(α,β)-SEMI OPEN SETS AND SOME NEW GENERALIZED SEPARATION AXIOMS

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ABSTRACT. Let (X,τ) be a topological space and $\alpha,\beta : P(X) \to P(X)$ be operators associated to τ, we introduce the concept of (α,β)-semi open sets and new generalized forms of separations by (α,β)-semi open sets. Also, we analyze the relations with some well known separation notions.

1. Introduction

The study of semi open sets was initiated by Levine [1]. S. N. Maheshwari in [4] introduced and studied a new separation axiom called semi separation axiom. C. Carpintero, E. Rosas and J. Vielma [6] introduced the concept of operators associated to a topology τ on the set X and α-semi open set. E. Rosas, J. Vielma, C. Carpintero and M. Salas [7] defined the α-semi T_i spaces for $i = 0, 1/2, 1, 2$, using the operator α and the α-semi open sets. In this paper, we introduce and study the notions of (α,β)-semi open sets and observe that these concepts generalize the semi generalized axioms of separation given in [8] in the case of any operator. Also, we obtain better results in comparision with the results obtained in [7],[8],and [9].

2. Preliminaries

In this section, we recall some of the basic definitions and some important results.

Definition 2.1. *Let (X,τ) be a topological space. We say that α is an ope rator associated to τ, if $\alpha:P(X)\to P(X)$ satisfies $U \subseteq \alpha(U)$, for all $U \in \tau$*

Definition 2.2. *Let (X,τ) be a topological space and $\alpha:P(X)\to P(X)$ be an operator associated to a topology τ. A subset $A \subseteq X$ is said to be α-semi open set if there exists $U \in\tau$ such that $U\subseteq A\subseteq\alpha(U)$.*

Definition 2.3. *Let (X,τ) be a topological space and $\alpha,\beta:P(X)\to P(X)$ be operators associated to a topology τ on X. We say that a subset $A\subseteq X$ is an(α,β)-semi open set if for each $x\in A$, there exists a β-semi open set V such that $x\in V$ and $\alpha(V)\subseteq A$. The complement of an (α,β)-semi open set is (α,β)-semi closed set.*

We can observe that when $\alpha = \beta = id$, we have that

$$A \text{ is } (\alpha,\beta)\text{-semi open set} \Leftrightarrow A \text{ is open}$$

If $\beta = id$ and α is arbitrary, then

$$A \text{ is } (\alpha,\beta)\text{-semi open set} \Leftrightarrow A \text{ is } \alpha\text{-open set}$$

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It is interesting to see that if β is any monotone operator (i.e, if $U \subseteq V$ then $\beta(U) \subseteq \beta(V)$), the collection of all β-semi open sets is just the collection of all (id,β)-semi open sets.

Definition 2.4. *Let (X,τ) be a topological space and $\alpha,\beta{:}P(X)\to P(X)$ be operators associated to a topology τ on X. We say that a subset $A\subseteq X$ is (α,β)-open set if for each $x\in A$, there exist open sets U, V such that $x\in U$, $x\in V$, and $\alpha(U) \cup \beta(V) \subseteq A$.*

The following theorem gives a relationship between (α, β)-semi open sets and (α, β)-open sets.

Theorem 2.1. *Let (X,τ) be a topological space and $\alpha,\beta{:}P(X)\to P(X)$ be operators associated to a topology τ on X. If $A\subseteq X$ is (α,β)-open set then A is (α,β)-semi open set and (β,α)-semi open set.*

Proof. Let $x \in A$ be given, there are open sets U, V such that $x \in U$, $x \in V$ and $\alpha(U) \cup \beta(V) \subseteq A$. Since all open sets U are β-semi open for any operator β, we obtain that U is a β-semi open set, $x \in U$ and $\alpha(U) \subseteq A$, therefore A is (α,β)-semi open set. Similarly, V is α-semi open set, $x \in V$, and $\beta(V) \subseteq A$, thus, A is also a (β,α)-semi open set. □

The following theorem gives the relationship between (α, β)-open sets and α-open sets.

