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A SIMPLE TWO-PLAYER TWO-SIDED GAMES OF DECEPTION

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ABSTRACT. Let X_1, X_2, Y_1, Y_2 are *i.i.d.* r.v.s obeying the same probability destribution. Player I [II] looks privately $(X_1, X_2) = (x_1, x_2)$, $[(Y_1, Y_2) = (y_1, y_2)]$. They choose a single common number θ, and player II opens the nearest number to θ among y_1 and y_2 and covers the other number. If II's opened number is $> (<)\theta$, then I gets as his payoff, the opened (covered) number. The problem is to determine θ under which the expected payoff $M_1(\theta)$, I can get, is maximized. The maximization problem for II is quite similar as for I. Because of symmetry between the two players, our problem essentially reduces to computation of $M_1(\theta)$ and finding the θ which maximizes $M_1(\theta)$. This game is solved for (1) uniform distribution on [0, 1], (2) exponential distribution on $[0, \infty)$, (3) normal distribution on $(-\infty, \infty)$, and (4) some other distributions around them.

1 Two-player One-sided Games of Deception. Two numbers x_1 and x_2 are chosen from $[0, 1]$ by means of independent bivariate uniform distribution on $[0, 1]^2$. Player I now looks at the numbers privately and chooses one of the two and opens it to Player II, and the other number is covered. Player II then accepts either one of the opened number or the covered number, and receives from player I the number he accepted. Player I (II) aims to minimize (maximize) the expected payoff to II.

In Baston and Bostock (Ref.[1]) it is proven that the strategies;

σ^* : Choose the nearest number to $\frac{1}{2}$ among x_1 and x_2, and open it. The other number is covered.

for I and

τ^* : Accept the opened (covered) number if it is $> (<)\frac{1}{2}$,

for II, constitute an optimal strategy-pair, and the value of the game is 7/12.

By Sakaguchi (Ref.[5]) it is proven that, if x_1 and x_2 are independent bivariate standard normal distribution in $(-\infty, \infty)^2$, then the above strategy-pair σ^* and τ^* with $\frac{1}{2}$ replaced by 0, is optimal, and the value of the game is $\frac{2-\sqrt{2}}{\sqrt{2\pi}} \approx 0.2337$.

In the present paper, we make an approach to consider the two-sided game, where each player aims to maximize his expected payoff he obtains from his opponent.

2 Two-player Two-sided Games of Deception. Let X_1, X_2, Y_1, Y_2 are *i.i.d.* r.v.s with an identical p.d.f. Player I observes (X_1, X_2) and chooses his decision number $\theta_1 \in [0, 1]$. Player II observes (Y_1, Y_2) and chooses his decision number $\theta_2 \in [0, 1]$. Each player's choice of his decision number is made independently of the opponent's choice.

Player I chooses the nearest number to θ_1 among x_1 and x_2 and open it and the other number is covered. Player II chooses the nearest number to θ_2 among y_1 and y_2 and opens it and the other number is covered. If II's opened number is $> (<)\theta_1$, then I gets II's opened (covered) number. If I's opened number is $> (<)\theta_2$, then II gets I's opened (covered)

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number. For the sake of symmetry it should be $\theta_1 = \theta_2 (= \theta$, say). Both players want to choose the optimal θ which maximizes the common expected payoff, they can get.

An example of non-simple two-sided games of deception is mentioned in Remark 3 of Section 6.

3a Uniform Destribution in $[0,1]$. We divide the plane $[0,1]^2$ by the four "quadrant"s by the two axis $y_1 = \theta$ and $y_2 = \theta$, and denote them $Q^{(1)}, Q^{(2)}, Q^{(3)}$ and $Q^{(4)}$, in the clock-wise order. We use this convention throughout this paper.

We consider the two case $0 \le \theta < \frac{1}{2}$ and $\frac{1}{2} \le \theta \le 1$. First let $0 \le \theta < \frac{1}{2}$, Player I can get his payoff shown as in Figure 1.

