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INTUITIONISTIC FUZZY IDEALS IN INTRA-REGULAR RINGS

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ABSTRACT. The aim of this paper is to characterise intra-regular ring R by using the concept of intuitionistic fuzzy left(right,bi,quasi) ideals of R.

1. INTRODUCTION

The theory of fuzzy set was initiated by Zadeh[11] and so many researchers were conducted on the generalizations of the notion of fuzzy sets. The idea of *intuitionistic fuzzy set* was first published by Atanassov [2, 3] as a generalization of the notion of fuzzy sets. In [5], Banerjee and Basnet applied the concept of intuitionistic fuzzy sets to the theory of rings, and introduced the notions of intuitionistic fuzzy subrings and intuitionistic fuzzy ideals of a ring. In [7], Hur et al. introduced the notions of intuitionistic fuzzy (completely) prime ideals and intuitionistic fuzzy weak completely prime ideals in a ring. Present aurhors[9] introduced the notions of intuitionistic fuzzy bi-ideal and quasi-ideal,and characterizations of regular rings are proved. In this paper we introduc the intrinsic product of intuitionistic fuzzy sets and intuitionistic fuzzy left(right,bi,quasi) ideals in a ring. Using such notions, we discuss characterizations of intra-regular ring.

2. PRELIMINARIES

Let R be a ring. Let A and B be subsets of R. Then the multiplication of A and B is defined as follows:

$$AB = \Big\{ \sum_{\text{finite}} a_i b_i \mid a_i \in A,\, b_i \in B \Big\}$$

An additive subgroup Q of a ring R is called a *quasi-ideal* of R if $QR \cap RQ \subseteq Q$, and an additive subgroup B of a ring R is called a *bi-ideal* of R if $BB \subseteq B$ and $BRB \subseteq B$.

As an important generalization of the notion of fuzzy sets in M, Atanassov [2, 3] introduced the concept of an *intuitionistic fuzzy set* (IFS for short) defined on a non-empty set M as objects having the form

$$A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle \mid x \in M\},$$

where the functions $\mu_A : M \to [0,1]$ and $\gamma_A : M \to [0,1]$ denote the *degree of membership* (namely $\mu_A(x)$) and the *degree of nonmembership* (namely $\gamma_A(x)$) of each element $x \in M$ to A respectively, and $0 \le \mu_A(x) + \gamma_A(x) \le 1$ for all $x \in M$.

Such defined objects are studied by many authors (see for example two journals: 1. *Fuzzy Sets and Systems* and 2. *Notes on Intuitionistic Fuzzy Sets*) and have many interesting applications not only in mathematics (see Chapter 5 in the book [4]).

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For the sake of simplicity, we shall use the symbol $A = (\mu_A, \gamma_A)$ for the IFS $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle \mid x \in M\}$. Let $A = (\mu_A, \gamma_A)$ and $B = (\mu_B, \gamma_B)$ be IFSs in a set M. We define

- $A \subseteq B \Leftrightarrow (\forall x \in M)\,(\mu_A(x) \leq \mu_B(x),\ \gamma_A(x) \geq \gamma_B(x))$.
- $A = B \Leftrightarrow A \subseteq B$ and $B \subseteq A$.
- $A \cap B = (\mu_A \wedge \mu_B, \gamma_A \vee \gamma_B)$.
- $A \cup B = (\mu_A \vee \mu_B, \gamma_A \wedge \gamma_B)$.
- $0_\sim = (0, 1)$ and $1_\sim = (1, 0)$.

