TRIANGULAR NORMED FUZZY SUBALGEBRAS OF \( BCK \)-ALGEBRAS

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Abstract. Using a \( t \)-norm \( T \), the notion of \( T \)-fuzzy topological subalgebras in \( BCK \)-algebras is introduced, and the fact that \( T \)-fuzzy subalgebras of a \( BCK \)-algebra \( X \) form a complete lattice is proved. Using a chain of subalgebras, a \( T \)-fuzzy subalgebra is established. Some of Foster’s results on homomorphic image and inverse image to \( T \)-fuzzy topological subalgebras are considered.

1. Introduction

A \( BCK \)-algebra is an important class of logical algebras introduced by Iséki and was extensively investigated by several researchers. The concept of fuzzy sets, which was introduced in [9], provides a natural framework for generalizing many of the concepts of general topology to what might be called fuzzy topological spaces. Foster [1] combined the structure of a fuzzy topological spaces with that of a fuzzy group, introduced by Rosenfeld [6], to formulate the elements of a theory of fuzzy topological groups. In [2], Jun, one of the present authors, introduced the concept of fuzzy topological \( BCK \)-algebras. Jun and Zhang [4] redefined the fuzzy subalgebra of a \( BCK \)-algebra with respect to a \( t \)-norm, and Jun [3] considered the direct product and \( t \)-normed product of fuzzy subalgebras of a \( BCK \)-algebra with respect to a \( t \)-norm. In the present paper, using triangular norm, we discuss the concept of fuzzy topological subalgebras in \( BCK \)-algebras. We verify \( T \)-fuzzy subalgebras of a \( BCK \)-algebra \( X \) form a complete lattice. Using a chain of subalgebras, we establish a \( T \)-fuzzy subalgebra. We apply some of Foster’s results on the homomorphic images and inverse images to a \( T \)-fuzzy subalgebra.

2. Preliminaries

An algebra \((X; *, 0)\) of type \((2, 0)\) is said to be a \( BCK \)-algebra if it satisfies: for all \( x, y, z \in X \),

(I) \((x * y) * (x * z) = (z * y) * (z * x)\),

(II) \((x * (x * y)) = y = 0\),

(III) \(x * x = 0\),

(IV) \(x * x = 0\) and \(y * x = 0\) imply \(x = y\).

Define a binary relation \(\leq\) on \(X\) by letting \(x \leq y\) if and only if \(x * y = 0\). Then \((X; \leq)\) is a partially ordered set with the least element \(0\). A subset \(S\) of a \( BCK \)-algebra \(X\) is called a subalgebra of \( X \) if \(x * y \in S\) whenever \(x, y \in S\). A mapping \(f : X \rightarrow X'\) of \( BCK \)-algebras is called a homomorphism if \(f(x * y) = f(x) * f(y)\) for all \(x, y \in X\). In any \( BCK \)-algebra \(X\), the following hold:

(P1) \((x * y) * z = (x * z) * y\).

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(P2) $x \ast y \leq x,$
(P3) $x \ast 0 = x,$
(P4) $(x \ast z) \ast (y \ast z) \leq x \ast y,$
(P5) $x \ast (x \ast y) = x \ast y,$
(P6) $x \leq y$ implies $x \ast z \leq y \ast z$ and $z \ast y \leq z \ast x,$

for all $x, y, z \in X.$ Generally, an aggregation operator is a mapping $F : I \rightarrow I^n$ $(n \geq 2),$ where $I = [0, 1].$ Essentially it takes a collection of arguments and provides an aggregated value. An important class of aggregation operators are the triangular norm operators, $t$-norm and $t$-conorm. These operators play a significant role in the theory of fuzzy subsets by generalizing the intersection (and) and union (or) operators, respectively. By a $t$-norm $T$ (see [7]) we mean a function $T : I \times I \rightarrow I$ satisfying the following conditions:

(T1) $T(x, 1) = x,$
(T2) $T(x, y) \leq T(x, z)$ whenever $y \leq z,$
(T3) $T(x, y) = T(y, x),$ 
(T4) $T(x, T(y, z)) = T(T(x, y), z),$

for all $x, y, z \in I.$ For a $t$-norm $T,$ let $\Delta_T$ denote the set of elements $\alpha \in I$ such that $T(\alpha, \alpha) = \alpha,$ that is, $\Delta_T := \{\alpha \in I \mid T(\alpha, \alpha) = \alpha\}.$