Theorem 2.2. *Let(X,τ) be a topological space and $\alpha,\beta{:}P(X)\to P(X)$ be operators associated to a topology τ on X. Then, A is (α,β)-open set if and only if A is α-open set and β-open set.*

Proof. (Sufficiency). Given $x \in A$ there exist open sets U, V such that $x \in U$, $x \in V$ and $\alpha(U) \cup \beta(V) \subseteq A$. It follows that U is an open set, $x \in U$ and $\alpha(U) \subseteq A$, therefore A is α-open set. Similarly, V is an open set, $x \in V$ and $\beta(V) \subseteq A$, therefore A is β-open set.
(Necessity). If A is an α-open set and β-open set, then for all $x \in A$ there exist open sets U, V such that $x \in U$, $x \in V$, $\alpha(U) \subseteq A$ and $\beta(V) \subseteq A$, which implies that $\alpha(U)\cup\beta(V) \subseteq A$; therefore, A is (α,β)-open set. □

The following example shows, that there exists a subset of a space X and operators α, β that are β-semi open set but are not (α,β)-open set.

Example 2.1. *Let $X = \{a, b, c\}$ and $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$. Consider the following associated operators α and β defined as:*

$$\alpha(A) = A \text{ if } b \in A \text{ and } \alpha(A) = Cl(A) \text{ if } b \notin A$$
$$\beta(A) = Cl(A) \text{ if } b \in A \text{ and } \beta(A) = A \text{ if } b \notin A$$

We can see that $\{b, c\}$ is β-semi open set, but it is not (α,β)-open set.

The following lemmas give information about some fundamental properties of the (α,β)-semi open sets ((α,β)-semi closed sets, resp.).

Lemma 2.3. *Let (X,τ) be a topological space and $\alpha,\beta{:}P(X)\to P(X)$ be ope rators associated to a topology τ on X. If $\{A_i : i \in I\}$ is a collection of (α,β)-semi open sets, then $\bigcup_{i\in I} A_i$ is (α,β)-semi open set.*

Proof. Given $x\in\bigcup_i A_i$, then $x\in A_j$ for some $j\in I$. In this case, there exists a β semi open set V_j such that $x\in V_j$ and $\alpha(V_j)\subseteq A_j\subseteq\bigcup_i A_i$. Therefore, given $x\in\bigcup_i A_i$, there exists a β semi open set V_j such that $\alpha(V_j)\subseteq\bigcup_i A_i$. This implies that $\bigcup_i A_i$ is an (α,β)-semi open set. □

Now using the above lemma and the De Morgan laws, we obtain the following corollary.

Corollary 2.4. *Let (X,τ) be a topological space and $\alpha,\beta:P(X)\to P(X)$ be operators associated to a topology τ on X. If $\{A_i : i \in I\}$ is a collection of (α,β)-semi closed sets, then $\bigcap_{i\in I} A_i$ is (α,β)-semi closed set.*

We can observe, that from these two properties, it is possible to define in a natural way the (α,β)-semi closure and the (α,β)-semi interior of a set $A\subseteq X$. They will be denoted by (α,β)-$sCl(A)$and (α,β)-$sInt(A)$, respectively.
In a topological space (X,τ) for which it have the associated operators $\alpha,\beta:P(X)\to P(X)$, we have in a natural way some properties that are well known as we can see in the following lemma.

Lemma 2.5. *Let (X,τ) be a topological space and $\alpha,\beta : P(X) \to P(X)$ be operators associated to a topology τ on X. Then:*

(a) $(\alpha,\beta) - sInt(A) \subseteq (\alpha,\beta) - sInt(B)$ *if* $A \subseteq B$;
(b) $(\alpha,\beta) - sCl(A) \subseteq (\alpha,\beta) - sCl(B)$ *if* $A \subseteq B$;
(c) A *is* $(\alpha,\beta) -$ *semi open set* $\Leftrightarrow$ $A = (\alpha,\beta) - sInt(A)$;
(d) B *is* $(\alpha,\beta) -$ *semi closed set* $\Leftrightarrow$ $B = (\alpha,\beta) - sCl(B)$;
(e) $x \in (\alpha,\beta) - sInt(A)$ *if and only if there exists an* $(\alpha,\beta) -$ *semi open set* G *such that* $x \in G \subseteq A$;
(f) $x \in (\alpha,\beta) - sCl(B)$ *if and only if for all subset* G $(\alpha,\beta) -$ *semi open set such that* $x \in G$, $G \cap B \neq \emptyset$;
(g) $X\backslash((\alpha,\beta) - sCl(A))) = (\alpha,\beta) - sInt(X\backslash A)$ *and* $X\backslash((\alpha,\beta) - sInt(A))) = (\alpha,\beta) - sCl(X\backslash A)$.