Definition 2.1. [5, 8] An IFS $A = (\mu_A, \gamma_A)$ in a ring R is called an *intuitionistic fuzzy subring* of R if it satisfies the following conditions:

(i) $(\forall x, y \in R)\ (\mu_A(x - y) \geq \min\{\mu_A(x), \mu_A(y)\})$,
(ii) $(\forall x, y \in R)\ (\mu_A(xy) \geq \min\{\mu_A(x), \mu_A(y)\})$,
(iii) $(\forall x, y \in R)\ (\gamma_A(x - y) \leq \max\{\gamma_A(x), \gamma_A(y)\})$,
(iv) $(\forall x, y \in R)\ (\gamma_A(xy) \leq \max\{\gamma_A(x), \gamma_A(y)\})$,

Definition 2.2. [5, 8] An IFS set $A = (\mu_A, \gamma_A)$ in a ring R is called an *intuitionistic fuzzy left* (resp. *right*) *ideal* of R if it satisfies the following conditions:

(i) $(\forall x, y \in R)\ (\mu_A(x - y) \geq \min\{\mu_A(x), \mu_A(y)\})$,
(ii) $(\forall x, y \in R)\ (\gamma_A(x - y) \leq \max\{\gamma_A(x), \gamma_A(y)\})$,
(iii) $(\forall a, x \in R)\ (\mu_A(ax) \geq \mu_A(x))$ (resp. $\gamma_A(xa) \leq \gamma_A(x)$).

If $A = (\mu_A, \gamma_A)$ is both an intuitionistic fuzzy left and intuitionistic fuzzy right ideal of a ring R, then $A = (\mu_A, \gamma_A)$ is called an *intuitionistic fuzzy ideal* of R.

Definition 2.3. [9] Let $A = (\mu_A, \gamma_A)$ and $B = (\mu_B, \gamma_B)$ be IFSs in a ring R. The *intrinsic product* of $A = (\mu_A, \gamma_A)$ and $B = (\mu_B, \gamma_B)$ is defined to be the IFS $A * B = (\mu_{A*B}, \gamma_{A*B})$ in R given by

$$\mu_{A*B}(x) := \bigvee_{x = \sum_{\text{finite}} a_i b_i} \min\left\{ \begin{array}{l} \mu_A(a_1), \mu_A(a_2), \cdots, \mu_A(a_m), \\ \mu_B(b_1), \mu_B(b_2), \cdots, \mu_B(b_m) \end{array} \right\}$$

$$\gamma_{A*B}(x) := \bigwedge_{x = \sum_{\text{finite}} a_i b_i} \max\left\{ \begin{array}{l} \gamma_A(a_1), \gamma_A(a_2), \cdots, \gamma_A(a_m), \\ \gamma_B(b_1), \gamma_B(b_2), \cdots, \gamma_B(b_m) \end{array} \right\}$$

if we can express $x = a_1b_1 + a_2b_2 + \cdots + a_mb_m$ for some $a_i, b_i \in R$ and for some positive integer m where each $a_ib_i \neq 0$. Otherwise, we define $A * B = 0_\sim$, i.e., $\mu_{A*B}(x) = 0$ and $\gamma_{A*B}(x) = 1$.

Definition 2.4. [9] An IFS $A = (\mu_A, \gamma_A)$ in a ring R is called an *intuitionistic fuzzy quasi-ideal* of R if

(i) $(\forall x, y \in R)\ (\mu_A(x - y) \geq \min\{\mu_A(x), \mu_A(y)\})$,
(ii) $(\forall x, y \in R)\ (\gamma_A(x - y) \leq \max\{\gamma_A(x), \gamma_A(y)\})$,
(iii) $(A * 1_\sim) \cap (1_\sim * A) \subseteq A$.

Definition 2.5. [9] An IFS $A = (\mu_A, \gamma_A)$ in a ring R is called an *intuitionistic fuzzy bi-ideal* of R if

(i) $(\forall x, y \in R)\ (\mu_A(x - y) \geq \min\{\mu_A(x), \mu_A(y)\})$,
(ii) $(\forall x, y \in R)\ (\gamma_A(x - y) \leq \max\{\gamma_A(x), \gamma_A(y)\})$,
(iii) $A * A \subseteq A$ and $A * 1_\sim * A \subseteq A$.

