher up, while indeed many routes may join and reinforce each other [CCP, p. 359].*

This essay, written for "A Century of Mathematics in America" and published by the American Mathematical Society, gives anything but just an American view. Rather, it reads like the manifesto of a world citizen who reports globally on the development and state of category theory, an area of mathematics that he never saw in isolation but always as a tool or mediator for gaining new mathematical insights, wherever they needed to be made. There are numerous other publications by Saunders Mac Lane, in which he describes the many mathematical developments he was involved in, especially the development of category theory; see particularly his 1965 article on "Categorical Algebra" [CA], his 1976 Retiring Presidential Address "Topology and Logic as a Source of Algebra" [TLSA], his 1995 survey article "Categories in Geometry, Algebra and Logic" [CGAL], and his own "Mathematical Autobiography" [MA], published only after his death. Taken in conjunction with the many excellent articles written about Saunders Mac Lane, such as Max Kelly's account [CT] of his encounters with Saunders that appeared in the 1979 Springer book "Saunders Mac Lane, Selected Papers" [SP] (edited by Irving Kaplansky, Saunders' first of a total of 39 successful Ph.D. students), there seems to be little that one could add. Hence, in this article, I can only attempt to give a very small glimpse of Saunders' personality and work, from the perspective of someone who met and observed him just at conferences during the last third of his life. And just like Max did at the time, I must apologize that this approach almost "necessarily includes more about myself than is decent".

Göttingen

The question that intrigued me first when gathering my thoughts for this article was how the young man, born on August 4, 1909 in small-town Connecticut, grew into the role of a mathematical world leader, with such a strong sense of caring for the well being of his domains of interest anywhere around the globe. Yes, as a student at Yale and Chicago he was deeply influenced by famous and broadly educated teachers such as E. H. Moore, but his stay in Göttingen 1931-1933 seems to have been pivotal in shaping the mathematician as we know and remember him today. At the dawn of the dark ages of Nazi rule, the Mathematical Institute in this pretty provincial town was for a very short time still *the* place to be in world mathematics. The giant of the time, David Hilbert, was still giving lectures and probably helped to shape the highly international perspective of the young Mac Lane. Of his teachers, Edmund Landau and Hermann Weyl seem to have had the greatest mathematical influence on the graduate student from America. In his short article "A Late Return to a Thesis in Logic" [LRTL] Mac Lane recalls how impressed he was with Edmund Landau's lectures on Dirichlet series, the style of which may have given him some unintended inspiration for the choice of his thesis topic:

> *As always, Landau's proofs were simply careful lists of one detail after the other,*

but he gave this detail with such exemplary care that I could both copy down in my notebook all the needed detail and enter in the margin some overarching description of the plan of his proof (a plan which he never directly revealed).

But, as he had remarked earlier,

a good proof consist[s] of more than just rigorous detail, because there [is] also an important element of plan for the proof - the crucial ideas, which, over and above the careful detail, make the proof function.

In the brief curriculum vitae at the end of Saunders' thesis "Abgekürzte Beweise im Logikkalkul" (Abbreviated Proofs in the Calculus of Logic) he thanks his advisor Paul Bernays "für seine Kritk" (for his criticism), "und vor allem Professor Weyl für seine Ratschläge und für die Anregung seiner Vorlesungen" (and most of all Professor Hermann Weyl for his advice and for the inspiration of his lectures), a very strong indication of the admiration Saunders must have felt for Hilbert's brilliant successor. Weyl is in fact listed officially as the "Referent" (referee) of the thesis, after Paul Bernays had been chased out by the Nazis, who could bank on far too many active and passive helpers within the university system, some of whom willing to implement their notorious ideologies at any cost. But already before these incredible events, that leveled a world class institution to average quality within a year and made foreigners like Mac Lane rush to finish their degrees (see his lively article "Mathematics at Göttingen under the Nazis" [MGN]), Weyl must have assumed the role of a co-supervisor, as is indicated when he writes in [LRTL]:

I had already started work on an earlier thesis idea, also in logic, early in the academic year 1932-33. I no longer know what was intended as the content of the thesis, but I do clearly recall that it did not find favor with either Professor Bernays or Professor Weyl when I explained it to them in February of 1933.