Proof. It is a direct consequence of the definitions of (α,β)-semi closure and (α,β)-semi interior. □

3. (α,β)-semi T_i Spaces

In this section, we introduce the generalized separation axioms using the notions of (α,β)-semi open sets, also we give some characterization of these types of spaces and study the existent relationships between them and other types of spaces well known.

Definition 3.1. *Let (X,τ) be a topological space and $\alpha,\beta : P(X) \to P(X)$ be operators associated to a topology τ on X. The space X is said to be :*

(i) (α,β)-semi T_0 if for each pair of points $x, y \in X$, $x \neq y$, there is a (α,β)-semi open set containing one of the points, but not the other one.
(ii) (α,β)-semi T_1 if for each pair of distinct points $x, y \in X$ there exist a pair of (α,β)-semi open sets, one of them containing x but not y and the other one containing y but not x.
(iii) (α,β)-semi T_2 if for each pair of distinct points $x, y \in X$ there exist disjoints (α,β)-semi open sets U and V, in X such that $x \in U$ and $y \in V$.

The following theorems characterize the spaces: (α,β)-semi T_0, (α,β)-semi T_1 and (α,β)-semi T_2. We shall observe that there exist some similarities to the usually well known cases. These last ones are more general in the sense that they include the usual cases and others due to the arbitrariness of the considered operators α and β. We can see that the $\alpha - T_i$ spaces ([7]), with $i = 0, 1, 2$, are the (α, i_d)-semi T_i spaces. And the $\alpha -$ semi T_i spaces ([7]), with $i = 0, 1, 2$, are the $(i_d, \alpha) -$ semi T_i spaces.

Theorem 3.1. *Let (X,τ) be a topological space and $\alpha,\beta : P(X) \to P(X)$ be operators associated to a topology τ on X. Then X is a (α,β)-semi T_0 space if and only if for any $x, y \in X$ such that $x \neq y$ we have that $(\alpha,\beta) - sCl(\{x\}) \neq (\alpha,\beta) - sCl(\{y\})$.*

Proof. (Sufficiency) Suppose that X is (α,β)-semi T_0 space, then for any pair of distinct points $x, y \in X$ there exists a (α,β)-semi open set U, such that $x \in U$ and $y \notin U$ or $y \in U$ and $x \notin U$. It follows that $(\alpha,\beta) - sCl(\{x\}) \neq (\alpha,\beta) - sCl(\{y\})$.
(Necessity) Suppose that $x, y \in X$ and $x \neq y$, imply that $(\alpha,\beta) - sCl(\{x\}) \neq (\alpha,\beta) - sCl(\{y\})$. It follows that, given $x \neq y$, there is a point $z \in X$ such that $z \in (\alpha,\beta) - sCl(\{y\})$ and $z \notin (\alpha,\beta) - sCl(\{x\})$ or $z \in (\alpha,\beta) - sCl(\{x\})$ and $z \notin (\alpha,\beta) - sCl(\{y\})$. If $z \in (\alpha,\beta) - sCl(\{y\})$ and $z \notin (\alpha,\beta) - sCl(\{x\})$, there exist an ($\alpha$,$\beta$)-semi open set V such that $y \in V$ and $V \cap \{x\} = \emptyset$. In case that $z \in (\alpha,\beta) - sCl(\{x\})$ and $z \notin (\alpha,\beta) - sCl(\{y\})$, there exist an ($\alpha$,$\beta$)-semi open set U such that $x \in U$ and $V \cap \{y\} = \emptyset$. This shows that that X is (α,β)-semi T_0. □

Theorem 3.2. *Let (X,τ) be a topological space and $\alpha, \beta : P(X) \to P(X)$ be operators associated to a topology τ on X. For the topological space (X,τ, the followings conditions are equivalent:*

(a) X is $(\alpha,\beta) -$ semi T_1 space.
(b) Each unitary set $\{x\}$, $x \in X$, is a (α,β)-semi closed set.
(c) Each subset of X is the intersection of all super sets (α,β)-semi open containing it.