Lemma 2.6. [9] *For an IFS $A = (\mu_A, \gamma_A)$ in a ring R, the following assertions are equivalent:*

(i) *$A = (\mu_A, \gamma_A)$ is an intuitionistic fuzzy subring of R.*
(ii) *$A = (\mu_A, \gamma_A)$ is an intuitionistic fuzzy subgroup of the additive group $(R, +)$ and $A * A \subseteq A$.*

Lemma 2.7. [9] *An IFS $A = (\mu_A, \gamma_A)$ in a ring R is an intuitionistic fuzzy left* (resp. *left*) *ideal of a ring R if and only if*

(i) $\mu_A(x-y) \geq \min\{\mu_A(x), \mu_A(y)\}$ *and* $\gamma_A(x-y) \leq \max\{\gamma_A(x), \gamma_A(y)\}$,
(ii) $1_\sim * A \subseteq A$ (resp. $A * 1_\sim \subseteq A$).

Lemma 2.8. [9] *Every intuitionistic fuzzy left* (resp. *right, two-sided*) *ideal of a ring R is an intuitionistic fuzzy quasi-ideal of R.*

Lemma 2.9. [9] *Any intuitionistic fuzzy quasi-ideal of a ring R is an intuitionistic fuzzy bi-ideal of R.*

For a subset X of a ring R,we denote $\widetilde{X} = \{\langle x, \mu_{\widetilde{X}}(x), \gamma_{\widetilde{X}}(x)\rangle \mid x \in R\}$ defined by

$$\mu_{\widetilde{X}}(x) := \begin{cases} 1 & \text{if } x \in X \\ 0 & \text{otherwise} \end{cases}$$

$$\text{and } \gamma_{\widetilde{X}}(x) := \begin{cases} 0 & \text{if } x \in X \\ 1 & \text{otherwise} \end{cases}$$

for all $x \in R$. For the sake of simplicity, we shall use the symbol $\widetilde{X} = (\mu_{\widetilde{X}}, \gamma_{\widetilde{X}})$ for the $\widetilde{X} = \{\langle x, \mu_{\widetilde{X}}(x), \gamma_{\widetilde{X}}(x)\rangle \mid x \in X\}$.

Lemma 2.10. *Let A and B be any subsets of a ring R. Then we have*

(i) $\widetilde{A} * \widetilde{B} = \widetilde{AB}$.
(ii) $\widetilde{A} \cap \widetilde{B} = \widetilde{A \cap B}$.

Proof. Straightforward. □

Lemma 2.11. *Let A be a nonempty subset of R, Then the following holds assertions.*

(i) *A is a subring of a ring R if and only if $\widetilde{A}$ is an intuitionistic fuzzy subring of R.*
(ii) *A is a left(right) ideal of R if and only if $\widetilde{A}$ is an intuitionistic fuzzy left(right) ideal of R.*
(iii) *A is a quasi ideal of R if and only if $\widetilde{A}$ is an intuitionistic fuzzy quasi ideal of R.*

Proof. Straightforward. □

3. Intra-regular Rings

Recall that a ring R is said to be *intra-regular* if for each element a of R, there exists elements x_i and y_i of R such that $a = \sum_{i=1}^{n} x_i a^2 y_i$.

Theorem 3.1. *A ring R is intra-regular if and only if $A \cap B \subseteq B * A$ for every intuitionistic fuzzy right ideal A of R and every intuitionistic fuzzy left ideal B of R.*

