α-SEMI CONNECTED AND LOCALLY α-SEMI CONNECTED PROPERTIES IN TOPOLOGICAL SPACES

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Abstract. In this paper we study the α-separation of sets, α-semicontinuity properties between topological spaces, α-semiconnected sets and α-locally semiconnected sets and prove some properties related to these topics. Also we study some forms of continuity.

In [2] , the concept of locally semi connected set is given. In this paper we introduce the concept of α semi connected sets, α locally semi connected sets and show that it generalizes the above concept when the operator α is the identity. Also we study some forms of continuity. Throughout this paper, we use the following notations; $Cl(A)$ denotes the usual closure and $Int(A)$ denotes the interior of a set.

Definition 1 [7].*Let (X, Γ) be a topological space and α be a map from $P(X)$ to $P(X)$ such that the following property is satisfied:*

for all $U \in \Gamma, U \subseteq \alpha(U)$. Then α is said to be an operator associated with Γ.

Definition 2 . *Let (X, Γ) be a topological space and α be an operator associated with Γ. We said that α is a monotone operator if for every pair of open sets U, V such that $U \subset V$, we have that $\alpha(U) \subset \alpha(V)$.*

Example 1 *We can observe that the closure operator is a monotone operator.*

Definition 3 . *Let (X, Γ) be a topological space and α be an operator associated with Γ. We say that α is an inversely additive operator if for every countable collection $\{U\}_{i \in I}$ of open sets, we have that $\cup_i \alpha(U_i\ ) \subset \alpha(\cup_i U_i\ )$.*

Example 2 *As example of an inversely additive operator, we can take the closure operator.*

The following theorem characterizes the monotone operator.

Theorem 1 *An inversely additive operator is equivalent to a monotone operator.*

Proof. Suppose that α is a monotone operator. Let $\{U\}_{i \in I}$ be a countable collection of open sets. we have that for each $i \in I, \alpha(U_i) \subset \alpha(\cup_i U_i)$. And we obtain easily that α is inversely additive. Now suppose that α is inversely additive, let $U \subset V$,then $\alpha(U) \subset \alpha(U) \cup \alpha(V) \subset \alpha(U \cup V) = \alpha(V)$ and the result follows.

Definition 4 *Let (X, Γ) be a topological space and α be an operator associated with Γ. A subset A of X is said to be α- semi open if there exists an open set $U \in \Gamma$ such that $U \subset A \subset \alpha(U)$. The complement of a α - semi open set is called a α - semi closed set.*

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Remark 1 *Observe that when α is the closure operator, the above definition agrees with the definition of semi-open set given by Levine in* [5]*. Also when α is the identity operator, the definition of α semi-open set agrees with the definition of open set.*

Remark 2 *Observe that for any operator α each open set is α semi-open.*

Example 3 *Let (X, Γ) be a topological space and α be an operator associated with Γ. Consider A a subset of X,where A is closed but not open and $Int(A) \neq \emptyset$. Define an operator β as follows: $\beta(V) = \alpha(V) \cup A$. Observe that A is β semi-open, but is not open.*

Definition 5 *Let (X, Γ) be a topological space and α be an operator associated with Γ. We say that, the subset A of X is α - regular open if $A = Int(\alpha(A))$.*

Example 4 *Let $X = \{a, b, c, d\}, \Gamma = \{\emptyset, X, \{a\}, \{a, b\}\}$*

$$\alpha(A) = \begin{cases} \emptyset & \text{if } A = \emptyset \\ \{a, b\} & \text{if } A = \{a, b\} \\ X & \text{if } A = \{a\} \text{ or } A = X \end{cases} \quad \text{and } \alpha(A) = A \text{ if } A \notin \Gamma$$

The set $\{a, b\}$ is α regular open,. $\{a, b, c\}$is α semi-open but is not α regular open.

Lemma 1 *If α is a monotone operator. A subset A of X is said to be α semi-open if and only if $A \subseteq \alpha(Int(A))$. Observe that when α is the closure operator, this definition agrees with the equivalence given in* [5]*.*

Proof. Suppose that $A \subseteq \alpha(Int(A))$, then we can see easily by definition that A is α semi-open. Conversely if A is α semi-open, then $U \subset A \subset \alpha(U)$ for some open set U. Now since $U \subset Int(A)$, we have that $\alpha(U) \subset \alpha(Int(A))$ and the result follows.