Saunders reports that, after this disappointment, he briefly tossed around the idea of leaving Göttingen for Vienna to work with Rudolph Carnap, but that he then experienced "a decisive spurt" in April 1933 and produced "an exuberant first draft (in English) of the thesis: long-winded, full of rash philosophical assertions" shortly afterwards. "The thesis itself (rewritten later, first in English and then translated into German) is more mathematical and businesslike", Saunders reports. Indeed, a careful reader of the thesis (which was defended on July 19, 1933, printed (as required in Germany) in 1934 and reprinted in [SP]), will recognize early traces of Mac Lane's trademark mathematical style. Although the goal of the thesis, namely to give an "analysis of the structure of logical proofs and the confirmation that this analysis will yield abbreviated proofs" (my translation of the German thesis text, [SP, pp. 56-57]), could have easily led him to a more formalistic presentation, as seen in Russell and Whitehead's "Principia Mathematica" and in many other works of the time, Mac Lane makes repeated efforts to always reconnect with the average mathematical reader with easy, but illustrative examples, a principle that is applied in all of his mathematical writings and that has made books like "Categories for the Working Mathematician" [CWM] so successful.

I cannot resist mentioning here one of the Göttingen anecdotes, which I remember Saunders telling me about with a big smile on his face, and which should be especially appreciated by all of us who struggle with the daily pitfalls when having to operate in a second language. In those days it was (and still is) quite customary in Germany for professors to invite some colleagues and their graduate students at least once a term for more or less informal after-dinner receptions, the purpose of which is to have free-ranging discussions and, as a kind of coincidental by-product, introduce the young men to the professors' daughters. Towards the end of one of those occasions, Saunders intended to offer to take one of the young women home, but ended up saying "Darf ich Sie *zu* Hause nehmen", meaning "May I take you *at* home".

I gather that this event must have predated the arrival of his fiancé Dorothy Jones in Göttingen, who typed his thesis and whom he married there. As Saunders tells us in [MA], she was very fond of traveling and may therefore have greatly helped to shape Saunders' decidedly international views. Even when confined to a wheelchair she accompanied her husband not only on conference travel but also on more cumbersome trips, such as a visit to China in 1981. Saunders would normally refer to her respectfully as Mrs. Mac Lane, just as he would refer to any significant result of his as that of Mac Lane. Although being very unassuming himself and usually staying with everybody else in low-cost conference accommodations (tolerating the common bathroom of student houses as late as 1993 at a MSRI conference at Berkeley – in his mid-eighties, that is!), he certainly made sure that Mrs. Mac Lane was appropriately cared for and entertained. When I co-organized a conference in 1981 at Gummersbach (Germany), one of my colleagues had to skip the lectures and spend the afternoon driving her around in search of local collectors' teaspoons.

There is another life-long fallout of Mac Lane's early studies and his Göttingen times, namely his profound interest in the philosophy and the foundations of mathematics, documented by brilliant writings such as "Proof, Truth and Confusion" [PTC], as well as his many contributions about the status of category theory vis-à-vis set theory. His 1986 book "Mathematics, Form and Function" [MFF] describes much of his general perception and philosophy of mathematics.

