ON THE NUMBER OF THE NON-EQUIVALENT KM-SPANNING SUBGRAPHS OF THE COMPLETE GRAPH WITH ORDER MK

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Received August 27, 2002

Abstract. Let \( m \) be greater than or equal to 2 and \( n \) be a multiple of \( m \). We will call a spanning subgraph whose components are \( K_m \) of the complete graph \( K_n \) a \( K_m \)-spanning subgraph of \( K_n \). The Dihedral group \( D_n \) acts on the complete graph \( K_n \) naturally. This action of \( D_n \) induces the action on the set of the \( K_m \)-spanning subgraphs of the complete graph \( K_n \). In [3], we calculated the number of the equivalence classes of the 1-regular spanning subgraphs of the complete graph \( K_n \) of even order \( n \) by this action by using Burnside’s Lemma. This is in the case \( m = 2 \). In this paper, we generalize this results and calculate the number of the non-equivalent \( K_m \)-spanning subgraphs of \( K_n \) for all \( m \) and \( n \).

Let \( m \) be greater than or equal to 2 and let \( n \) be a multiple of \( m \). Let \( \{v_0; v_1; \ldots; v_{n-1}\} \) be the vertices of the complete graph \( K_n \). The action to \( K_n \) of the Dihedral group \( D_n = \{1; 2; \ldots; n; n+1\} \) is defined by

\[
\frac{1}{2} i (v_k) = v_{(k+i) \mod n} \text{ for } 0 \leq i \leq n-1; 0 \leq k \leq n-1.
\]

\[
\frac{3}{4} s (v_k) = v_{(n+i+k) \mod n} \text{ for } 0 \leq i \leq n-1; 0 \leq k \leq n-1.
\]

We call a spanning subgraph whose components are \( K_m \) of the complete graph \( K_n \) a \( K_m \)-spanning subgraph of \( K_n \). Let \( X_{n,m}^m \) be the set of the \( K_m \)-spanning subgraphs of \( K_n \). Then the above action induces the action on \( X_{n,m}^m \) of the Dihedral group \( D_n \).

For example, the equivalence classes of \( X_{3,6}^3 \) are given with the next figure.

The equivalence classes of \( X_{3,9}^3 \) are given with the next figure.

We calculate the number of the equivalence classes by this group action. These computations can be done by using Burnside’s lemma.
Theorem 1. (Burnside's lemma) Let $G$ be a group of permutations acting on a set $S$. Then the number of orbits induced on $S$ is given by

$$\frac{1}{|G|} \sum_{g \in G} \text{fix}(g)$$

where $\text{fix}(g) = \text{fix}(S \cdot x) = xg$.

Notation 1. An integer function $\varphi(p, q)$ is defined by

$$\varphi(p, q) = \begin{cases} 1 & \text{if } p \equiv 0 \pmod{q} \\ 0 & \text{otherwise} \end{cases}$$

Notation 2. For each integer $i$ such that $0 \leq i \leq n - 1$, let $d = (n; i)$ and $R_{d,i}^m$ be

$$R_{d,i}^m = \sum_{d = p \prod_{j=1}^s p_j}^{d} \prod_{j=1}^{s_l} (p_j)^{s_j} \prod_{j=1}^{s_r} (p_j)^{s_j}$$

Notation 3. $S_{n;0}^m; 0 \cdot i \cdot n \cdot 1$ is given by the following recursive formula:

- If $n$ is odd then $S_{n,k}^m = S_{n;0}^m$ for $1 \cdot k \cdot n \cdot 1$.
- If $n$ is even then $S_{n;2k}^m = S_{n;0}^m$ for $1 \cdot k \cdot n \cdot 1$ and $S_{n;2k+1}^m = S_{n;1}^m$ for $1 \cdot k \cdot n \cdot 1$.