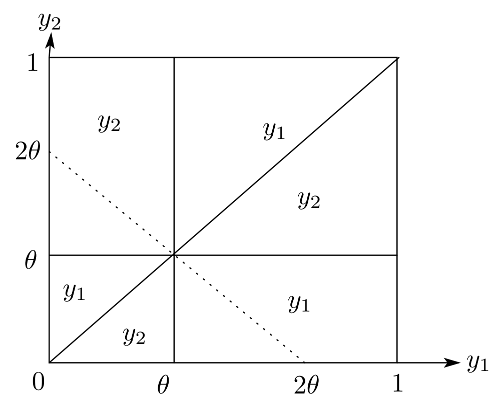

Figure 1. Case $0 \le \theta < \frac{1}{2}$

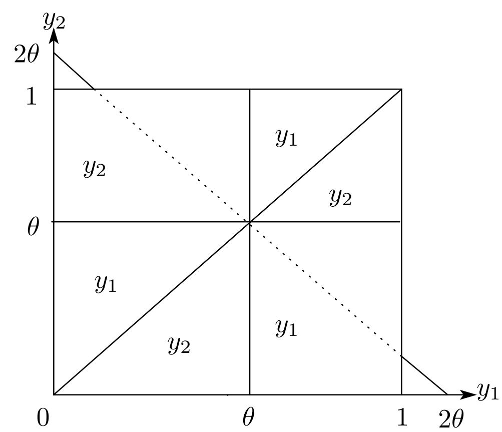

Figure 2. Case $\frac{1}{2} \le \theta \le 1$

The proof is as follows. In the upper-left of $Q^{(1)}, y_1 + y_2 > 2\theta$ and $\theta < y_1 < y_2$ hold, and hence II opens y_1, and I gets y_1. In the lower-right of $Q^{(1)}, y_1 + y_2 > 2\theta$ and $y_1 > y_2 > \theta$, and hence II opens y_2 and I gets y_2. In the upper-right of $Q^{(2)}, y_1 + y_2 > 2\theta$ and $y_2 < y_1 \wedge \theta$, and so II opens y_2, and I gets the covered y_1. In the lower-left of $Q^{(2)}, y_1 + y_2 < 2\theta$ and $y_1 > y_2 \vee \theta$, and so II opens y_1 and I gets y_1. In $Q^{(3)}$ and $Q^{(4)}$ similar arguments can be made and the result is as shown in Figure 1.

I's total expected reward is

$$2\left[\int_\theta^1 dy_2 \int_\theta^{y_2} y_1 dy_1 + \int_0^\theta dy_2 \int_\theta^1 y_1 dy_1 + \int_0^\theta dy_2 \int_0^{y_2} y_1 dy_1\right]. \tag{3.1}$$

The reason of why (3.1) is 2 times of $[\cdots]$ is that "y_1 domain" and "y_2 domain" are located symmetric about the straight line $y_1 = y_2$, and y_1 and y_2 are *i.i.d.* distributed.
Here, the first (second, third) term gives the reward from $Q^{(1)}(Q^{(2)}, Q^{(3)})$. The sum of these three is equal to

$$2\left[\left(\frac{1}{6} - \frac{1}{2}\theta^2 + \frac{1}{3}\theta^3\right) + \frac{1}{2}(\theta - \theta^3) + \frac{1}{6}\theta^3\right] = \frac{1}{3} + \theta - \theta^2 \tag{3.2}$$

which is concave, increasing with values $\frac{1}{3}$ at $\theta = 0$ and $\frac{7}{12}$ at $\theta = \frac{1}{2}$.

Now secondly, let $\frac{1}{2} \le \theta \le 1$. We repeat the same arguments as in the case $0 \le \theta < \frac{1}{2}$ (see Figure 2) and we obtain the same result $\frac{1}{3} + \theta - \theta^2$, which is concave, decreasing with values $\frac{7}{12}$ at $\theta = \frac{1}{2}$ and $\frac{1}{3}$ at $\theta = 1$.