The Clash

At the Prague conference on "Categorical Topology" in August of 1988, Saunders Mac Lane gave a talk on the "Development and Clash of Ideas", examining various mathematical milestones of his lifetime in terms of whether they came about as an important development within a given discipline, or whether they arose as the result of a (positive) clash of ideas from different disciplines. At many occasions he in fact referred to the birth of the notions of category, functor and natural transformation as a clash between algebra and topology. Having returned from Göttingen to the U.S. in 1933, while holding fellow- and instructorships consecutively at Yale, Harvard, Cornell and Chicago, and finally assuming an assistant professorship at Harvard in 1938, Saunders quite quickly shifted his interests away from logic to predominantly (but by no means exclusively) algebra, as witnessed most visibly by his 1941 book "A Survey of Modern Algebra" [SMA] with Garrett Birkhoff which, a decade after van der Waerden's "Moderne Algebra", changed undergraduate courses in algebra fundamentally and made abstract algebra "both accessible and attractive" (as Mac Lane's student Alfred Putnam writes in the Preface to [SP]; various editions and translations appeared as late as 1997, in addition to the 1967 book on "Algebra" [A]). The initial clash of algebra and topology, personified by Mac Lane and Eilenberg, occurred at a Michigan conference on topology in 1940, when Saunders lectured on group extensions. In "Samuel Eilenberg and Categories" [SEC], he recalls what happened:

> *I set out the description of a group extension by means of factor sets and computed the group of such extensions for the case of an interesting abelian factor group defined for any prime p and given by generators a_n with $pa_{n+1} = a_n$ for all n. When I presented this result in my lecture, Sammy immediately pointed out that I had found Steenrod's calculation of the homology group of the p-adic solenoid. This solenoid, already studied by Sammy in Poland, can be described thus: Inside a*

torus T_1, wind another torus T_2 p-times, then another torus T_3 p-times inside T_2, and so on. What is the cohomology of the final intersection? Sammy observed that the Ext *group I had calculated gave exactly Steenrod's calculation of the homology of the solenoid! The coincidence was highly mysterious. Why in the world did a group of abelian group extensions come up in homology? We stayed up all night trying to find out "why". Sammy wanted to get to the bottom of this coincidence.*

Mac Lane and Eilenberg were about to find the "Universal Coefficient Theorem", which forms the core of their first joint paper "Group extensions and homology" [GEH], published "with the steady encouragement of Lefschetz" (Saunder's words) in the Annals of Mathematics in 1942. Then, in a letter to Sammy, dated May 10, 1942, Saunders points to the similar behaviour of various mathematical objects which they had considered in their paper (see [SEC]):

This indicates that it is possible to give a precise definition of a natural isomorphism *between functions of groups. Then it will be possible to have all the isomorphisms in any such investigation proved at once to be* natural.

With the term "natural" transformation already around in vector space theory (associated with maps like the one from a space to its double dual), Eilenberg and Mac Lane "purloined" (Saunders' word) the terms "category" from Kant and "functor" from Carnap (who had used it in a different sense in his book on "Logical Syntax of Language"), and published their first paper on the beginnings of category theory, first as a short note on "Natural isomorphisms in group theory" [NIGT] in 1942, and then as a longer paper (that takes a decisive step away from groups) in their "General theory of natural equivalences" [GTNE] of 1945. As Saunders recalls in [SEC],

That paper was certainly off beat, but happily it was accepted for publication. At the time, Sammy stated firmly that this would be the only paper needed for category theory. Probably what he had in mind was that the trio of notions - category, functor, and natural transformation was enough to make good applications possible.

Why should we marvel at the creation of three relatively easy notions that seem to be just instances of a language, rather than a theory? After all, given that "naturality" was already around, a critic may claim that almost anybody may have been able to find the right notions for the structure given by a generalized monoid with a partially defined binary operation and the appropriate structure-preserving maps between them! But Eilenberg and Mac Lane's revolutionary achievement does not lie in the mere creation of yet another algebraic structure, but in daring to think of proper classes as objects with structure themselves and to study the appropriate maps between such monsters, just as if these were homomorphisms of groups, and in finally embedding naturality into this framework. The boldness of this intellectual step is comparable only to Georg Cantor's daring to think different degrees of infinity. Of course, being themselves embedded in the perceived safe haven of set theory, they had to overcome their own scruples to venture beyond it. In fact, they try to bypass them when they state in typical Mac Lanean pragmatism (in Section 6 "Foundations" of [GTNE]):

Hence we have chosen to adopt the intuitive standpoint, leaving the reader free to insert whatever type of logical foundation (or absence thereof) he may prefer.