If $m$ is odd then

$$\begin{align*}
S_{m,0}^m & = 1 \\
S_{2m,1}^m & = 2^m \\
S_{n,0}^m & = \frac{m!}{2^m} S_{n,m;0} \\
S_{n,1}^m & = \frac{m!}{2^m} S_{n,m;1} \\
S_{m;1}^m & = \frac{m!}{2^m} S_{n,m;1} + \frac{(m+1)!}{k!(m+1-k)!} S_{n^2;0}^m \\
S_{n^2;0}^m & = \frac{m!}{2^m} S_{n,m;0} + \frac{(m+1)!}{k!(m+1-k)!} S_{n^2;1}^m \\
S_{n^2;1}^m & = \frac{m!}{2^m} S_{n,m;1} + \frac{(m+1)!}{k!(m+1-k)!} S_{n^2;2}^m \\
S_{n^2;2}^m & = \frac{m!}{2^m} S_{n,m;2} + \frac{(m+1)!}{k!(m+1-k)!} S_{n^2;2}^m
\end{align*}$$

If $m$ is even then

$$\begin{align*}
S_{m,0}^m & = S_{m;1}^m = 1 \\
S_{2m,1}^m & = \frac{m!}{2^{2m}} \\
S_{n,0}^m & = \frac{m!}{2^{2m}} S_{n,m;0} \\
S_{n,1}^m & = \frac{m!}{2^{2m}} S_{n,m;1} \\
S_{m;1}^m & = \frac{m!}{2^{2m}} S_{n,m;1} + \frac{(m+1)!}{k!(m+1-k)!} S_{n^2;2}^m \\
S_{n^2;1}^m & = \frac{m!}{2^{2m}} S_{n,m;2} + \frac{(m+1)!}{k!(m+1-k)!} S_{n^2;2}^m \\
S_{n^2;2}^m & = \frac{m!}{2^{2m}} S_{n,m;2} + \frac{(m+1)!}{k!(m+1-k)!} S_{n^2;2}^m
\end{align*}$$
Our main Theorem is the following:

Theorem 2. The number of the non-equivalent $K_m$-spanning subgraphs of the complete graph $K_n$ is given by the following formula:

If $n$ is odd then

$$\frac{1}{2n} \sum_{i=0}^{\lfloor n/2 \rfloor} R_{n,i}^m + n \sum_{i=0}^{\lfloor n/2 \rfloor} S_{n,0}^m$$

If $n$ is even then

$$\frac{1}{2n} \sum_{i=0}^{\lfloor n/2 \rfloor} R_{n,i}^m + \frac{n}{2} \sum_{i=0}^{\lfloor n/2 \rfloor} (S_{n,0}^m + S_{n,1}^m)$$

We must determine the numbers of the fixed points of each permutation $1/2$ and $3/4$ to prove the Theorem by using Burnside's Lemma.

Lemma 1. The number of the $K_m$-spanning subgraphs of $K_n$ is

$$\sum_{k=1}^{n} \frac{\mu_m}{m^m}$$

This is the number of the fixed points of $1/2$.

Proof. Since the number of ways to select $m$ items from a collection of $n$ items is $\binom{n}{m}$, the number of ways to partition $n$ items into subsets of size $m$ is $\sum_{i=1}^{n} \frac{\mu_m}{m^m}$. Then the number of ways to select $n$ groups of size $m$ from a collection of $n$ items is $\sum_{i=1}^{n} \left(\frac{\mu_m}{m^m}\right)^i$. Then we have the results.

Remark 1. It is easily checked that $R_{n,0}^m$ is equal to $\sum_{k=1}^{n} \frac{\mu_m}{m^m}$. If $(n,i)=1$ then the number of the fixed points of $3/4$ is one.

Notation 4. Let $M_n^m$ be the union of $G_0; G_1; \cdots; G_{n=m}, 1$, where $G_i$ be the complete graph whose vertices are $v_0; v_1; \cdots; v_{n=m}; v_{j+2n=m}; \cdots; v_{j+(m-1)n=m}$ for $0 < j < n=m$. Since $M_n^m$ is a $K_m$-spanning subgraph of $K_n$ which is fixed by $1/2$ and contain a component $C$ whose vertices are $v_0; v_1; v_2; \cdots; v_{n=m}$, $0 < k_1 < k_2 < \cdots < k_m < n$. Since $(n,i)=1$, there is an integer $s$ such that $s \equiv 1 \pmod{n}$. Then $\frac{s}{2} = k_1 + k_2 \cdots + k_m \equiv 2 \pmod{2}$. If $s$ is not equal to $k_1$ then $k_2$ must be greater than $k_1$ by the assumption of $0 < k_1 < k_2 < \cdots < k_m < n$. Since $\frac{s}{2} = k_1 + k_2 \cdots + k_m \equiv 2 \pmod{2}$ and $0 < k_1 < k_2 < \cdots < k_m < n$, this is impossible. Then we have $k_1 = k_2$. We assume that $k_i = j_k$ for $j \cdot s$ and prove that $k_{s+1} = (s+1)k_i.$ Since $\frac{s}{2} = k_1 + k_2 \cdots + k_m \equiv 2 \pmod{2}$ and $(s+1)k_1$ is greater than or equal to $k_{s+1}$, if $(s+1)k_1 > k_{s+1}$ then $\frac{s}{2} = k_1 + k_2 \cdots + k_m \equiv 2 \pmod{2}$ and $k_{s+1} < 2k_1$. This is impossible. Then we have $k_{s+1} = (s+1)k_1$. We finally prove that $m k_1 = n$. Since
Remark 2. By the following lemmas we will see that \( S_{n,1}^m \) agrees with the one which is given in Notation 3.