The following example shows that the condition of monotone can not be removed from the above lemma.

Example 5 *Let $X = \{a, b, c, d\}, \Gamma = \{\emptyset, X, \{a\}, \{a, b\}\}$*

$$\alpha(A) = \begin{cases} \emptyset & \text{if } A = \emptyset \\ \{a, b\} & \text{if } A = \{a, b\} \\ X & \text{if } A = \{a\} \text{ or } A = X \end{cases} \quad \text{and } \alpha(A) = A \text{ if } A \notin \Gamma$$

Observe that α is not a monotone operator, the set $\{a, b, c\}$ is α semi open but $\{a, b, c\}$ is not contained in $\alpha(Int(\{a, b, c\})) = \alpha(\{a, b\}) = \{a, b\}$.

¿From [3], we have the following equivalence when α is the closure operator

Lemma 2 *If α is the closure operator, then a subset S of X is α- semi closed if and only if there exists a closed subset F of X such that $Int(F) \subset S \subset F$.*

Proof: Suppose that $Int(F) \subset S \subset F$ for some closed set F, then $X - F \subset X - S \subset X - Int(F)$. We claim that, $X - Int(F) \subseteq Cl(X - F)$. Let $x \in X - Int(F)$, then $x \notin Int(F)$, this implies that for all neighborhood θ_x of x, $\theta_x \cap (X - F) \neq \phi$, from this, $x \in Cl(X - F)$ and $X - S$ is α semi open. Conversely, if S is α semi closed; then $X - S$ is α semi open, therefore there exists an open set U such that, $U \subset X - S \subset Cl(U)$. We claim that, $Int(X - U) \subset X - Cl(U)$. Let $x \in Int(X - U)$, then there exists a neighborhood θ_x of x such that, $\theta_x \subset (X - U)$, this implies that, $\theta_x \cap U = \phi$, therefore $x \notin Cl(F)$. From this, we obtain that x $\in X - Cl(U)$and the result follows.

Lemma 3 *If α is the closure operator. A subset S of X is α - semi closed if and only if $Int(\alpha(S)) \subseteq S$.*

Proof. Suppose that S is α semi closed. From the above lemma, there exists a closed set F in X such that $Int(F) \subset S \subset F$. ¿From this, we obtain that $Int(\alpha(\mathrm{S})) \subset Int(F) \subset S$, and the result follows. Conversely if $Int(\alpha(S)) \subseteq S$,then $Int(\alpha(S)) \subseteq S \subseteq \alpha(S)$, since α is the closure operator the result follows.

At this point there is a natural question: Is it possible to characterize α semi closed set for any operator α. The answer is not yet, because it is necessary to know which properties have to have the operator α.

Definition 6 *A subset S of (X, Γ) is said to be α - semi regular if it is both α - semi open and α -semi closed.*

Denote by: α -SO(X) the family of all α semi open sets of X.
α - SR(X) the family of all α semi closed sets of X.
α -SO(x) the family of all α semi open sets of X containing x.
α - SR(x) the family of all α semi closed sets of X containing x.

Lemma 4 *If α is a monotone operator, then the union of all α - semi open sets contained in the set S is α - semi open and it is denoted by α- $sInt(S)$.*

Proof. Let $\{U_i\}_{i\in I}$ a collection of α - semi open sets contained in S, then for each $i \in I$, there exists an open set V_i such that, $V_i \subseteq U_i \subseteq \alpha(V_i)$. Therefore we obtain that $\cup V_i \subseteq \cup U_i \subseteq \cup\alpha(V_i) \subset \alpha(\cup V_i)$, and the result follows.

The following example shows that the condition of monotone can not be removed from the above lemma.