(Spotting this sentence only recently for the first time reminded me that I made an almost identical disclaimer in my own Ph. D. thesis.) In fact, they go on to explain that "the concept of a category is essentially an auxiliary one", and that one could avoid it since, in practice, one is dealing only with a few objects at a time. In my own experience, if you hide quantification over a proper class by saying "let G be a group", nobody even thinks about that, but "let G be an object of the category of groups" still sends shivers through a big part of your general mathematical audience.

The die-hard belief that set theory provides the sole foundation of mathematics still prevents the notion of category to enjoy unreserved general acceptance. Saunders, being always very concerned about staying in touch with mainstream mathematics, has written repeatedly about the "Foundations for categories and sets", for instance, under this title, in [FCS] in 1969. Before embracing topos theory more consequently he favoured the (very limited) use of Grothendieck universes, in order to reconcile set-theoretic problems naturally arising when forming functor categories, and the like, and this is what the reader finds in [CWM]. But the second edition of [CWM], in addition to the old section on "Foundations" of the main text, contains also a new three-page appendix, again entitled "Foundations", in which he summarizes topos-theoretic axioms for sets and their standing vis-à-vis Zermelo-Fraenkel. (An elementary account of such an approach is given in the book by F. W. Lawvere and R. Rosebrugh, entitled "Sets for Mathematics", Cambridge 2003).

As a mathematical achievement, the creation of bold notions that open the gates for a new theory becomes, after some time, easily underestimated. Like Cantor's first steps into new territory, also Eilenberg and Mac Lane's categories had to face not only dismissal but often outright opposition. In fact, they did not make it into Bourbaki, who instead stuck with an ill-fated general notion of mathematical structure. Saunders writes in [MA]:

> *At the time, we sometimes called our subject "general abstract nonsense". We didn't really mean the nonsense part, and we were proud of its generality.*

Unfortunately, even today many mathematicians use that label to not only portray category theory as a mere "language", but to claim to "know category theory" essentially based on familiarity with the original three notions - a manifestation of ignorance comparable really only to someone claiming knowledge of set theory after having learned naively the language of sets, and how to form unions and power sets.

The Big Leaps

While equal credit must be given to Eilenberg and Mac Lane for the creation of the trio of basic notions, it was Saunders alone who took the decisive step of transforming a category from (in Max Kelly's words) "a rather structureless domain or codomain of a functor [to] become something more tangible and individual" [CT]. Having assumed his final regular academic appointment at the University of Chicago in 1947, in his "Duality of Groups" [DG] of 1950 (preceded by the short 1948 note "Groups, Categories and Dualities" [GCD]) he takes the big leap of characterizing cartesian and free products of groups by their categorical properties. The objects $G \times H$ and $G * H$, that look so different when (within traditional foundations) one tries to say what they "physically" are, became all of the sudden so similar (in fact, dual to each other) when one concentrates on what they do within the league of all groups. This was the starting point of a fundamentally novel breed of thinking that lies at the core of category theory and that was bound to blossom in many variations: Step inside a category and characterize various players solely by means of their interaction with their teammates! The apparent defect of being able to characterize mathematical objects or constructions in this way uniquely "only up to (a unique) isomorphism" actually turns into an undeniable asset, since it frees us from having to stare at irrelevant differences. In fact, since nobody wants to seriously distinguish between, for example, the objects $\mathbf{R}$

and $\mathbf{R} \times\{0\}$ (as embedded into $\mathbf{R}^2$), so that most people would declare that the two objects should be "identified", only few seem to realize that they are actually adopting the categorical viewpoint.