Lemma 4. If \( n \) is odd then the number of the fixed points of \( \frac{1}{2} \) is equal to the number of the fixed points of \( \frac{1}{2} \) for all \( 1 \cdot k \cdot n \).
Proof. We assume that \( k \) is even. Let \( H \) be a \( K_m \)-spanning subgraph of \( K_n \) \( \ast \)xed by \( \frac{n}{2} \). Then it is easily verified that \( \frac{n}{2} (H) \) is a \( K_m \)-spanning subgraph of \( K_n \) \( \ast \)xed by \( \frac{n}{2} \). Conversely, if \( H \) is a \( K_m \)-spanning subgraph of \( K_n \) \( \ast \)xed by \( \frac{n}{2} \) then \( \frac{n}{2} (H) \) is a \( K_m \)-spanning subgraph of \( K_n \) \( \ast \)xed by \( \frac{n}{2} \). Next we assume that \( k \) is odd. Let \( H \) be a \( K_m \)-spanning subgraph of \( K_n \) \( \ast \)xed by \( \frac{n}{2} \). Then it is easily verified that \( \frac{n+1}{2} (H) \) is a \( K_m \)-spanning subgraph of \( K_n \) \( \ast \)xed by \( \frac{n}{2} \). Conversely, if \( H \) is a \( K_m \)-spanning subgraph of \( K_n \) \( \ast \)xed by \( \frac{n}{2} \) then \( \frac{n+1}{2} (H) \) is a \( K_m \)-spanning subgraph of \( K_n \) \( \ast \)xed by \( \frac{n}{2} \). Then we have the results.

Similarly, we have the next Lemma.

**Lemma 5.** If \( n \) is even then the number of the \( \ast \)xed points of \( \frac{n}{2} \) is equal to the number of the \( \ast \)xed points of \( \frac{n}{2} 2d \) for all \( 1 \cdot d \cdot n=2, \ldots, 1 \) and the number of the \( \ast \)xed points of \( \frac{n}{2} \) is equal to the number of the \( \ast \)xed points of \( \frac{n}{2} 2d+1 \) for all \( 1 \cdot d \cdot n=2, \ldots, 1 \).

**Lemma 6.** If \( n \) is odd and \( m \) is odd then

\[
S_{m,0}^n = \frac{1}{\mu \frac{1}{2}} \quad \text{and} \quad S_{n,0}^m = \frac{1}{m^{2} \frac{1}{2}} \quad \text{if } n \cdot 2m
\]

Proof. The \( K_m \)-spanning subgraph of \( K_m \) is \( K_m \) and \( K_m \) is \( \ast \)xed by \( \frac{n}{2} \). Then we have \( S_{m,0}^n = 1 \). We assume that \( n \cdot 2m \). Let \( H \) be a \( K_m \)-spanning subgraph of \( K_n \) \( \ast \)xed by \( \frac{n}{2} \). Let \( C \) be the component of \( H \) which contains vertex \( v_0 \). If \( C \) naturally becomes \( K_m \)-spanning subgraph of \( K_{n_1} \) \( \ast \)xed by \( \frac{n}{2} \) when we change the name of the vertices. Conversely, let \( H \) be a \( K_m \)-spanning subgraph of \( K_{n_1} \) \( \ast \)xed by \( \frac{n}{2} \). Since \( n_1 \cdot m \) is even, the axis of the line symmetry is not passing any vertices. If we take one vertex of \( K_m \) in the position of \( v_0 \) of the graph which we will construct and divide the remaining vertices of \( K_m \) into halves and distribute them between the vertices of \( H \) permitting redundancy and symmetrically regarding the axis then the resulting graph becomes a \( K_m \)-spanning subgraph of \( K_n \) \( \ast \)xed by \( \frac{n}{2} \). The number of ways to distribute the vertices of \( K_m \) is \( \frac{1}{m^{2} \frac{1}{2}} \). Then we have the results.