Thus we can state

Theorem 1 *In Case* 3a, *the optimal choice is* $\theta^* = \frac{1}{2}$ *and the common* OPR *(optimal reward) is* 7/12.

3b Triangular Distribution in $[0,1]$. The p.d.f. is $f(x) = 2x, x \in [0,1]$. Mean value is 2/3. Figures 1 and 2 remain unchanged as in section 3a. We explain in the following the values in $Q^{(3)}$ and $Q^{(4)}$. In the upper-left of $Q^{(3)}$ in Figure 1, we have $y_1 + y_2 < 2\theta$ and $y_1 < y_2 < \theta$ and so II opens y_2 and I gets the covered number y_1. In the lower-right of $Q^{(3)}, y_1 + y_2 < 2\theta$ and $\theta > y_1 > y_2$, and so II opens y_1 and I gets the covered number y_2. In the upper-right of $Q^{(4)}$ we have $y_1 + y_2 > 2\theta$ and $y_1 < y_2 \wedge \theta$, and so II opens y_1 and I gets the covered number y_2. In the upper-left of $Q^{(4)}$ we have $y_1 + y_2 < 2\theta$ and $y_2 > y_1 \vee \theta$, and so II opens y_2 and I gets y_2. The result is as shown in Figure 1.

I's total expected reward is

$$(3.3) \qquad 8\left[\int_\theta^1 y_2 dy_2 \int_0^{y_2} y_1^2 dy_1 + \int_0^\theta y_2 dy_2 \int_\theta^1 y_1^2 dy_1 + \int_0^\theta y_2 dy_2 \int_0^{y_2} y_1^2 dy_1\right]$$

$$= 8\left[\left(\frac{1}{15} - \frac{1}{6}\theta^3 + \frac{1}{10}\theta^5\right) + \frac{1}{6}(\theta^2 - \theta^3) + \frac{1}{15}\theta^5\right] = 8\left(\frac{1}{15} + \frac{1}{6}\theta^2 - \frac{1}{6}\theta^3\right)$$

which is increasing and convex-concave (with the point of inflexion $\theta = 1/3$), with values $\frac{8}{15} \approx 0.5333$ at $\theta = 0$, $\frac{256}{405} \approx 0.6321$ at $\theta = \frac{1}{3}$, and 0.7 at $\theta = \frac{1}{2}$.

As for Figure 2, *i.e.* the case $\frac{1}{2} \le \theta \le 1$, computation is made similarly, and we find that I's total expected reward remains unchanged and is (3.3) again. However, this (3.3) for $\frac{1}{2} \le \theta \le 1$ is unimodal, concave with the maximal value $\frac{296}{405} \approx 0.7309$ at $\theta = \frac{2}{3}$. I's total expected reward as a function of $\theta \in [0,1]$ is shown by Figure 3.

