THE WORLD OF MATHEMATICS HAS LOST A TOWERING FIGURE

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Saunders Mac Lane, longtime Professor at the University of Chicago, died on April 14, 2005, at the age of 95. We can only briefly mention some of the many ways in which he will be missed.

He had labored tirelessly to raise the level of support for scientific research and for science education in the United States: as President of the Mathematical Association of America in the early 1950's; as President of the American Mathematical Society in the early 1970's; and later as Vice-president of the National Academy of Sciences.

His enduring legacies, however, are twofold: the deep variety of the advances made possible by his own direct contributions to mathematical research, and the legions of students who have learned over the past 65 years from his popular textbooks on algebra and its applications.

In those textbooks Saunders Mac Lane and his collaborators produced original and understandable expositions of the most recent advanced results. The modern algebra that had been developed in the 1920's by Artin and Noether, was then diffused by van der Waerden, but only at an advanced level; Mac Lane & Birkhoff treated it for beginners in 1941. Mac Lane's books "Homology" and "Categories for the Working Mathematician" are still unexcelled as expositions of their respective fields. The developing field of topos theory received a "first introduction" in his 1991 book with Ieke Moerdijk.

Saunders Mac Lane's doctoral dissertation, at the Goettingen of Hilbert, Noether, Weyl, and Bernays, was in logic. But his best-known contributions were in algebraic topology.

Algebraic topology became necessary in the epoch of Betti and Volterra, who had noted that the behavior of electromagnetic fields and elastic fluids may depend on qualitative features of the shape of bodies and containers, and that those qualitative features require measurement by quantities of a new type. After several years of development, Brouwer in the 1920's gathered Hopf, Noether, and Vietoris in Holland for intensive research, from which the abelian nature of these quantities clearly emerged. Hurewicz, de Rham, Steenrod, and many others continued the advance into the 1930's and beyond, whereas Whitney distinguished one important class of these qualitative quantities as "cohomology".

Mac Lane entered algebraic topology through his friend Samuel Eilenberg. Together they constructed the famous Eilenberg-Mac Lane spaces and proved in 1942 that these spaces "represent cohomology". That seemingly technical result of geometrical algebra actually required several striking methodological advances:
(a) cohomology is a "functor" (a specific kind of dependence on change of spaces along maps);
(b) the category where these functors are defined has (as its maps) equivalence classes of continuous maps (rather than the continuous maps themselves) where deformations of maps serve to establish the equivalences;
(c) in any category, any fixed object K determines a special "representable" functor that assigns (to any X) the set [X,K] of maps from X to K. Most functors are not of that form and thus it is remarkable that the particular cohomological functors H*of interest turned out to enjoy a natural isomorphism

$$H^*(X) = [X, K]$$

but only for the above category (b) and only for the spaces K of the kind constructed for H* by Eilenberg and Mac Lane.

This specific advance required the general concepts of category, functor, and naturality, which had been invented in the same year by the same Eilenberg and Mac Lane! Gradually, their category theory began to revolutionize many fields of mathematics; its concepts were used to unify and simplify and even, as in the above original example, to make possible accurate formulations of results.

However, in a real sense, Mac Lane never moved away from his initial interest in logic. His last book is entitled "Sheaves in Geometry and Logic" for good reason. Already before going to study in Goettingen in the early 1930's, he had traveled from Yale to Chicago to study with E.H. Moore (the author of "General Analysis"). That continuing quest for understanding accompanied him through his whole life and led to fundamental work that made possible a qualitative broadening and deepening of logic. As the serious study of the general aspects of all exact thinking, Logic could not remain confined to the narrow "symbolic" tradition that had made presentations (or "languages") seem more fundamental than the algebras presented, nor could it remain confined to the narrow Frege tradition which maintained that concepts are mere properties. Taking due account of the achievements of those traditions, the Eilenberg-Mac Lane tradition has systematized the construction of actual geometric and conceptual objects (for which properties can then be defined, and whose algebraic aspect sometimes needs presentations). In that way the goal of a conscious guiding vision for the development of unified mathematics, which Moore a century ago could only fragmentarily realize, has become tangible.

Mac Lane's philosophy is based on geometry, but a geometry grown so broad and multi-layered that it incorporates the earlier logic. With this recognition of the unity of the principles of logic and geometry, and with the conceptual tools we inherit from his generation, we can hope to fulfill the central dream of Mac Lane (and of Moore), to convey deeper understanding of the developing mathematics to students and mathematical scientists.

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