Note that every $t$-norm $T$ has a useful property:

(p7) $T(\alpha, \beta) \leq \min(\alpha, \beta)$ for all $\alpha, \beta \in I.$

A $t$-norm $T$ on $I$ is said to be continuous if $T$ is a continuous function from $I \times I$ to $I$ with respect to the usual topology. A fuzzy set $\mu$ in a set $L$ is said to satisfy imaginable property if $\text{Im}(\mu) \subseteq \Delta_T.$

3. Triangular normed fuzzy subalgebras

**Definition 3.1** [4] A fuzzy set $\mu$ in a BCK-algebra $X$ is called a fuzzy subalgebra of $X$ with respect to a $t$-norm $T$ (briefly, a $T$-fuzzy subalgebra of $X$) if it satisfies the inequality $\mu(x \ast y) \geq T(\mu(x), \mu(y))$ for all $x, y \in X.$

**Example 3.2** Let $X = \{0, a, b, c\}$ be a BCK-algebra with the following Cayley table:

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>a</th>
<th>b</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
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<td>0</td>
</tr>
<tr>
<td>a</td>
<td>a</td>
<td>0</td>
<td>0</td>
<td>a</td>
</tr>
<tr>
<td>b</td>
<td>b</td>
<td>a</td>
<td>0</td>
<td>b</td>
</tr>
<tr>
<td>c</td>
<td>c</td>
<td>c</td>
<td>c</td>
<td>0</td>
</tr>
</tbody>
</table>

Let $T_m$ be a $t$-norm defined by $T_m(\alpha, \beta) = \max(\alpha + \beta - 1, 0)$ for all $\alpha, \beta \in [0, 1].$

(1) Define a fuzzy set $\mu : X \rightarrow [0, 1]$ by

$\mu(x) := \begin{cases} 
0.7 & \text{if } x \in \{0, b\}, \\
0.07 & \text{otherwise}.
\end{cases}$

Then $\mu$ is a $T_m$-fuzzy subalgebra of $X$ which is not imaginable.

(2) Let $\nu$ be a fuzzy set in $X$ defined by

$\nu(x) := \begin{cases} 
1 & \text{if } x \in \{0, b\}, \\
0 & \text{otherwise}.
\end{cases}$

Then $\nu$ is an imaginable $T_m$-fuzzy subalgebra of $X.$
Proposition 3.3 Let $T$ be a $t$-norm on $I$. If a fuzzy set $\mu$ in a BCK-algebra $X$ is an imaginable $T$-fuzzy subalgebra of $X$, then $\mu(0) \geq \mu(x)$ for all $x \in X$.

**Proof.** For every $x \in X$, we have $\mu(0) = \mu(x * x) \geq T(\mu(x), \mu(x)) = \mu(x)$.

Proposition 3.4 Let $S$ be a subalgebra of a BCK-algebra $X$ and let $\mu$ be a fuzzy set in $X$ defined by

$$
\mu(x) := \begin{cases} 
\alpha & \text{if } x \in S, \\
\beta & \text{otherwise,}
\end{cases}
$$

for all $x \in X$, where $\alpha, \beta \in [0,1]$ with $\alpha \geq \beta$. Then $\mu$ is a $T_m$-fuzzy subalgebra of $X$. If $\alpha = 1$ and $\beta = 0$, then $\mu$ is imaginable, where $T_m$ is the $t$-norm in Example 3.2.

**Proof.** Let $x, y \in X$. If $x, y \in S$, then

$$
T_m(\mu(x), \mu(y)) = T_m(\alpha, \alpha) = \max\{2\alpha - 1, 0\} = \begin{cases} 
2\alpha - 1 & \text{if } \alpha \geq \frac{1}{2} \\
0 & \text{if } \alpha < \frac{1}{2}
\end{cases} \leq \alpha = \mu(x * y).
$$

If $x \in S$ and $y \notin S$ (or, $x \notin S$ and $y \in S$), then

$$
T_m(\mu(x), \mu(y)) = T_m(\alpha, \beta) = \max\{\alpha + \beta - 1, 0\} = \begin{cases} 
\alpha + \beta - 1 & \text{if } \alpha + \beta \geq 1 \\
0 & \text{otherwise}
\end{cases} \leq \beta = \mu(x * y).
$$