The second big leap accomplished in [DG] is the realization that a category may need additional properties or structure to afford results of substance. Mac Lane comes close to giving the notion of abelian category (the definite form of which was given by Buchsbaum in 1955, albeit under a different name), and he proves a representation theorem, with the help of a predecessor of Grothendieck's famous AB5 axiom. In defining his "abelian bicategories" Saunders also presents a definition equivalent to the notion of factorization system in general categories, as promoted primarily by Freyd and Kelly more than twenty years later, a fact that I had to learn the very hard way more than forty years after Mac Lane's paper. When in an article with M. Korostenski I remarked that the roots of factorization systems were "already present in Isbell's work" (since John Isbell was the first to use the "unique diagonalization property", the weak form of which became known as the right or left "lifting property" in Quillen's model categories), immediately after its appearance I received a hastily scribbled and almost illegible note from Saunders in the mail, dated April 9, 1993, that left me feeling devastated:

Dear Walter
Just saw your joint paper with Korostenski JPAA 85 (1993) 57
What do you mean!
Factorization system in Isbell 1957
They were in Mac Lane Duality for Groups BULL AMS 1950
Look it up!!!
Saunders

I never forgave myself for having relied just on secondary sources in this matter. Fortunately Saunders did. Like to many other writers of articles or presenters at conferences, the eagle eye of the field had taught me a lesson, but he also quickly turned to his kind and cheerful self after the job was done.

Throughout the 1950s and early 1960s abelian category theory found its definite form through the works of Alexandre Grothendieck, Peter Gabriel and Peter Freyd. Meanwhile Mac Lane pursued primarily his monumental work on homology and cohomology theory that, inter alia, boosted a total of fifteen joint papers with Eilenberg and culminated in the 1963 publication of his book on "Homology" [H]. The axiomatization of these subjects through category theory is a major milestone of twentieth century mathematics. But that year turned also out to be a vintage year for great progress in category theory itself, featuring the appearance of the mimeographed notes of SGA IV by Grothendieck and his school (that redefined algebraic geometry altogether), of Bill Lawvere's Ph.D. thesis (which introduces algebraic theories as the ultimate setting for the syntax of Birkhoff's general algebras, and which makes the first, but decisive steps towards a categorical foundation of mathematics), of Peter Freyd's adjoint functor theorem (that showed the ubiquity of this brilliant categorical concept, invented by Daniel Kan in 1958), of Charles Ehresmann's "Catégories Structurées" (that, among many other things, introduces category objects in categories), and last, but not least, of Mac Lane's first coherence theorem in "Natural associativity and commutativity" [NAC]. His famous pentagon diagram for the associativity isomorphisms of tensor products took centre stage in many of his talks during the following years, and he must be largely credited for having initiated the development of monoidal categories, and therefore of enriched category theory and of higher-dimensional category theory, irrespective of the fact that the subject seemed to be somewhat in the air: Jean Bénabou's "Catégories avec multiplication" appeared that very same year.

But 1963, also the year of Cohen's independence proof for the continuum hypothesis, was just the beginning of a decade of extraordinary activity in category theory. The first widely-circulated book on the "Theory of Categories" by Barry Mitchell appeared in 1965, and the proceedings of the "Conference on Categorical Algebra" held the same year in La Jolla gave first testimony to the many facets of category theory, a new discipline that was beginning to grow far beyond the domains of its origins and that was attracting the interest of researchers around the globe. The year 1967 turned out to be another vintage year for category theory, especially as it applies to homotopy theory, featuring Jon Beck's Ph. D. thesis (which made monad theory *the* hot topic of the time, yielding many unexpected discoveries within a short period of time, such as Ernie Manes' proof of the monadicity of compact Hausdorff spaces over sets), the influential book by Gabriel and Zisman on categories of fractions, and Dan Quillen's Springer Lecture Notes (which reshaped abstract homotopy theory). Then, in 1969, Bill Lawvere and Myles Tierney defined the notion of elementary topos and opened the doors for the discovery of fascinating connections between geometry and logic. In his Prague lecture, Saunders labeled the event as a "triple clash" of the elementary theory of sets, of Cohen's forcing methods, and of axiomatic sheaf theory. The general excitement that followed may have prevented people from fully appreciating other important developments of the time, such as the appearance of Peter Gabriel and Friedrich Ulmer's Springer Lecture Notes on "Lokal präsentierbare Kategorien" in 1971, perhaps not just because of the language barrier, but also because the message that "size matters" may have run against the current of getting away from sets. In that year also two articles by Oswald Wyler as well as Guillaume Brümmer's Ph.D. thesis appeared that made the people around Horst Herrlich try to create a categorical framework for point-set topology. Meanwhile, in Prague, the school around Vera Trnková and Aleš Pultr, also firmly based on set-theoretical foundations, dug deeper into their study of set-valued functors and were about to embark on the study of iterative methods that quickly drew category theory into the domain of theoretical computer science. In 1972 John Isbell published his "Atomless parts of spaces", the starting point of the theory of "locales" (or complete Heyting algebras) which, in a sense, reconciles topos theory and general topology. Ross Street's "Formal theory of monads" appeared the same year, making him, together with Max Kelly, the core of what became known as the Australian School in category theory.