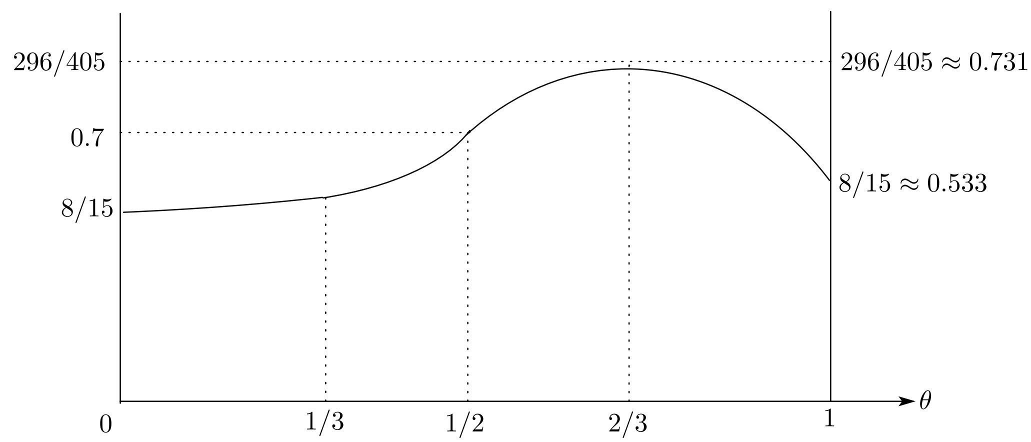

Figure 3. The function of $\theta \in [0,1]$

Theorem 2 *In Case* 3b, *the optimal choice is* $\theta^* = \frac{2}{3}$ *and the common* OPR *is* $\frac{296}{405} \approx 0.7309$.

Moreover, we can show in the same way that

Theorem 2′ *In Case* 3b′, *where* $f(x) = 2\overline{x}$, *in* $[0,1]$, *the optimal choice is* $\theta^* = \frac{1}{3}$, *and the common* OPR *is* $\frac{161}{405} \approx 0.39753$.

It is interesting that $\frac{296}{405} - \frac{161}{405} = \frac{1}{3}$.

4a Exponential Distribution in $[0, \infty)$. The p.d.f. is $f(x) = e^{-x}, x \in [0, \infty)$. Mean value is 1. The analysis goes just as before and we obtain Figure 4, which shows the reward I can get.

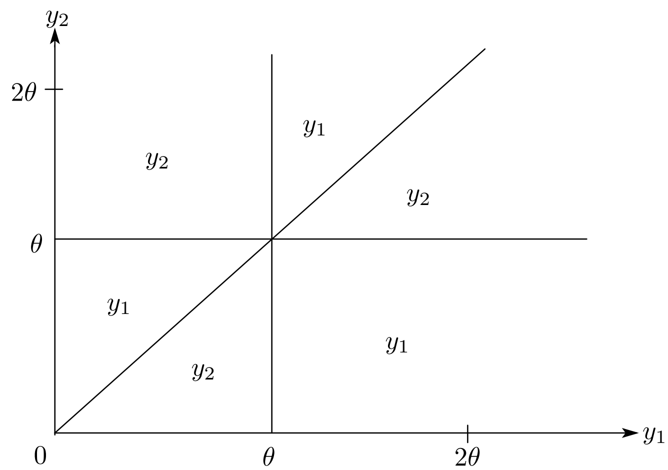

Figure 4. The reward player II can get.

The computation as made in (2.1) is 2 times of

$$\int_\theta^\infty e^{-y_2}dy_2 \int_\theta^{y_2} y_1 e^{-y_1}dy_1 + \int_0^\theta e^{-y_2}dy_2 \int_\theta^\infty y_1 e^{-y_1}dy_1 + \int_0^\theta e^{-y_2}dy_2 \int_0^{y_2} y_1 e^{-y_1}dy_1 \tag{4.1}$$

where the first integral is

$$\int_\theta^\infty -(1+y_2)e^{-2y_2}dy_2 + (1+\theta)e^{-2\theta},$$

the second integral is $(1+\theta)\left(e^{-\theta} - e^{-2\theta}\right)$, and third integral is

$$\left(1 - e^{-\theta}\right) + \int_0^\theta \left\{-(1+y_2)e^{-2y_2}\right\} dy_2.$$

Hence the sum is

$$\begin{aligned}\int_0^\infty -(1+y_2)e^{-2y_2}dy_2 \quad &+ \quad (1+\theta)e^{-2\theta} + (1+\theta)\left(e^{-\theta} - e^{-2\theta}\right) + \left(1 - e^{-\theta}\right)\\ &= \quad -\frac{3}{4} + \left(1 + \theta e^{-\theta}\right) = \frac{1}{4} + \theta e^{-\theta},\end{aligned}$$

which is maximized at $\theta = 1$. Thus we have

Theorem 3 *In Case* 4a*, the optimal choice is* $\theta^* = 1$*, and the common* OPR *is* $\frac{1}{2} + 2e^{-1} \approx 1.23576$.