If $x \notin S$ and $y \notin S$, then

$$
T_m(\mu(x), \mu(y)) = T_m(\beta, \beta) = \max\{2\beta - 1, 0\} = \begin{cases} 
2\beta - 1 & \text{if } \beta \geq \frac{1}{2} \\
0 & \text{otherwise}
\end{cases} \leq \beta = \mu(x * y).
$$

Hence $\mu$ is a $T_m$-fuzzy subalgebra. Assume that $\alpha = 1$ and $\beta = 0$. Then

$$
T_m(\alpha, \alpha) = \max\{2\alpha - 1, 0\} = 1 = \alpha,
$$

$$
T_m(\beta, \beta) = \max\{2\beta - 1, 0\} = 0 = \beta.
$$

Thus $\alpha, \beta \in \Delta_{T_m}$, that is, $\text{Im}(\mu) \subseteq \Delta_{T_m}$ and so $\mu$ is imaginable. This completes the proof.

Proposition 3.5 If $\mu_i$, $i \in I$, is a $T$-fuzzy subalgebra of a BCK-algebra $X$, then $\bigcap_{i \in I} \mu_i$ is also a $T$-fuzzy subalgebra of $X$ where $\bigcap_{i \in I} \mu_i$ is defined by $\left(\bigcap_{i \in I} \mu_i\right)(x) = \inf_{i \in I} \mu_i(x)$ for all $x \in X$.

**Proof.** For any $x, y \in X$, we have $\mu_i(x) \geq \inf_{i \in I} \mu_i(x)$ and $\mu_i(y) \geq \inf_{i \in I} \mu_i(y)$. Hence for every $i \in I$,

$$
T(\mu_i(x), \mu_i(y)) \geq T(\inf_{i \in I} \mu_i(x), \inf_{i \in I} \mu_i(y)),
$$

where $T$ is a triangular norm on $I$.
and so \( \inf_{i \in I} T(\mu_i(x), \mu_i(y)) \geq T(\inf_{i \in I} \mu_i(x), \inf_{i \in I} \mu_i(y)) \). It follows that

\[
(\bigcap_{i \in I} \mu_i)(x \ast y) = \inf_{i \in I} \mu_i(x \ast y) \\
\geq \inf_{i \in I} T(\mu_i(x), \mu_i(y)) \\
\geq T(\inf_{i \in I} \mu_i(x), \inf_{i \in I} \mu_i(y)) \\
= T((\bigcap_{i \in I} \mu_i)(x), (\bigcap_{i \in I} \mu_i)(y)).
\]

Obviously, \((\bigcap_{i \in I} \mu_i)(0) = (\bigcap_{i \in I} \mu_i)(1)\). This completes the proof.

It follows that the \( T \)-fuzzy subalgebras of a \( BCK \)-algebra \( X \) form a complete lattice. In this lattice, the inf of a set of \( T \)-fuzzy subalgebras \( \mu_i \) is just \( \bigcap \mu_i \), while their sup is the least \( \mu \), i.e., the intersection of \( \mu \)'s, which contains \( \bigcup \mu_i \), where \((\bigcup_{i \in I} \mu_i)(x) = \sup \mu_i(x)\) for all \( x \in L \).

**Theorem 3.6** Let \( T \) be a \( t \)-norm on \( I \) and let \( \mu \) be a fuzzy set in a \( BCK \)-algebra \( X \) with \( \text{Im} \{ \mu \} = \{ \alpha_1, \alpha_2, \cdots, \alpha_n \} \), where \( \alpha_i < \alpha_j \) whenever \( i > j \). Suppose that there exists a chain of subalgebras of \( X \):

\[
G_0 \subset G_1 \subset \cdots \subset G_n = X
\]

such that \( \mu(G_k) = \alpha_k \), where \( G_k = G_k \setminus G_{k-1} \) and \( G_{-1} = \emptyset \) for \( k = 0, 1, \ldots, n \). Then \( \mu \) is a \( T \)-fuzzy subalgebra of \( X \).