Proof. $(a) \Rightarrow (b)$. Let X be $(\alpha,\beta) -$ semi T_1 space. Given $y \in X - \{x\}$, then $y \notin \{x\}$, by hypothesis there are (α,β)-semi open sets $U, V \subseteq X$ such that $x \in U$, $y \notin U$ and $y \in V$, $x \notin V$. Therefore, $y \in V \subseteq X - \{x\}$, because $V \cap \{x\} = \emptyset$. It follows that $X - \{x\}$ is a (α,β)-semi open set and, therefore, $\{x\}$ is (α,β)-semi closed set.

$(b) \Rightarrow (c)$. Let us suppose that each $\{x\}, x \in X$, is (α,β)-semi closed set. Given $A \subseteq X$ and $D(A) = \bigcap\{S : A \subseteq S \text{ and } S \text{ is } (\alpha,\beta) \text{ semi open set}\}$. In general, $A \subset D(A)$. Suppose that $x \notin A$. Then $A \subseteq X - \{x\}$ and $X - \{x\}$ is a (α,β)-semi open because $\{x\}$ is (α,β)-semi closed. Therefore, $x \notin D(A)$ and hence $D(A) \subseteq A$. In consequence $A = D(A)$.

$(c) \Rightarrow (a)$. Let $D(x) = \{S : x \in S \text{ and } S \text{ is } (\alpha,\beta) \text{ semi open}\}$. By hypothesis, $\{x\} = \bigcap_{S \in D(x)} S$, therefore if $y \neq x$ then $y \notin \bigcap_{S \in D(x)} S$ and there is (α,β)-semi open set S such that $x \in S$ and $y \notin S$, in analogue form, if $x \notin \bigcap_{S' \in D(y)} S'$ and there is (α,β)-semi open set S' such that $y \in S'$ and $x \notin S'$. It said that X is a (α,β)-semi T_1 space. □

From the above definitions, we can see easily the following relations

$$(\alpha,\beta) - semiT_2 \quad \Rightarrow \quad (\alpha,\beta) - semiT_1 \quad \Rightarrow \quad (\alpha,\beta) - semiT_0.$$

In the same way, we can introduce the notions of semi regularity and (α,β)-semi T_3 spaces, using (α,β)- semi open sets.

Definition 3.2. *Let (X,τ) be a topological space and $\alpha, \beta : P(X) \to P(X)$ be operators associated to a topology τ on X. X is said to be (α,β)-semi regular space if whenever A is (α,β)-semi closed set in X and $x \notin A$, there are disjoint (α,β)- semi open sets U and V with $x \in U$ and $A \subseteq V$.*

The following proposition characterize the (α,β)-semi regular spaces.

Theorem 3.3. *Let (X,τ) be a topological space and $\alpha, \beta : P(X) \to P(X)$ be operators associated to a topology τ on X. The following are equivalent:*
(a) X is (α,β)-semi regular.
(b) If U is (α,β)-semi open set and $x \in U$, there is (α,β)-semi open set V such that $x \in V$ and $(\alpha,\beta) - sCl(V) \subseteq U$.
(c) Each $x \in X$ has a neighborhood base consisting of (α,β)-semi closed sets.

Proof. The proof follows in the same way to the case of semi regular spaces. □

Definition 3.3. *Let (X,τ) be a topological space and $\alpha,\beta : P(X) \to P(X)$ be operators associated to a topology τ on X. X is said to be a (α,β)-semi T_3 space, if X is a (α,β)-semi regular and (α,β)-semi T_1.*

Clearly every (α,β)-semi T_3 space is $(\alpha,\beta)-$ semi T_2.

4. (α,β)-GENERALIZED SEMI CLOSED SETS AND $(\alpha,\beta)-$ SEMI $T_{1/2}$ SPACES

We recall that if $A \subseteq X$ and α,β:$P(X)\to P(X)$ are associated operators to a topology τ on X, then the $(\alpha,\beta)-sCl(A)$ is (α,β)-semi closed set. In consequence, we can introduce the notions of (α,β)-semi $T_{1/2}$ spaces in a natural way, using the (α,β)-generalized semi closed sets. Also we can study the relations with other spaces that we have studied before.