4b Another Exponential Distribution in $[0, \infty)$. The p.d.f. is $f(x) = xe^{-x}, x \in [0, \infty)$. Mean value is 2. The analysis goes just as before and we obtain Figure 4 again. The computation as in (4.1) is now 2 times of

$$\int_\theta^\infty y_2 e^{-y_2} dy_2 \int_\theta^{y_2} y_1^2 e^{-y_1} dy_1 + \int_0^\theta y_2 e^{-y_2} dy_2 \int_\theta^\infty y_1^2 e^{-y_1} dy_1 + \int_0^\theta y_2 e^{-y_2} dy_2 \int_0^{y_2} y_1^2 e^{-y_1} dy_1 \tag{4.2}$$

where the first integral is

$$\int_\theta^\infty y_2 e^{-y_2} dy_2 \Big[(-y^2 - 2y - 2)e^{-y}\Big]_\theta^{y_2} = \int_\theta^\infty - \left(y_2^3 + 2y_2^2 + 2y_2\right) e^{-2y_2} dy_2 + (1+\theta)(\theta^2 + 2\theta + 2)e^{-2\theta},$$

the second integral is

$$\left\{1 - (1+\theta)e^{-\theta}\right\}(\theta^2 + 2\theta + 2)e^{-\theta} = (\theta^2 + 2\theta + 2)\left\{e^{-\theta} - (1+\theta)e^{-2\theta}\right\},$$

and the third integral is

$$\int_0^\theta y_2 e^{-y_2} dy_2 \Big[(-y^2 - 2y - 2)e^{-y}\Big]_0^{y_2} = \int_0^\infty - \left(y_2^3 + 2y_2^2 + 2y_2\right) e^{-2y_2} dy_2 + 2\left\{1 - (1+\theta)e^{-\theta}\right\}.$$

Hence the sum is

$$- \int_0^\infty (y_2^3 + 2y_2^2 + y_2)e^{-2y_2} dy_2 + (\theta^2 + 2\theta + 2)e^{-\theta} + 2\left\{1 - (1+\theta)e^{-\theta}\right\} = -1 + \left[2 + \left\{\theta^2 + 2\theta + 2 - 2(1+\theta)\right\} e^{-\theta}\right] = 1 + \theta^2 e^{-\theta},$$

which is maximized at $\theta = 2$.

For the computations of these integrals we used the formulas

$$\int te^{-t} dt = -(1+t)e^{-t}, \quad \int t^2 e^{-t} dt = -(t^2 + 2t + 2)e^{-t}, \tag{4.3}$$

$$\int te^{-2t} dt = -\frac{1}{4}(1+t)e^{-t} \text{ and } \int t^2 e^{-2t} dt = -\frac{1}{8}(t^2 + 2t + 2)e^{-t}.$$

Thus we obtain

Theorem 4 *In Case* 4*b, the optimal choice is* $\theta^* = 2$*, and the common* OPR *is* $2(1 + 4e^{-2}) \approx 3.0827$.

5a Normal Distribution in $(-\infty, \infty)$. We consider the case $\theta \geq 0$. The case $\theta < 0$ is not needed to consider, because of symmetry of the p.d.f.. The reward player II can get is shown, just as in the preceding distribution, by Figure 5.