Example 6 *Let X be the real line with the usual topology and α be the operator defined as follows:*

$$\alpha(A) = \begin{cases} A & \text{if } 0 \in A \\ Cl(A) & \text{if } 0 \notin A \\ \{1\} & \text{if } A = \emptyset \end{cases}$$

Observe that α is not a monotone operator, the sets $(-1, 1)$, $\{1\}$ are α semi open but $(-1, 1]$ is not α semi open

Corollary 1 *If α is a monotone operator, the intersection of all α-semi closed sets of X containing the set S is α - semi closed. It is called the α - semi closure of S and is denoted by α-$sCl(S)$.*

Corollary 2 *If A is a subspace of X and α is an operator associated with the topology of X, then the α-$sCl_A(S) = A \cap (\alpha\text{-}sCl(S))$.*

Proof. Let $\{U_i\}_{i\in I}$ be a collection of α - semi closed sets containing S. We need to show that $\grave{X}\backslash(\cap_{i\in I} U_i)$ is an α - semi open set. Using the above lemma the result follows.

Remark 3 *Observe that if α is a monotone operator associated with Γ and A is a subset of X, then A is α-semi closed if and only if α-$sCl(A) = A$.*

Theorem 2 *If A, B are two subsets of a topological space (X, Γ), α is an operator associated with Γ and $A \subset B$, then α -$sCl(A) \subset \alpha$ -$sCl(B)$.*

Proof. By definition

Now we introduce the notion of α-semi-connected set.

Definition 7 *Two non-empty subsets A, B of a topological space (X, Γ) are said to be α - semi-separated if and only if $(\alpha\text{-}sCl(A)) \cap\ B = A \cap\ (\alpha\ \text{-}\ sCl(B))\ = \emptyset$.*

Definition 8 *In a topological space (X, Γ), a set which can not be expressed as the union of two α - semi-separated sets is said to be α - semi-connected set.*

The topological space (X, Γ) is said to be a α - semi-connected if and only if X is α - semi-connected.

Note: We can observe that when α is the identity operator, the definition of $\alpha-$semi separated set agrees with the definition of separated set in the usual sense and therefore the definition of α - semi-connected set generalizes the definition of connected set.

Theorem 3 *A space X is α - semi-connected if and only if the only subsets of X that are both α-semi open and α-semi closed in X are the empty set and X itself.*

Proof: If A is a nonempty proper subset of X which is both α - semi-open and α-semi-closed in X, then the sets $U = A$ and $V = X \setminus A$ constitute an α-semi separation of X. Conversely, if U and V form an α-semi separation of X and $X = U \cup V$, then U is nonempty and different from X, since $U \cap V \subseteq U \cap (\alpha\ \text{-s}Cl(V)) = (\alpha\ \text{-s}Cl(U)) \cap V = \phi$, we obtain that both sets are α-semi open and α-semi closed.

Theorem 4 *If A is α - semi-connected and $A \subset C \cup D$ where C and D are α - semi-separated, then either $A \subset C$ or $A \subset D$.*

Proof. We write $A = (A \cap C) \cup (A \cap D)$. Observe that, $(A \cap C) \cap (\alpha\text{-s}Cl(A) \cap \alpha\text{-s}Cl(D)) \subseteq C \cap (\alpha\text{-s}Cl(D))$. Since C and D are α- semi-separated, $C \cap (\alpha\text{-s}Cl(D)) = \emptyset$. Similarly $(A \cap D) \cap (\alpha\text{-s}Cl(A) \cap\ \alpha\text{-s}Cl(C)) = \emptyset$. So if both $A \cap C \neq \emptyset$ and $A \cap D \neq \emptyset$, then A is not α-semi-connected. This shows that either $A \cap C = \emptyset$ or $A \cap D = \emptyset$. This shows that $A \subset C$ or $A \subset .D$

Theorem 5 *The union E of any family $(C_i)_{i \in I}$ of α -semi-connected sets having a non-empty intersection is an α - semi-connected set.*

Proof. Suppose that $E = A \cup B$, where A and B form a α-semi-separation of E. By hypothesis, we may choose a point $x \in \cap_{\mathrm{i} \in \mathrm{I}} C_{\mathrm{i}}$. Then x must belong either a subset A or a subset B. Since A, B are disjoint, we must have $C_{\mathrm{i}} \subset A$ for all i $\in$I, and so $E \subset A$. From this we obtain that B $= \emptyset$,which is a contradiction.This proves the theorem.