**Proof.** Let \( x, y \in X \). If \( x \) and \( y \) belong to the same \( \tilde{G}_k \), then \( \mu(x) = \mu(y) = \alpha_k \) and \( x \ast y \in G_k \). Hence

\[
\mu(x \ast y) \geq \alpha_k = \min \{ \mu(x), \mu(y) \} \geq T(\mu(x), \mu(y)).
\]

Assume that \( x \in \tilde{G}_i \) and \( y \in \tilde{G}_j \) for every \( i \neq j \). Without loss of generality we may assume that \( i > j \). Then \( \mu(x) = \alpha_i < \alpha_j = \mu(y) \) and \( x \ast y \in G_i \). It follows that

\[
\mu(x \ast y) \geq \alpha_i = \min \{ \mu(x), \mu(y) \} \geq T(\mu(x), \mu(y)).
\]

Consequently, \( \mu \) is a \( T \)-fuzzy subalgebra of \( X \).

4. Fuzzy Topological Subalgebras

In this section, for the convenience of notation, we use symbols \( A, B, \cdots, \) etc. instead of fuzzy sets \( \mu, \nu, \cdots, \) etc., that is, \( A, B, \cdots \) are fuzzy sets with membership functions \( \mu_A, \mu_B, \cdots, \) respectively. Let \( B \) be a fuzzy set in \( Y \) with membership function \( \mu_B \). The **inverse image** of \( B \), denoted \( f^{-1}(B) \), is the fuzzy set in \( X \) with membership function \( \mu_{f^{-1}(B)}(x) = \mu_B(f(x)) \) for all \( x \in X \). Conversely, let \( A \) be a fuzzy set in \( X \) with membership function \( \mu_A \). Then the image of \( A \), denoted \( f(A) \), is the fuzzy set in \( Y \) such that

\[
\mu_{f(A)}(y) = \sup_{z \in f^{-1}(y)} \mu_A(z) \quad \text{if} \quad f^{-1}(y) \neq \emptyset,
\]

and 0 otherwise.

A **fuzzy topology** on a set \( X \) is a family \( T \) of fuzzy sets in \( X \) which satisfies the following conditions:
Proposition 4.4

Let \( \mu \) be a function if for each open fuzzy set \( X \), \( \mu \) is continuous on \( X \). The induced fuzzy topology is denoted by \( T_{\mu} \), and the pair \( (X, T_{\mu}) \) is called a fuzzy subspace of \( (X, T) \). Let \( (X, T) \) and \( (Y, U) \) be two fuzzy topological spaces. A mapping \( f \) of \( (X, T) \) into \( (Y, U) \) is fuzzy continuous if for each open fuzzy set \( U \) in \( U \) the inverse image \( f^{-1}(U) \) is in \( T \). Conversely, \( f \) is fuzzy open if for each open fuzzy set \( V \) in \( T \), the image \( f(V) \) is in \( U \). Let \( (X, T_{\mu}) \) and \( (Y, U_{\mu}) \) be fuzzy subspaces of fuzzy topological spaces \( (X, T) \) and \( (Y, U) \) respectively, and let \( f \) be a mapping \( (X, T) \to (Y, U) \). Then \( f \) is a mapping of \( (X, T_{\mu}) \) into \( (Y, U_{\mu}) \) if \( f(A) \subset B \). Furthermore \( f \) is relatively fuzzy continuous if for each open fuzzy set \( V' \) in \( U_{\mu} \), the intersection \( f^{-1}(V') \cap A \) is in \( T_{\mu} \). Conversely, \( f \) is relatively fuzzy open if for each open fuzzy set \( U' \) in \( T_{\mu} \), the image \( f(U') \) is in \( U_{\mu} \).

Lemma 4.1

[1] Let \( (A, T_A) \), \( (B, U_B) \) be fuzzy subspaces of fuzzy topological spaces \( (X, T) \), \( (Y, U) \) respectively, and let \( f \) be a fuzzy continuous mapping of \( (X, T) \) into \( (Y, U) \) such that \( f(A) \subset B \). Then \( f \) is a relatively fuzzy continuous mapping of \( (A, T_A) \) into \( (B, U_B) \).