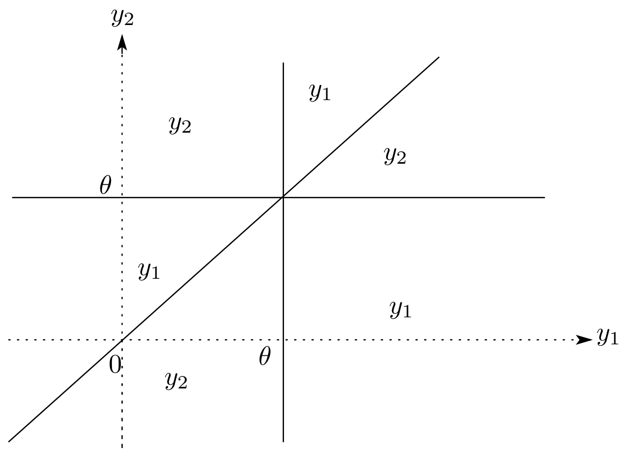

Figure 5. Case $\theta \geq 0$.

I's total expected reward is 2 times of

$$\int_\theta^\infty \phi(y_2)dy_2 \int_\theta^{y_2} y_1\phi(y_1)dy_1 + \Phi(\theta)\int_\theta^\infty y_1\phi(y_1)dy_1 + \int_{-\infty}^\theta y_1\phi(y_1)(\Phi(\theta)-\Phi(y_1))dy_1, \tag{5.1}$$

where

$$\phi(t) \equiv \frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}t^2} \text{ and } \Phi(t) \equiv \int_{-\infty}^t \phi(s)ds.$$

The first integral is

$$\left[-\phi(t)\overline{\Phi}(t)\right]_\theta^\infty - \int_\theta^\infty (\phi(t))^2\,dt = \phi(\theta)\overline{\Phi}(\theta) - \int_\theta^\infty (\phi(t))^2\,dt,$$

the second integral is $\Phi(\theta)\left[-\phi(t)\right]_\theta^\infty = \Phi(\theta)\phi(\theta)$, and the third integral is

$$\left[-\phi(t)\left(\Phi(\theta)-\Phi(t)\right)\right]_{t=-\infty}^\theta - \int_{-\infty}^\theta (\phi(t))^2\,dt = -\int_{-\infty}^\theta (\phi(t))^2\,dt.$$

Hence the sum is

$$\phi(\theta) - \int_\theta^\infty (\phi(t))^2\,dt - \int_{-\infty}^\theta (\phi(t))^2\,dt \tag{5.2}$$

and its derivative is

$$-\theta\phi(\theta) + (\phi(\theta))^2 - (\phi(\theta))^2 = -\theta\phi(\theta) < 0.$$

Therefore the sum is maximized at $\theta = 0$, and has the value

$$\phi(0) - \int_0^\infty (\phi(t))^2\,dt - \int_{-\infty}^0 (\phi(t))^2\,dt = \frac{1}{\sqrt{2\pi}} - 2\cdot\frac{1}{4\sqrt{\pi}} = \frac{2-\sqrt{2}}{2\sqrt{2\pi}}$$

$\left(\text{Note that } \int_0^\infty (\phi(t))^2\,dt = \frac{1}{4\sqrt{\pi}}\right)$. Thus we arrive at

Theorem 5 *In Case* 5a*, the optimal choice is* $\theta^* = 0$*, and the common* OPR *is* $\frac{2-\sqrt{2}}{\sqrt{2\pi}} \approx 0.2337$.

5b. Symmetric Exponential Distribution in $(-\infty, \infty)$. The p.d.f. is $f(x) = \frac{1}{2}e^{-|x|}$. The mean value is 0. We consider the case $\theta \geq 0$ only. The reward I can get is shown by Figure 5, again.