The Worker

Saunders found himself in the midst of these developments and countless others, participating vigorously in most of them, and trying to make his careful selection of what seemed worthy of being treated in his 1971 book [CWM], and what did not. "Categories for the worker", as it was quickly dubbed by the 1968 generation (I first heard the expression from John Isbell but don't know whether he is responsible for it; Saunders' short title was "Categories work") makes the most elementary notion of terminal (or, dually, initial) object in a category central to the theory. By drawing from a wide array of examples and illustrations, he is never in danger of losing touch with the average reader. So very much unlike the long journeys into the land of abstraction by the protagonists of the great two (but disjoint) French schools, lead by Grothendieck and Ehresmann, he never stretches the patience of the average reader. While having great respect for their achievements, one senses the absence of "mathematical chemistry" between Saunders and any of the two. In fact, Grothendieck's fibred categories, in spite of having been popularized with English readers by John Gray in the La Jolla proceedings, are worth only one sentence in [CWM], and Ehresmann's double categories fare only slightly better. However, the second edition of 1997 treats them in more detail, as part of a new chapter on "Structures in categories", which in fact leads the reader into many domains of current research in category theory. Enriched categories get mentioned in that chapter only briefly and, surprisingly, Gabriel and Ulmer's more hands-on notion of locally presentable category remains absent altogether. Again and again, we see a man ready to make choices, and stand by them.

Peter Johnstone's demanding book on "Topos Theory" of 1977 was the first universally respected text on the subject, preceded only by informal reports (by Anders Kock and Gavin Wraith in 1971, for example), journal articles (such as Peter Freyd's 1972 "Aspects of Topoi") and by Gavin Wraith's Springer Lecture Notes of 1975. The more elementary book by Robert Goldblatt of 1979 gained wide circulation but was met with considerable reservations by the experts. In the 1985 book on "Toposes, Triples and Theories" by Michael Barr and Charles Wells, topos theory appears only as part of a three-theme story, side by side with monads and Ehresmann sketches. Meanwhile the subject had grown into other exciting branches, in the form of synthetic differential geometry (Lawvere, Kock, Dubuc, Lavendhomme, Moerdijk, Reyes) and of the lambda calculus (Lambek, Seely, Scott). With nobody in the large community of highly talented experts stepping forward to write a book that would treat the geometric and logical aspects of the subject with equal weight, Saunders Mac Lane, now in his eighties, set out to do just that. In Ieke Moerdijk, born almost half a century after he was, he found a coauthor, highly gifted in both aspects, so that in 1992 "Sheaves in Geometry and Logic" [SGL] was ready to go. In [MA] Saunders writes about the collaboration: "The novelty of the subject very much required our joint efforts and our combined knowledge." Like his [CWM], the book was written while many aspects of the theory were still in flux, and it makes topos theory both accessible and relevant to the proverbial working mathematician, overall a truly astonishing accomplishment.