Proposition 4.2

Let \( T \) be a \( t \)-norm and let \( f : X \to Y \) be a homomorphism of \( BCK \)-algebras. If \( G \) is a \( T \)-fuzzy subalgebra of \( Y \) with the membership function \( \mu_G \), then the inverse image \( f^{-1}(G) \) of \( G \) under \( f \) with the membership function \( \mu_{f^{-1}(G)} \) is a \( T \)-fuzzy subalgebra of \( X \).

Proof.

For any \( x, y \in X \), we have

\[
\mu_{f^{-1}(G)}(x * y) = \mu_G(f(x * y)) = \mu_G(f(x) * f(y)) \\
\geq T(\mu_G(f(x)), \mu_G(f(y))) \\
= T(\mu_{f^{-1}(G)}(x), \mu_{f^{-1}(G)}(y)).
\]

Hence \( f^{-1}(G) \) is a \( T \)-fuzzy subalgebra of \( X \).

Definition 4.3

[8] A \( t \)-norm \( T \) on \( I \) is called a continuous \( t \)-norm if \( T \) is a continuous function from \( I \times I \) to \( I \) with respect to the usual topology.

Proposition 4.4

[4] Let \( T \) be a continuous \( t \)-norm and let \( f \) be a homomorphism of a \( BCK \)-algebra \( X \) onto a \( BCK \)-algebra \( Y \). If a fuzzy set \( F \) with the membership function \( \mu_F \) is a \( T \)-fuzzy subalgebra of \( X \), then the image \( f(F) \) of \( F \) under \( f \) with the membership function \( \mu_{f(F)} \) is a \( T \)-fuzzy subalgebra of \( Y \).

For any \( BCK \)-algebra \( X \) and any element \( a \in X \) we use \( a_r \) to denote the self-map of \( X \) defined by \( a_r(x) = x * a \) for all \( x \in X \).

Definition 4.5

Let \( T \) be a fuzzy topology on a \( BCK \)-algebra \( X \). For a \( t \)-norm \( T \), let \( F \) be a \( T \)-fuzzy subalgebra of \( X \) with the induced topology \( T_F \). Then \( F \) is called a \( T \)-fuzzy topological subalgebra of \( X \) if for each \( a \in X \) the mapping \( a_r : x \mapsto x * a \) of \( (F, T_F) \to (F, T_F) \)
is relatively fuzzy continuous.

**Theorem 4.6** Let $T$ be a $t$-norm. Given $BCK$-algebras $X$, $Y$ and a homomorphism $f : X \to Y$, let $\mathcal{U}$ and $\mathcal{T}$ be fuzzy topologies on $Y$ and $X$ respectively such that $\mathcal{T} = f^{-1}(\mathcal{U})$. Let $G$ be a $T$-fuzzy topological subalgebra of $Y$ with the membership function $\mu_G$. Then $f^{-1}(G)$ is a $T$-fuzzy topological subalgebra of $X$ with the membership function $\mu_{f^{-1}(G)}$.

**Proof.** Note from Proposition 4.2 that $f^{-1}(G)$ is a $T$-fuzzy subalgebra of $X$. It is sufficient to show that for each $a \in X$ the mapping

$$a_r : x \mapsto x \ast a \text{ of } (f^{-1}(G), \mathcal{T}_{f^{-1}(G)}) \to (f^{-1}(G), \mathcal{T}_{f^{-1}(G)})$$

is relatively fuzzy continuous. Let $U$ be an open fuzzy set in $\mathcal{T}_{f^{-1}(G)}$ on $f^{-1}(G)$. Since $f$ is a fuzzy continuous mapping of $(X, \mathcal{T})$ into $(Y, \mathcal{U})$, it follows from Lemma 4.1 that $f$ is a relatively fuzzy continuous mapping of $(f^{-1}(G), \mathcal{T}_{f^{-1}(G)})$ into $(G, \mathcal{U}_G)$. Note that there exists an open fuzzy set $V \in \mathcal{U}_G$ such that $f^{-1}(V) = U$. The membership function of $a_r^{-1}(U)$ is given by

$$\mu_{a_r^{-1}(U)}(x) = \mu_V(a_r(x)) = \mu_V(x \ast a) = \mu_{f^{-1}(V)}(x \ast a) = \mu_V(f(x \ast a)) = \mu_V(f(x) \ast f(a)).$$