I's total expected reward is $2 \times \frac{1}{4}$ times of

$$(5.3)\quad \iint\limits_{\theta<y_1<y_2<\infty} y_1 e^{-(y_1+y_2)} dy_1 dy_2 + \left\{ \int_0^\theta e^{-y_2} dy_2 + \int_{-\infty}^0 e^{y_2} dy_2 \right\} \left\{ \int_\theta^\infty y_1 e^{-y_1} dy_1 \right\}$$
$$+ \left\{ \iint\limits_{-\infty<y_1<0<y_2<\theta} y_1 e^{y_1-y_2} dy_1 dy_2 + \iint\limits_{0<y_1<y_2<\theta} y_1 e^{-(y_1+y_2)} dy_1 dy_2 \right.$$
$$\left. + \iint\limits_{-\infty<y_1<y_2<0} y_1 e^{y_1+y_2} dy_1 dy_2 \right\},$$

i.e., the sum of the reward in $Q^{(1)}, Q^{(2)}$ and $Q^{(3)}$, in this order. For computation of these integrals, we used the formulas (4.3).

The part in $Q^{(1)}$ is

$$\int_\theta^\infty e^{-y_2} dy_2 \int_\theta^{y_2} y_1 e^{-y_1} dy_1 = \int_\theta^\infty \left\{ (1+\theta)e^{-\theta} - (1+y_2)e^{-y_2} \right\} e^{-y_2} dy_2$$
$$= \left(\frac{1}{2} + \theta \right) e^{-2\theta} - \frac{1}{4}(1+\theta)e^{-\theta},$$

the part in $Q^{(2)}$ is $(1+\theta)\left(2 - e^{-\theta}\right) e^{-\theta}$, and the part in $Q^{(3)}$ is

$$-\left(1 - e^{-\theta}\right) + \int_0^\infty \left\{ 1 - (1+y_2)e^{-y_2} \right\} e^{-y_2} dy_2 + \int_{-\infty}^0 -(1-y_2)e^{2y_2} dy_2$$
$$= e^{-\theta} - \int_0^\infty (1+t)e^{-2t} dt - \left(\frac{1}{2} - \frac{1}{4} \right) = e^{-\theta} - 1.$$

Hence the sum of the three parts is

$$(5.4)\quad \left\{ \left(\frac{1}{2} + \theta \right) e^{-2\theta} - \frac{1}{4}(1+\theta)e^{-\theta} \right\} + (1+\theta)\left(2 - e^{-\theta}\right) e^{-\theta} + \left(e^{-\theta} - 1\right)$$
$$= \left(\frac{11}{4} + \frac{7}{4}\theta \right) e^{-\theta} - \frac{1}{2} e^{-2\theta} - 1.$$

Its derivative satisfies

$$-e^{-\theta} \left(1 + \frac{7}{4}\theta - e^{-\theta} \right) \left\{ \begin{matrix} = \\ < \end{matrix} \right\} 0, \text{ if } \theta \left\{ \begin{matrix} = \\ > \end{matrix} \right\} 0.$$

Therefore we obtain

Theorem 6 *In Case* 5b*, the optimal choice is* $\theta^* = 0$ *and the common* OPR *is* $2 \times \frac{1}{4} \times \left(\frac{11}{4} - \frac{3}{2} \right) = \frac{5}{8} (= 0.625)$.

6 Three Remarks

Remark 1 According to our theorems 1∼6, player's optimal choice θ^* is equal to the expected value of each r.v.. The choice is made to the effect that the information concerning the *covered* r.v. obtained by the opponent, becomes least.

Suppose that player II doesn't employ any deception strategy *i.e.*, he chooses Y_1 and Y_2 with probability 1/2 each, and opens it. If player I knows this policy of his opponent, then player I can get $E_f(Y \vee \mu_f)$, where $\mu_f = E_f(Y)$. It is clear that

$$E_f[Y \vee \mu_f] = \begin{cases} \displaystyle\int_0^1 \left(y \vee \frac{1}{2}\right) dy = \frac{5}{8}, & \text{if } f(y) = 1, \forall y \in [0,1] \\ \displaystyle\int_0^\infty (y \vee 1)\ e^{-y} dy = 1 + e^{-1} \approx 1.368, & \text{if } f(y) = e^{-y}, \forall y \in [0,\infty) \\ \displaystyle\int_0^\infty y\ \phi(y)\ dy = \frac{1}{\sqrt{2\pi}} \approx 0.399, & \text{if } f(y) = \phi(y), \forall y \in (-\infty,\infty), \end{cases}$$