Theorem 6 *If C is a α - semi-connected set and $C \subset \alpha\text{-}sCl(E) \subset \alpha\text{-}sCl(C)$, then α-$sCl(E)$ is α - semi-connected set.*

Proof. If $\alpha\text{-s}Cl(E)$ is not α-semi-connected, we can write $\alpha\text{-s}Cl(E) = A \cup B$, where $A \neq \emptyset, B \neq \emptyset$, $A \cap (\alpha\text{-s}Cl(B)) = \emptyset$,and $(\alpha\text{-s}Cl(A)) \cap B = \emptyset$. By theorem 4, we must have $C \subset A$ or $C \subset B$. Without loss of generality, let us suppose $C \subset A$, it follows by Theorem 2 that $\alpha\text{-s}Cl(C) \subset \alpha\text{-s}Cl(A)$ therefore, $(\alpha\text{-s}Cl(C)) \cap B \subset (\alpha\text{-s}Cl(A)) \cap B = \emptyset$. On the other hand $B \subset \alpha\text{-s}Cl(E) \subset \alpha\text{-s}Cl(C)$ and $\alpha\text{-s}Cl(C) \cap B = B$, we must have $B = \emptyset$. And the result follows.

Theorem 7 *Let (X, Γ) be a topological space, α be a monotone operator associated with Γ and A be an open set. Then A is α-semi connected if and only if $(A, \Gamma_{/A})$ is α-semi connected.*

Proof: Suppose that A is not α-semi connected. Let H and K be an α semi-separation of A, then; H and K are α semi-separated sets in any X containing A, since $(\alpha\text{-s}Cl(H) \cap K = (\alpha\text{-s}Cl(H)) \cap A \cap K = ((\alpha\text{-s}Cl(H)) \cap A) \cap K = (\alpha\text{-s}Cl_A(H)) \cap K = \emptyset$, and similarly $(\alpha\text{-s}Cl(K) \cap H = (\alpha\text{-s}Cl_A(K) \cap H = \emptyset$ Conversely, if H and K is an α-semi separation of A and $A = H \cup K$, then we have $\alpha\text{-s}Cl_A(H) = (\alpha\text{-s}Cl(H)) \cap A = (H \cup K) \cap (\alpha\text{-s}Cl(H)) = (H \cap (\alpha\text{-s}Cl(H)) \cup (K \cap (\alpha\text{-s}Cl(H))) = H$. And hence H is α-semi closed in A. Similarly K is α-semi closed in A. Since A is an open set, we obtain that $K = A \setminus H$ and $H = A \setminus K$ are α-semi open in A. The result follows.

Definition 9 *Let (X, Γ) be a topological space, α be a monotone operator associated with Γ and $x \in X$. The α- semi component of x denoted by α-S.C(x), is the union of all α-semi-connected subsets of X containing x.*

We can see from Theorem 5 that the set α-S.C(x) is α-semi-connected.

Theorem 8 *Let (X, Γ) be a topological space and α be a monotone operator associated with Γ. Then the following are satisfied:*

a. Each α-semi-component α-S.C(x) is a maximal α- semi-connected set in X.
b. The set of all α- semi-components of a point of X form a partition of X.
c. Each α-S.C(x) is α-semi-closed.

Proof: a. Follows from the definition.

b. Let α-S.C(x) and α-S.C(y) be two α- semi-components of distinct points x and y in X. If $(\alpha\text{-S.C}(x)) \cap (\alpha\text{-S.C}(y)) \neq \emptyset$, then by theorem 5 $(\alpha\text{-S.C}(x)) \cup (\alpha\text{-S.C}(y))$ is α- semi-connected set, but $(\alpha\text{-S.C}(x)) \subseteq (\alpha\text{-S.C}(x)) \cup (\alpha\text{-S.C}(y))$, this contradicts the fact that α-S.C(x) is maximal. Now for any point $x \in X$, $x \in (\alpha\text{-S.C}(x))$ and $\cup_{x \in X} \{x\} \subset \cup_{x \in X} (\alpha\text{-S.C}(x))$. This implies that $X \subseteq \cup_{x \in X} (\alpha\text{-S.C}(x)) \subseteq X$. Therefore, $\cup_{x \in X} (\alpha\text{-S.C}(x)) = X$.

c. For any point $x \in X$, $\alpha\text{-s}Cl(\alpha\text{-S.C}(x))$ is α- semi-connected, but α-S.C(x) is the maximal α- semi-connected set containing x, therefore $\alpha\text{-s}Cl(\alpha\text{-S.C}(x)) \subseteq \alpha\text{-S.C}(x)$. But $\alpha\text{-S.C}(x) \subseteq \alpha\text{-s}Cl(\alpha\text{-S.C}(x))$, in consequence $\alpha\text{-s}Cl(\alpha\text{-S.C}(x)) = \alpha\text{-S.C}(x)$. And the result follows.