Definition 4.1. *Let(X,τ) be a topological space and α,β:$P(X)\to P(X)$ be operators associated to a topology τ on X. $A \subseteq X$ is said to be (α,β)-generalized semi closed set if the $(\alpha,\beta)-sCl(A) \subseteq S$ for all $(\alpha,\beta)-$ semi open set S such that $A \subseteq S$.*

Definition 4.2. *Let (X,τ) be a topological space and α,β:$P(X)\to P(X)$ be operators associated to a topology τ on X. X is said to be (α,β)-semi $T_{1/2}$ space if all (α,β)-generalized semi closed set is $(\alpha,\beta)-$ semi closed set.*

Observe that when β is a monotone operator and $\alpha = id$, then the (α,β)-generalized semi closed sets are the β-generalized semi closed sets, therefore the (α,β)-semi $T_{1/2}$ spaces are the β-semi $T_{1/2}$ spaces.

Theorem 4.1. *Let (X,τ) be a topological space and α,β:$P(X)\to P(X)$ be operators associated to a topology τ on X. X is $(\alpha,\beta)-$ semi $T_{1/2}$ space if and only if for each $x \in X$, $\{x\}$ is $(\alpha,\beta)-$ semi open set or $(\alpha,\beta)-$ semi closed set.*

Proof. (Sufficiency) Suppose that X is an $(\alpha,\beta)-$ *semi* $T_{1/2}$ space and there exists $x \in X$ such that $\{x\}$ is neither $(\alpha,\beta)-$ semi open set nor $(\alpha,\beta)-$ semi closed set. Since X is the only (α,β)-semi open set that contain $X-\{x\}$, then $X-\{x\}$ is (α,β)-generalized semi closed set, therefore (α,β)-semi closed, this implies that $\{x\}$ is (α,β)-semi open set. This is a contradiction.
(Neccesity) Let A be a (α,β)-generalized semi closed set and $x \in (\alpha,\beta)-sCl(A)$. If $\{x\}$ is (α,β)-semi open set, then $\{x\} \cap A \neq \emptyset$, therefore $x \in A$. If $\{x\}$is an (α,β)-semi closed set and $x \notin A$, then $X-\{x\}$ is (α,β)-semi open and $A \subseteq X-\{x\}$. Since A is (α,β)-generalized semi closed, the $(\alpha,\beta)-sCl(A) \subseteq X-\{x\}$ and $x \notin (\alpha,\beta)-sCl(A)$. This is contrary to $x \in (\alpha,\beta)-sCl(A)$. Hence $x \in A$ and A is (α,β)-semi closed. □

From the above theorem, we obtain inmediately the following corollary.

Corollary 4.2. *Let (X,τ) be a topological space and $\alpha,\beta : P(X) \to P(X)$ be operators associated to a topology τ on X. X is $(\alpha,\beta)-semiT_{1/2}$ space if and only if each subset of X, is the intersection of all $(\alpha,\beta)-$ semi open sets and (α,β)-semi closed sets containing it.*

Theorem 4.3. *Let (X,τ) be a topological space and $\alpha,\beta : P(X) \to P(X)$ be operators associated to a topology τ on X. Every (α,β)-semi $T_{1/2}$ space is a $(\alpha,\beta)-$ semi T_0 space.*

Proof. Let x, y be any pair of distinct points of X. By theorem 5.1, the singleton $\{x\}$ is a (α,β)-semi open or (α,β)-semi closed. If $\{x\}$ is a (α,β)-semi open, $x \in \{x\}$ and $y \notin \{x\}$. If $\{x\}$ is a (α,β)-semi closed, then $X-\{x\}$ is a (α,β)-semi open, $y \in X-\{x\}$ and $x \notin X-\{x\}$. Therefore, (X,τ) is $(\alpha,\beta)-semiT_0$. □

The following example shows that there exist $(\alpha,\beta)-$*semi* T_0 spaces that are not $(\alpha,\beta)-$*semi* $T_{1/2}$ spaces.