these of which are *grater* than

$$\frac{7}{12}\ \text{(in Th.1)},\ \ \frac{1}{2} + 2e^{-1} \approx 1.236\ \text{(in Th.3) and}\ \frac{2-\sqrt{2}}{\sqrt{2\pi}} \approx 0.2337\ \text{(in Th.5)}$$

respectively, in this order. We interprete that this means that II can deceive his opponent by enploying the deception strategy as mentioned Section 2. The situation is the same as for player I.

Remark 2 Let $(X_1, X_2)[(Y_1, Y_2)]$ be a bivariate correlated r.v. with p.d.f. $f(x_1, x_2)$ $[f(y_1, y_2)]$. For example

$$f(x_1, x_2) = 1 + \gamma(1 - 2x_1)(1 - 2x_2), \quad (x_1, x_2) \in [0,1]^2 \tag{6.1}$$

with $\gamma, |\gamma| \le 1$, is a given constnt, and the correlation coefficient is $(1/3)\,\gamma$. This is one of the simplest one that has identical uniform marginals and correlated components. See Ref.[3, 5].
Also

$$f(x_1, x_2) = e^{-(x_1+x_2)} \left\{1 + \gamma(2e^{-x_1} - 1)(2e^{-x_2} - 1)\right\}, \quad (x_1, x_2) \in [0,\infty)^2, \tag{6.2}$$

where $\gamma, |\gamma| \le 1$, has identical exponential marginals $f(x) = e^{-x}$ and the correlation coefficient is $(1/4)\,\gamma$. This is one of the simplest one that has identical exponential marginals and correlated components.

Finally, bivariate normal distribution

$$f(x_1, x_2) = \frac{1}{2\pi\sqrt{1-\rho^2}} \exp\left[-\frac{1}{2(1-\rho^2)}\left(x_1^2 - 2\rho x_1 x_2 + x_2^2\right)\right], \quad (x_1, x_2) \in (-\infty,\infty)^2, \tag{6.3}$$

where $\rho, |\rho| < 1$, is the correlation coefficient. This has the identical marginal p.d.f. $\phi(x) \equiv \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$.

The two-sided games of deception for bivariate distributions (6.1), (6.2) and (6.3) are an interesting problem to be solved (see Ref.[5]).

Remark 3 We give an example of non-simple two-sided games of deception. Let X_1 and $X_2[Y_1$ and $Y_2]$ be *i.i.d.* r.v.s with p.d.f. $f(x)[g(y)]$. $f(x)$ and $g(y)$ are different p.d.f.s. Player I (II) chooses a number $\theta_1(\theta_2)$ and he opens the number nearest to $\theta_1(\theta_2)$ among $x_1, x_2(y_1, y_2)$, and covers the other number. Each player gets as his reward his opponent's $\left\{\begin{array}{c}\text{opened}\\ \text{covered}\end{array}\right\}$ number if it is $\left\{\begin{array}{c}> \theta_1 \vee \theta_2\\ < \theta_1 \wedge \theta_2\end{array}\right\}$ and $\frac{1}{2}$ (opened + covered), if otherwise. I and II want to choose θ_1 and θ_2, respectively, under which each player maxmizes the expected reward he can get. The three cases (1) $f(x) \equiv 1$ and $g(x) = 2x, x \in [0,1]$, (2) $f(x) = e^{-x}$ and $g(x) = xe^{-x}, x \in [0,\infty)$, (3) $f(x) = \phi(x)$ and $g(x) = \frac{1}{2}e^{-|x|}, x \in (-\infty,\infty)$, would be interesting.

Remark 4 There are a number of interesting researches around the games of deception. Among these are Ref.[2, 3, 4, 6 and 7].

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