Proof. Assume that R is an intra-regular ring. Let $A = (\mu_A, \gamma_A)$ and $B = (\mu_B, \gamma_B)$ be IFSs in a ring R and let $a \in R$. Since R is a intra-regular ring, there exist elements $x_i, y_i \in R$

such that $x = \sum x_i a^2 y_i = \sum (x_i a)(a y_i)$. Then we have

$$\begin{aligned}
\mu_{(A*B)}(a) &= \bigvee_{a=\sum_{\text{finite}} p_i q_i} \min\{\mu_A(p_1), \mu_A(p_2), \cdots, \mu_A(p_m), \mu_B(q_1), \mu_B(q_2), \cdots, \mu_B(q_m)\} \\
&\geq \min\left\{ \begin{array}{l} \mu_A(x_1 a), \mu_A(x_2 a), \cdots, \mu_A(x_m a), \\ \mu_B(a y_1), \mu_B(a y_2), \cdots, \mu_B(a y_m) \end{array} \right\} \\
&\geq \min\{\mu_A(a), \mu_B(a)\} \\
&= (\mu_A \wedge \mu_B)(a)
\end{aligned}$$

and

$$\begin{aligned}
\gamma_{(A*B)}(a) &= \bigwedge_{a=\sum_{\text{finite}} p_i q_i} \max\{\gamma_A(p_1), \gamma_A(p_2), \cdots, \gamma_A(p_m), \gamma_B(q_1), \gamma_B(q_2), \cdots, \gamma_B(q_m)\} \\
&\leq \max\left\{ \begin{array}{l} \gamma_A(x_1 a), \gamma_A(x_2 a), \cdots, \gamma_A(x_m a), \\ \gamma_B(a y_1), \gamma_B(a y_2), \cdots, \gamma_B(a y_m) \end{array} \right\} \\
&\leq \max\{\gamma_A(a), \gamma_B(a)\} \\
&= (\gamma_A \vee \gamma_B)(a).
\end{aligned}$$

Hence $\mu_{A*B}(x) \geq (\mu_A \wedge \mu_B)(x)$ and $\gamma_{A*B}(x) \leq (\gamma_A \vee \gamma_B)(x)$ for all $x \in R$. Therefore $A \cap B \subseteq B * A$. Suppose the necessary condition holds. Let a be any element of R. We consider the principal left ideal $[a)$ and the principal right ideal $(a]$. By Lemma 2.11, $\widetilde{[a)} = (\mu_{\widetilde{[a)}}, \gamma_{\widetilde{[a)}})$ is an intuitionistic fuzzy left ideal and $\widetilde{(a]} = (\mu_{\widetilde{(a]}}, , \gamma_{\widetilde{(a]}})$ is an intuitionistic fuzzy right ideal. Then we have

$$\mu_{\widetilde{(a][a)}}(a) = \mu_{\widetilde{(a]}*\widetilde{[a)}}(a) \geq (\mu_{\widetilde{(a]}} \wedge \mu_{\widetilde{[a)}})(a) = \min\{\mu_{\widetilde{(a]}}(a), \mu_{\widetilde{[a)}}(a)\} = \min\{1, 1\} = 1$$

and

$$\gamma_{\widetilde{(a][a)}}(a) = \gamma_{\widetilde{(a]}*\widetilde{[a)}}(a) \leq (\gamma_{\widetilde{(a]}} \vee \gamma_{\widetilde{[a)}})(a) = \max\{\gamma_{\widetilde{(a]}}(a), \gamma_{\widetilde{[a)}}(a)\} = \max\{0, 0\} = 0.$$

Using the above results, we have

$$a \in (a][a) = (na + Ra)(ma + aR) = (m + n)a + (na)(aR) + Ra(ma) + (Ra)(aR).$$

and so $a = \sum_{i=1}^{n} x_i a^2 y_i$ for some x_i and y_i of R. Therefore R is an intra-regular ring. □

Lemma 3.2. [10] *A ring R is regular and intra-regular if and only if every quasi-ideal of R is idempotent.*

Theorem 3.3. *For a ring R, the following conditions are equivalent:*

(i) *R is a regular and intra-regular ring.*
(ii) *$A * A = A$ for every intuitionistic fuzzy bi-ideal A of R.*
(iii) *$A * A = A$ for every intuitionistic fuzzy quasi-ideal A of R.*