Saunders' mathematical activities didn't stop here. Finding good axioms for sets was one of the issues that remained on his agenda until as late as 2000, when he gave the opening talk at the Como meeting on a theme in the domain of algebraic set theory, an area that is still very actively pursued today. It seems that he could never abandon a subject which, in his mind, had not found a satisfactory setting. In [CCP] he concludes on the one hand

> *that the ZFC axiomatics is a remarkable conceptual triumph, but that the axiom system is far too strong for the task of explaining the role of the elementary notion of a "collection". It is also curious that most mathematicians can readily recite (and use) the Peano axioms for the natural numbers, but would be hard put to it to list all the axioms of ZFC.*

On the other hand, he never had any illusions about quick acceptance of any system that would radically break with traditions, as he also says in [CCP]:

> *And set theory without elements is still unpalatable to those trained from infancy to think of sets with elements: Habit is strong, and new ideas hard to accept.*

Final Salute

In this article I have not only neglected to even touch upon many parts of Saunders Mac Lane's mathematical work, which includes, among many other things, contributions to combinatorics, analysis and theoretical physics, but also failed to elaborate on his omnipresence in and strong engagements with editorial boards, professional societies, agencies, and policy-making bodies, be it as president or just letter writer, always ready to take a stand

and "to do something about it" whenever he saw problem. At the same time he kept in touch with almost anybody working anywhere in his wide array of mathematical interests, and he made the time to promote his causes anywhere around the globe. For example, after remarrying in 1987, he made a trip with his wife Osa Skotting to several parts of the former Soviet Union, which he vividly describes in [MA]. Having visited Tbilisi, Georgia, he decided to help publish George Janelidze's work on categorical Galois theory in the West and thereby started to integrate a highly capable but isolated group of researchers into the international arena. Coincidently, I first met George shortly afterwards, at a conference in Baku, Azerbaijan, and was happy to serve as a (somewhat illegal) letter carrier for his manuscripts. On November 3, 1987, Saunders found the time to send me a neatly typed letter, something that must have been just one of a hundred items on his daily agenda:

> *Dear Walter,*
> *Many thanks for your letter and the article by Janelidze on "Pure Galois Theory in Categories". He is a talented mathematician, and I am delighted to have the actual ms - I had tried hard to reconstruct his ideas from my notes.*
> *I hope that your sabbatical year goes well.*
> *Cordially,*
> *Saunders.*

How he managed to maintain such overwhelming level of activity on all ends over such a long period of time truly escapes me. Strict work attitudes and willingness to spend long hours at his desk, even throughout the night, make for necessary, but not sufficient conditions, and so does Saunders' mathematical endurance, to which Sammy Eilenberg alludes in [RIAG]:

> *One of the many qualities that I admire in Saunders is his capacity for orderly and intelligent computing. Not just ordinary 2-3 page calculations where the result is known in advance, but real computational fishing expeditions running to 30-40 pages. I admire this talent all the more since my own capacities in this direction are nil; a computation running more than five lines usually defeats me.*

Still, where did he take the needed physical and, perhaps more importantly, inner strength from? His fundamentally positive and cheerful nature comes to mind, and readers of [MA] also sense how much he appreciated the support that he received from both his first and second wives.

Last, but not least, it helps to be born a leader, a role that he could assume in any situation, and with great ease. At the 1999 Category Theory conference in Coimbra, Portugal, at which we celebrated Saunders' ninetieth birthday, he still apeared strong, followed an immense conference program, and took centre stage at the social events. He did not skip the Wednesday afternoon walk on a sandy hillside, in burning early-afternoon sunshine of July that made everybody thirsty and exhausted, and he was just at his best at the wonderful early-evening reception at a nearby castle that followed. Nobody could have written a better movie script for this scene: Our tour bus stops at the foot of the castle's hill, Saunders gets out first and just loves how he is royally welcomed by the mayor of the local town (who happened to be the brother of the conference organizer, Manuela Sobral, and who must have been carefully instructed on how to say the right things), and then the two of them lead the pack uphill to the castle, as if the day had just started. Also unforgettable the scene after the conference dinner two nights later, held in a grand palace in the countryside. The guests were entertained for quite a while by three typical Coimbra fado singers in their traditional black gowns, who were very surprised when, after the birthday bash, Saunders joined them on stage, put on one of the gowns and, to everybody's delight, chimed in.