**Lemma 7.** If \( n \) is even and \( m \) is odd then

\[
S_{n,0}^m = \frac{1}{m^{2} \frac{1}{2}} \quad \text{if } n \cdot 2m
\]

Proof. Let \( H \) be a \( K_m \)-spanning subgraph of \( K_n \) \( \ast \)xed by \( \frac{n}{2} \). Since \( n \) is even, the axis of \( \frac{n}{2} \) passes \( v_0 \) and \( v_{2d} \). Let \( C \) be the component of \( H \) which contains vertex \( v_{2d} \). Since \( m \) is odd, \( C \) does not contain the vertex \( v_0 \). If \( C \) naturally becomes \( K_m \)-spanning subgraph of \( K_{n_1} \) \( \ast \)xed by \( \frac{n}{2} \) when we change the name of the vertices. Conversely, let \( H \) be a \( K_m \)-spanning subgraph of \( K_{n_1} \) \( \ast \)xed by \( \frac{n}{2} \). Since \( n_1 \cdot m \) is odd, the axis of \( \frac{n}{2} \) passes the vertex \( v_0 \). If we take one vertex of \( K_m \) in the position of \( v_{2d} \) of the graph which we will construct and divide the remaining vertices of \( K_m \) into halves and distribute them between the vertices of \( H \) permitting redundancy and symmetrically regarding the axis then the resulting graph becomes a \( K_m \)-spanning subgraph of \( K_n \) \( \ast \)xed by \( \frac{n}{2} \). The number of ways to distribute the vertices of \( K_m \) is \( \frac{1}{m^{2} \frac{1}{2}} \). Then we have the results.
Lemma 8. If \( n \) is even and \( m \) is odd then
\[
S_{2m;1}^m = 2^{m_1} \\
S_{n;1}^m = \sum_{k=0}^{\lfloor n/2 \rfloor} \frac{\binom{n-k}{m-k} \cdot \binom{m}{k} \cdot \binom{m-k}{n-2k} \cdot \binom{n-2k}{m-2k}}{k! (m-k)! (n-2k)! (m-2k)!},
\]
and
\[
S_{2m;1}^m = \sum_{k=0}^{\lfloor n/2 \rfloor} \frac{\binom{n-k}{m-k} \cdot \binom{m}{k} \cdot \binom{m-k}{n-2k} \cdot \binom{n-2k}{m-2k}}{k! (m-k)! (n-2k)! (m-2k)!},
\]
if \( n \neq 4m \).

Proof. We assume that \( n = 2m \). If we take one vertex of \( K_m \) in the position of \( v_2 \) and one vertex of another \( K_m \) in the position of \( v_{2+1} \) of the graph which we will construct and distribute the remaining vertices of two \( K_m \) to both sides of the perpendicular bisector of \( v_{2+1} \) and \( v_{2+1} \) permitting redundancy and symmetrically regarding the line then the resulting graph becomes a \( K_m \)-spanning subgraph of \( K_{2m} \) xed by \( \frac{3}{4} \). The number of ways to distribute the vertices of two \( K_m \) is
\[
\sum_{k=0}^{\lfloor n/2 \rfloor} \frac{\binom{n-k}{m-k} \cdot \binom{m}{k} \cdot \binom{m-k}{n-2k} \cdot \binom{n-2k}{m-2k}}{k! (m-k)! (n-2k)! (m-2k)!} = 2^{m_1}.
\]
We assume that \( n \neq 4m \). Let \( H \) be a \( K_m \)-spanning subgraph of \( K_n \) xed by \( \frac{3}{4} \). Since \( n \) is even, the axis of \( \frac{3}{4} \) does not pass any vertices. Since \( m \) is odd, there is no component which contains both \( v_2 \) and \( v_{2+1} \). Let \( C_1 \) be a component which contains vertex \( v_2 \) and \( C_2 \) be a component which contains vertex \( v_{2+1} \). \( H \cap C_1 \cap C_2 \) naturally becomes \( K_m \)-spanning subgraph of \( K_{n_1} 2m \) xed by \( \frac{3}{4} \) when we change the name of the vertices. Conversely, let \( H \) be a \( K_m \)-spanning subgraph of \( K_{n_1} 2m \) xed by \( \frac{3}{4} \). Since \( n_1 2m \) is even, the axis of \( \frac{3}{4} \) does not pass any vertices. If we take one vertex of \( K_m \) in the position of \( v_2 \) and one vertex of another \( K_m \) in the position of \( v_{2+1} \) of the graph which we will construct and distribute the remaining vertices of two \( K_m \) between the vertices of \( H \) permitting redundancy and symmetrically regarding the axis then the resulting graph becomes a \( K_m \)-spanning subgraph of \( K_n \) xed by \( \frac{3}{4} \). The number of ways to distribute the vertices of two \( K_m \) is
\[
\sum_{k=0}^{\lfloor n/2 \rfloor} \frac{\binom{n-k}{m-k} \cdot \binom{m}{k} \cdot \binom{m-k}{n-2k} \cdot \binom{n-2k}{m-2k}}{k! (m-k)! (n-2k)! (m-2k)!}.
\]
Then we have the results.