Proof. (i) ⇒ (ii). Let $A = (\mu_A, \gamma_A)$ be any intuitionistic fuzzy bi-ideal of R. Then $A * A \subseteq A$. To prove the opposite inclusion, let a be any element of R. Since R is a regular and intra-regular ring , there exists an element x, y_i and z_i in R such that $a = axa$ and $a = \sum_{i=1}^{n} y_i a^2 z_i$. Then

$$a = axa = axaxa = ax(\sum y_i a a z_i)xa = \sum (axy_i a)(a z_i x a).$$

Thus we have

$$\begin{aligned}\mu_{A*A}(a) &= \bigvee_{a=\sum\limits_{\text{finite}} p_i q_i} \min\left\{\begin{array}{l}\mu_A(p_1), \mu_A(p_2), \cdots, \mu_A(p_m), \\ \mu_A(q_1), \mu_A(q_2), \cdots, \mu_A(q_m)\end{array}\right\} \\ &\geq \min\left\{\begin{array}{l}\mu_A(axy_1a), \mu_A(axy_2a), \cdots, \mu_A(axy_ma), \\ \mu_A(az_1xa), \mu_A(az_2xa), \cdots, \mu_A(az_mxa)\end{array}\right\} \\ &\geq \min\{\mu_A(a), \mu_A(a)\} \\ &= \mu_A(a)\end{aligned}$$

and

$$\begin{aligned}\gamma_{A*A}(a) &= \bigwedge_{a=\sum\limits_{\text{finite}} p_i q_i} \max\left\{\begin{array}{l}\gamma_A(p_1), \gamma_A(p_2), \cdots, \gamma_A(p_m), \\ \gamma_A(q_1), \gamma_A(q_2), \cdots, \gamma_A(q_m)\end{array}\right\} \\ &\leq \max\left\{\begin{array}{l}\gamma_A(axy_1a), \gamma_A(axy_2a), \cdots, \gamma_A(axy_ma), \\ \gamma_A(az_1xa), \gamma_A(az_2xa), \cdots, \gamma_A(az_mxa)\end{array}\right\} \\ &\leq \max\{\gamma_A(a), \gamma_A(a)\} \\ &= \gamma_A(a),\end{aligned}$$

and so $A \subseteq A * A$. Therefore we obtain $A * A = A$.

(ii)$\Rightarrow$ (iii) Since any intuitionistic fuzzy quasi-ideal of R is an intuitionistic fuzzy bi-ideal of R by Lemma 2.11 , the implication (ii)$\Rightarrow$ (iii) is valid.

(iii)$\Rightarrow$ (i). Let Q be any quasi-ideal of R, and a any element of Q. By Lemma 2.11 , $\widetilde{Q}$ is an intuitionistic fuzzy quasi-ideal of R. Then we have $\mu_{\widetilde{Q^2}}(a) = \mu_{\widetilde{Q}*\widetilde{Q}}(a) = \mu_{\widetilde{Q}}(a) = 1$ and $\gamma_{\widetilde{Q^2}}(a) = \gamma_{\widetilde{Q}*\widetilde{Q}}(a) = \gamma_{\widetilde{Q}}(a) = 0$ and so $a \in Q^2$, that is, $Q \subseteq Q^2$. Since the reverse inclusion always holds, we obtain $Q^2 = Q$. It follows from Lemma 3.2 , R is a regular and intra-regular ring. □

Lemma 3.4. [9] *A ring R is regular if and only if $A * B = A \cap B$ for every intuitionistic fuzzy right ideal $A = (\mu_A, \gamma_A)$ of R and every intuitionistic fuzzy left ideal $B = (\mu_B, \gamma_B)$ of R.*