As $G$ is a $T$-fuzzy topological subalgebra of $Y$, the mapping

$$b_r : y \mapsto y \ast b \text{ of } (G, \mathcal{U}_G) \to (G, \mathcal{U}_G)$$

is relatively fuzzy continuous for each $b \in Y$. Hence

$$\mu_{b_r^{-1}(U)}(x) = \mu_V(f(x) \ast f(a)) = \mu_V(f(a) \ast f(x)) = \mu_{f(a)_r^{-1}(V)}(f(x)) = \mu_{f^{-1}(f(a)_r^{-1}(V))}(x),$$

which implies that $a_r^{-1}(U) = f^{-1}(f(a)_r^{-1}(V))$ so that

$$a_r^{-1}(U) \cap f^{-1}(G) = f^{-1}(f(a)_r^{-1}(V)) \cap f^{-1}(G)$$

is open in the induced fuzzy topology on $f^{-1}(G)$. This completes the proof.

We say that the membership function $\mu_G$ of a $T$-fuzzy subalgebra $G$ of a $BCK$-algebra $X$ is $f$-invariant if, for all $x, y \in X$, $f(x) = f(y)$ implies $\mu_G(x) = \mu_G(y)$.

**Theorem 4.7** Let $T$ be a continuous $t$-norm. Given $BCK$-algebras $X$, $Y$ and a homomorphism $f$ of $X$ onto $Y$, let $\mathcal{T}$ and $\mathcal{U}$ be fuzzy topologies on $X$ and $Y$, respectively, such that $f(\mathcal{T}) = \mathcal{U}$. Let $F$ be a $T$-fuzzy topological subalgebra of $X$. If the membership function $\mu_F$ of $F$ is $f$-invariant, then $f(F)$ is a $T$-fuzzy topological subalgebra of $Y$.

**Proof.** By Proposition 4.4, $f(F)$ is a $T$-fuzzy subalgebra of $Y$. Hence it is sufficient to show that the mapping

$$b_r : y \mapsto y \ast b \text{ of } (f(F), \mathcal{U}_{f(F)}) \to (f(F), \mathcal{U}_{f(F)})$$

is relatively fuzzy continuous for each $b \in Y$. Note that $f$ is relatively fuzzy open; for if $U' \in \mathcal{T}_F$, there exists $U \in \mathcal{T}$ such that $U' = U \cap F$ and by the $f$-invariance of $\mu_F$, $f(U') = f(U) \cap f(F) \in \mathcal{U}_{f(F)}$. 

$$f(U') = f(U) \cap f(F) \in \mathcal{U}_{f(F)}.$$
Let $V'$ be an open fuzzy set in $\mathcal{U}(F)$. Since $f$ is onto, for each $b \in Y$ there exists $a \in X$ such that $b = f(a)$. Hence
\[
\mu_{f^{-1}(b^{-1}_r(V'))}(x) = \mu_{f^{-1}(f(a)^{-1}_r(V'))}(x) = \mu_{f^{-1}(f(a)^{-1}_r(V'))}(f(x)) \\
= \mu_{V'}(f(a)_r(f(x))) = \mu_{V'}(f(x) \ast f(a)) \\
= \mu_{V'}(f(x \ast a)) = \mu_{f^{-1}(V')}(x \ast a) \\
= \mu_{f^{-1}(V')}(a_r(x)) = \mu_{a^{-1}_r(f^{-1}(V'))}(x),
\]
which implies that $f^{-1}(b^{-1}_r(V')) = a^{-1}_r(f^{-1}(f(V')))$. By hypothesis, $a_r : x \mapsto x \ast a$ is a relatively fuzzy continuous mapping: $(F, T_F) \rightarrow (F, T_F)$ and $f$ is a relatively fuzzy continuous mapping: $(F, T_F) \rightarrow (f(F), \mathcal{U}(F))$. Hence
\[
f^{-1}(b^{-1}_r(V')) \cap F = a^{-1}_r(f^{-1}(V')) \cap F
\]
is open in $T_F$. Since $f$ is relatively fuzzy open,
\[
f(f^{-1}(b^{-1}_r(V'))) \cap F = b^{-1}_r(V') \cap f(F)
\]
is open in $\mathcal{U}(F)$. Consequently, $f(F)$ is a $T$-fuzzy topological subalgebra of $Y$.

References


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