Lemma 9. If \( n \) is even and \( m \) is even then
\[
S_{m;0}^m = \sum_{k=0}^{\lfloor n/2 \rfloor} \frac{\binom{n-k}{m-k} \cdot \binom{m}{k} \cdot \binom{m-k}{n-2k} \cdot \binom{n-2k}{m-2k}}{k! (m-k)! (n-2k)! (m-2k)!},
\]
and
\[
S_{n;0}^m = \sum_{k=0}^{\lfloor n/2 \rfloor} \frac{\binom{n-k}{m-k} \cdot \binom{m}{k} \cdot \binom{m-k}{n-2k} \cdot \binom{n-2k}{m-2k}}{k! (m-k)! (n-2k)! (m-2k)!}.
\]

Proof. The \( K_m \)-spanning subgraph of \( K_m \) is \( K_m \) and \( K_m \) is xed by \( \frac{3}{4} \). Then we have \( S_{m;0}^m = 1 \). We assume that \( n \neq 2m \). Let \( H \) be a \( K_m \)-spanning subgraph of \( K_n \) xed by \( \frac{3}{4} \). Since \( n \) is even, the axis of \( \frac{3}{4} \) passes \( v_0 \) and \( v_2 \). Let \( C \) be the component of \( H \) which contains vertex \( v_0 \) and \( v_2 \). \( H \cap C \) naturally becomes \( K_m \)-spanning subgraph of \( K_{n_1} m \) xed by \( \frac{3}{4} \) when we change the name of the vertices. Conversely, let \( H \) be a \( K_m \)-spanning subgraph of \( K_{n_1} m \) xed by \( \frac{3}{4} \). Since \( n_1 m \) is even, the axis of \( \frac{3}{4} \) does not pass any vertices. If we take two vertices of \( K_m \) in the positions of \( v_0 \) and \( v_2 \) of the graph which we will construct and divide the remaining vertices of \( K_m \) into halves and distribute them between the vertices of \( H \) permitting redundancy and symmetrically regarding the axis then the resulting graph becomes a \( K_m \)-spanning subgraph of \( K_n \) xed by \( \frac{3}{4} \). The number of ways to distribute the vertices of \( K_m \) is
\[
\sum_{k=0}^{\lfloor n/2 \rfloor} \frac{\binom{n-k}{m-k} \cdot \binom{m}{k} \cdot \binom{m-k}{n-2k} \cdot \binom{n-2k}{m-2k}}{k! (m-k)! (n-2k)! (m-2k)!}.
\]
Then we have the results.
Lemma 10. If \( n \) is even and \( m \) is even then

\[
S_{m,1}^n = 1
\]

and

\[
S_{2m,1}^n = 2^{m-1} + \frac{\mu_{2m-2}}{m-2}
\]

and

\[
S_{n,1}^n = \frac{\mu_{n-2}}{m-2} S_{n,1}^n + \frac{n!}{m-2} \sum_{k=0}^{m-2} \frac{1}{k!} \left( \frac{n}{2} \right) \frac{1}{(n-2m)!} \text{ if } n \neq 3m
\]

Proof. The \( K_m \)-spanning subgraph of \( K_m \) is \( K_m \) and \( K_m \) is \( \frac{1}{2} \) xed by \( \frac{1}{2} \). Then we have \( S_{m,1}^n = 1 \). We assume that \( n \neq 3m \). We study two kinds of constitutions that compose \( K_m \)-spanning subgraphs of \( K_n \) \( \sim \) xed by \( \frac{1}{2} \) inductively.