Example 4.1. *Let $X=\{a,b,c,d\}$ and $\tau=\{\emptyset,X,\{a\},\{b\},\{a,b\}\{a,b,c\}\}$. Consider α and β two associated operators defined as follows:*

$$\alpha(A)=A \text{ and } \beta(A)=Cl(A)$$

then, X is a $(\alpha,\beta)-$ semi T_0 space that is not $(\alpha,\beta)-$ semi $T_{1/2}$ space.

Theorem 4.4. *Let (X,τ) be a topological space and $\alpha,\beta:P(X)\to P(X)$ be operators associated to a topology τ on X. Then all $(\alpha,\beta)-$ semi T_1 space is a $(\alpha,\beta)-$ semi $T_{1/2}$ space*

Proof. By Theorem 4.2, for each $x\in X$, the singleton $\{x\}$ is (α,β)-semi closed. Therefore, by theorem 5.1, (X,τ) is (α,β)-semi $T_{1/2}$. □

The following example shows that the existence of a $(\alpha,\beta)-$ *semi* $T_{1/2}$ spaces that is not a $(\alpha,\beta)-$ *semi* T_1 space.

Example 4.2. *Let $X=\{a,b,c\}$ and $\tau=\{\emptyset,X,\{a\},\{b\},\{a,b\}\}$. Consider the following two associated operators α and β defined as follows:*

$$\alpha(A)=A \text{ if } A=\{a\} \text{ or } \{b\},\ \alpha(A)=X \text{ in other case and } \beta(A)=A$$

then, X is an $(\alpha,\beta)-$ semi $T_{1/2}$ space that is not $(\alpha,\beta)-$ semi T_1.

Definition 4.3. *Let $(X,\tau,\alpha,\beta),(Y,\psi,\sigma,\theta)$ be two spaces with associated ope rators. A function $f:(X,\tau)\to(Y,\psi)$ is said to be $((\alpha,\beta),(\sigma,\theta))$ irresolute if the inverse image of each (σ,θ)-semi open set $U\subseteq Y$, is an (α,β)-semi open set in X*

Definition 4.4. *Let $(X,\tau,\alpha,\beta),(Y,\psi,\sigma,\theta)$ be two spaces with associated operators. A function $f:(X,\tau)\to(Y,\psi)$ is said to be $((\alpha,\beta),(\sigma,\theta))$ semi closed if the direct image of each (α,β)-semi closed set in X is a (σ,θ)-semi closed set in Y*

Theorem 4.5. *Let $(X,\tau,\alpha,\beta),(Y,\psi,\sigma,\theta)$ be two spaces with associated ope rators and $f:(X,\tau)\to(Y,\psi)$ be an $((\alpha,\beta),(\sigma,\theta))$ irresolute and $((\alpha,\beta),(\sigma,\theta))$ semi closed bijective function. X is an $(\alpha,\beta)-$ semi $T_{1/2}$ space, if and only if (Y,ψ) is a $(\sigma,\theta)-$ semi $T_{1/2}$ space.*

Proof. (Sufficiency). Suppose that X is an (α,β) semi $T_{1/2}$ space. Let $y\in Y$, then there exists $x\in X$ such that $y=f(x)$, also

$$f(\{x\})=\{y\} \quad \text{and} \quad f(X\setminus\{x\})=Y\setminus\{y\},$$

because f is a bijective function. It follows from Theorem 5.1, that the singleton point $\{x\}$ is (α,β)-semi open or semi-closed. Now using the fact that f is $((\alpha,\beta),(\sigma,\theta))$- semi closed function, we obtain that the singleton point $\{y\}$ is a(σ,θ)-semi open or (σ,θ)-semi-closed. It follows that Y is a (σ,θ) semi $T_{1/2}$ space.
(Neccesity). Suppose that Y is a (σ,θ) semi $T_{1/2}$ space. Let $x\in X$, there exists $y\in Y$ such that $y=f(x)$, then $x=f^{-1}(y)$. Similarly to the above case, we have that

$$f^{-1}(\{y\})=\{x\} \quad \text{and} \quad f^{-1}(Y\setminus\{y\})=X\setminus\{x\}.$$

Using this fact and Theorem 5.1, we obtain that $\{x\}$ is (α,β)-semi open or (α,β)-semi-closed. Thus X is (α,β) semi $T_{1/2}$. □

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