Theorem 3.5. *For a ring R, the following conditions are equivalent:*

(i) *R is regular and intra-regular.*
(ii) *$A \cap B \subseteq (A * B) \cap (B * A)$ for every intuitionistic fuzzy bi-ideals A and B of R.*
(iii) *$A \cap B \subseteq (A * B) \cap (B * A)$ for every intuitionistic fuzzy bi-ideal A and every intuitionistic fuzzy quasi-ideal B of R*
(iv) *$A \cap B \subseteq (A * B) \cap (B * A)$ for every intuitionistic fuzzy quasi-ideals A and B of R.*
(v) *$A \cap B \subseteq (A * B) \cap (B * A)$ for every intuitionistic fuzzy quasi-ideal A and intuitionistic fuzzy left ideal B of R.*
(vi) *$A \cap B \subseteq (A * B) \cap (B * A)$ for every intuitionistic fuzzy right ideal A and intuitionistic fuzzy left ideal B of R.*

Proof. (i) $\Rightarrow$ (ii). Assume that R is a regular and intra-regular ring. Let $A = (\mu_A, \gamma_A)$ and $B = (\mu_B, \gamma_B)$ be any intuitionistic fuzzy bi-ideals of R, and a be any element of R. Since R is regular and intra-regular, there exists elements x_i ,y_i and z_i in R such that

$a = \sum(ax_iy_ia)(az_ix_ia)$. Then we have

$$\begin{aligned}\mu_{A*B}(a) &= \bigvee_{a=\sum_{\text{finite}} p_iq_i} \min\left\{\begin{array}{l}\mu_A(p_1), \mu_A(p_2), \cdots, \mu_A(p_m),\\ \mu_B(q_1), \mu_B(q_2), \cdots, \mu_B(q_m)\end{array}\right\}\\ &\geq \min\left\{\begin{array}{l}\mu_A(ax_1y_1a), \mu_A(ax_2y_2a), \cdots, \mu_A(ax_my_ma),\\ \mu_B(az_1x_1a), \mu_B(az_2x_2a), \cdots, \mu_B(az_mx_ma)\end{array}\right\}\\ &\geq \min\{\mu_A(a), \mu_B(a)\}\\ &= (\mu_A \wedge \mu_B)(a)\end{aligned}$$

and

$$\begin{aligned}\gamma_{A*B}(a) &= \bigwedge_{a=\sum_{\text{finite}} p_iq_i} \max\left\{\begin{array}{l}\gamma_A(p_1), \gamma_A(p_2), \cdots, \gamma_A(p_m),\\ \gamma_B(q_1), \gamma_B(q_2), \cdots, \gamma_B(q_m)\end{array}\right\}\\ &\leq \max\left\{\begin{array}{l}\gamma_A(ax_1y_1a), \gamma_A(ax_2y_2a), \cdots, \gamma_A(ax_my_ma),\\ \gamma_B(az_1x_1a), \gamma_B(az_2x_2a), \cdots, \gamma_B(az_mx_ma)\end{array}\right\}\\ &\leq \max\{\gamma_A(a), \gamma_B(a)\}\\ &= (\gamma_A \vee \gamma_B)(a),\end{aligned}$$

and hence $A \cap B \subseteq A * B$. Similary $A \cap B \subseteq B * A$. Therefore $A \cap B \subseteq (A * B) \cap (B * A)$

(ii) $\Rightarrow$ (iii) $\Rightarrow$ (iv) $\Rightarrow$ (v) $\Rightarrow$ (vi). Straightforward.

(vi) $\Rightarrow$ (i). Let A and B be any intuitionistic fuzzy right ideal and any intuitionistic fuzzy left ideal of R respectively. Then we have $A \cap B \subseteq (A*B) \cap (B*A) \subseteq B*A$. It follows from Lemma 3.4 that R is an intra-regular ring. Similarly we can prove $A \cap B \subseteq (A*B) \cap (B*A) \subseteq (A*B)$. By Lemma 2.7 we see that $A*B \subseteq A*1_\sim \subseteq A$ and $A*B \subseteq 1_\sim * B \subseteq B$ so that $A*B \subset A \cap B$. Hence $A*B = A \cap B$ From Lemma 3.4 it follows that R is regular ring. Therefore R is regular and intra-regular $\square$

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