The first method is the following:
Let \( H \) be a \( K_m \)-spanning subgraph of \( K_{n+1} \) \( \sim \) xed by \( \frac{1}{2} \). Since \( n+1 \) \( m \) is even, the axis of \( \frac{1}{2} \) does not pass any vertices. If we take two vertices of \( K_m \) in the positions of \( v_0 \) and \( v_1 \) of the graph which we will construct and divide the remaining vertices of \( K_m \) into halves and distribute them between the vertices of \( H \) permitting redundancy and symmetrically regarding the axis then the resulting graph becomes a \( K_m \)-spanning subgraph of \( K_n \) \( \sim \) xed by \( \frac{1}{2} \). The number of ways to distribute the vertices of \( K_m \) is \( \frac{n-2}{m-2} \). Similarly, if we take two vertices of \( K_m \) in the positions of \( v_2 \) and \( v_3 \) of the graph which we will construct and divide the remaining vertices of \( K_m \) into halves and distribute them between the vertices of \( H \) permitting redundancy and symmetrically regarding the axis then the resulting graph becomes a \( K_m \)-spanning subgraph of \( K_n \) \( \sim \) xed by \( \frac{1}{2} \). The number of ways to distribute the vertices of \( K_m \) is \( \frac{n-2}{m-2} \). Accordingly, it is possible to \( 2 \frac{n-2}{m-2} \) \( K_m \)-spanning subgraph of \( K_n \) \( \sim \) xed by \( \frac{1}{2} \) as a whole with these constitutions.

The second method is the following:
Let \( H \) be a \( K_m \)-spanning subgraph of \( K_{n+2} \) \( \sim \) xed by \( \frac{1}{2} \). Since \( n+2 \) \( m \) is even, the axis of \( \frac{1}{2} \) does not pass any vertices. If we take one vertex of \( K_m \) in the position of \( v_0 \) and one vertex of another \( K_m \) in the position of \( v_2 \) of the graph which we will construct and distribute the remaining vertices of \( K_m \) between the vertices of \( H \) permitting redundancy and symmetrically regarding the axis then the resulting graph becomes a \( K_m \)-spanning subgraph of \( K_n \) \( \sim \) xed by \( \frac{1}{2} \). The number of ways to distribute the vertices of two \( K_m \) is \( \frac{n-2}{m-2} \). Similarly, if we take one vertex of \( K_m \) in the position of \( v_0 \) and one vertex of another \( K_m \) in the position of \( v_2 \) of the graph which we will construct and distribute the remaining vertices of two \( K_m \) between the vertices of \( H \) permitting redundancy and symmetrically regarding the axis then the resulting graph becomes a \( K_m \)-spanning subgraph of \( K_n \) \( \sim \) xed by \( \frac{1}{2} \). The number of ways to distribute the vertices of two \( K_m \) is \( \frac{n-2}{m-2} \). Therefore, by this construction, we can construct

\[
2 \frac{n-2}{m-2} \text{ } \text{ } K_m \text{-spanning subgraph of } K_n \text{ } \sim \text{ xed by } \frac{1}{2} \text{. By these two constructions, we can construct}
\]

\[
2 \frac{n-2}{m-2} S_{n,1}^m + 2 \frac{n-2}{m-2} \sum_{k=0}^{m-2} \frac{1}{k!} \left( \frac{n}{2} \right) \frac{1}{(n-2m)!} \text{ if } n \neq 3m
\]

\( K_m \)-spanning subgraphs of \( K_n \) \( \sim \) xed by \( \frac{1}{2} \). Clearly there are doubling two pieces of each. Also, it is clear to be able to compose all the \( K_m \)-spanning subgraphs of \( K_n \) \( \sim \) xed by \( \frac{1}{2} \)
by these methods. We assume that \( n \) is equal to 2m. Then we can similarly construct all 
\( K_m \)-spanning subgraphs of \( K_{2m} \) fixed by \( \frac{m}{2} \) by these two constructions if we set \( H \) be an empty graph in the case of the second constitution. We have the results.

Then we completely proved Theorem 2.

Remark 3. We calculated the non-equivarent \( K_4 \)-spanning subgraphs of \( K_n \), \( n \leq 16 \) by computer. The numbers agreed with the numbers that are given by Theorem 2. The results is as follows:

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References


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