That summer in Portugal the field that Saunders had helped to create and that he had nurtured over such a long period of time, celebrated its leader and his great accomplishments, for the last time in his presence. But certainly not for the last time.

References

[SMA] G. Birkhoff and S. Mac Lane, *A Survey of Modern Algebra,* Macmillan (New York 1941); fourth edition (New York, 1977).

[RIAG] S. Eilenberg, Some remarks on the interface of algebra and geometry. In [SP], pp. 525-526.

[GEH] S. Eilenberg and S. Mac Lane, Group extensions and homology, *Ann. of Math.* **43** (1942) 757-831.

[NIGT] S. Eilenberg and S. Mac Lane, Natural isomorphisms in group theory, *Proc. Nat. Acad. Sci. USA* **28** (1942) 537-543.

[GTNE] S. Eilenberg and S. Mac Lane, General theory of natural equivalences, *Trans. Amer. Math. Soc.* **58** (1945) 231-294.

[SP] I. Kaplansky (editor), *Saunders Mac Lane. Selected Papers,* Springer (New York, 1979).

[CT] Max Kelly, Saunders Mac Lane and category theory. In [SP], pp. 527-543.

[GCD] S. Mac Lane, Groups, categories and duality, *Proc. Nat. Acad. Sci. USA* **34** (1948) 263-267.

[DG] S. Mac Lane, Duality for groups, *Bull. Amer Math. Soc.* **56** (1950) 485-516.

[H] S. Mac Lane, *Homology,* Springer (New York, 1963).

[NAC] S. Mac Lane, Natural associativity and commutativity, *Rice Univ. Studies* **49** (1963) 28-46.

[CA] S. Mac Lane, Categorical Algebra, *Bull. Amer. Math. Soc.* **71** (1965) 40-106

[A] S. Mac Lane and G. Birkhoff, *Algebra,* AMS Chelsea Publishing (Providence, 1967); third edition (Providence, 1999).

[FCS] S. Mac Lane, Foundations for categories and sets. In: *Springer Lecture Notes in Math.* **92** (Berlin, 1969), pp. 146-164.

[CWM] S. Mac Lane, *Categories for the Working Mathematician,* Springer (New York, 1971); second edition (New York,1997).

[TLSA] S. Mac Lane, Topology and logic as a source of algebra, *Bull. Amer. Math. Soc.* **82** (1976) 1-40.

[LRTL] S. Mac Lane, A late return to a thesis in logic. In [SP], pp. 63-66.

[PTC] S. Mac Lane, Proof, truth and confusion. The 1982 Ryerson Lecture, University of Chicago (Chicago, 1982).

[CCP] S. Mac Lane, Concepts and categories in perspective, in: A Century of Mathematics in America, Part I, *Hist. Math* **1** (1988) 323-365 (Amer. Math. Soc., Providence, 1988). Addendum in Part III, *Hist. Math.***3** (1988) 439-441.

[MGN] S. Mac Lane, Mathematics at Göttingen under the Nazis, *Notices Amer. Math. Soc.* **42** (1995) 1134-1138. Corrections in: *Notices Amer. Math. Soc.* **43** (1996) 405-406.

[CGLA] S. Mac Lane, Categories in geometry, algebra and logic, *Math. Japonica* **42** (1995) 169-178.

[SEC] S. Mac Lane, Samuel Eilenberg and categories, *J. Pure Appl. Algebra* **168** (2002) 127-131.

[MA] S. Mac Lane, *Saunders Mac Lane. A Mathematical Autobiography,* A K Peters (Wellesley, 2005).

[SGL] S. Mac Lane and Ieke Moerdijk, *Sheaves in Geometry and Logic. A First Introduction to Topos Theory.* Springer (New York, 1992).

Department of Mathematics and Statistics,
York University, Toronto, Canada M3J1P3.