Definition 10 *A topological space (X, Γ) is called locally α semi-connected at the point $x \in X$ if and only if for every α semi open set U containing x, there exist an α semi-connected open set A such that $x \in A \subseteq U$. (X, Γ) is locally α semi-connected if and only if it is locally α semi-connected at every point of X.*

We can see easily that every α locally semi-connected topological space is α locally connected but the converse is not true as shown by the following example.

Example 7 *Consider $X = \{a, b, c\}$ and $\Gamma = \{X, \emptyset, \{a\}, \{a, b\}\}$. Define α as the closure operator. Then we obtain the following:*

α -SO$(X) = \{X, \emptyset, \{a\}, \{a, b\} \{a, c\}\}$. α -SR$(X) = \{X, \emptyset, \{b\} \{b, c\}, \{c\}\}$. Observe that $\{a, c\}$ is α semi open, but there no α open subset of $\{a, c\}$ exists containing c and so X is not α locally semi connected at c. Therefore X is not α locally semi connected. Note that X is α locally connected.

Example 8 *It is easy to see that α locally semi connectedness does not imply α semi connectedness as we show as follows:*

Let $X = \{a, b, c\}$ and $\Gamma = \{X, \emptyset, \{a\}, \{a, b\}, \{a, c\}, \{c\}\}$. Define α as the closure operator. Then we obtain the following:

α -SO$(X) = \{X, \emptyset, \{a\}, \{a, b\}, \{a, c\}, \{c\}\}$.

The α semi open sets containing a are: $\{a\}, \{a, b\}, \{a, c\}, X$. Observe that $\{a\}$ is an open set, and therefore, this implies that it is α - semi open, therefore X is α locally semi connected at the point a. In the same way, we show that X is α locally semi connected at the points b and c. But X is not α semi connected as we show: Let $A = \{a, b\}$ and $B = \{c\}$, then α-s$Cl(A) = \{a, b\}$ and α-s$Cl(B) = \{c\}$, $(\alpha\text{-s}Cl(A)) \cap B = \emptyset$ and $A \cap (\alpha\text{-s}Cl(B)) = \emptyset$, in this way A and B are two α semi separated sets and so X is not α semi connected .

Theorem 9 *Let (X, Γ) be a topological space and α be a monotone operator associated with Γ. X is α-locally semi-connected if and only if each α- semi-component of α semi open set are open sets.*

Proof: Suppose that (X, Γ) is α-locally semi-connected. Let $A \subset X$ be a α semi -open set and B be a α- semi-component of A. If $y \in B$, then $y \in A$, therefore, there is a α-semi-connected open set U such that $y \in U \subseteq A$. Since B is a α- semi-component of y and U is α- semi-connected, we have that $y \in U \subseteq B$, therefore B is open. Reciprocally if $x \in X$, and A is an α semi -open set containing x, let B be a α- semi-component of A such that $x \in B$. Since B is a α- semi-connected open set, $x \in B \subseteq A$. And the result follows.

Definition 11 *A mapping $f\colon (X, \Gamma) \longmapsto (Y, \Psi)$ is said to be (α, β) - semi-continuous if for each β-semi open set V in Y, $f^{-1}(V)$ is α- semi-open in X.*

Remark 4 *We can see easily that, for any operators α,β associated with Γ and Ψ respectively. If f is a continuous map then f is (α, β) - semi-continuous. Also, if f is a semi-continuous map in the sense of Levine, then f is (Cl, id)-semi-continuous.*

Remark 5 *We can observe that the definition of (α, β) - semi-continuous mappings generalize the definition of irresolute mappings given in* [6].

Theorem 10 *If $f\colon (X, \Gamma) \longmapsto (Y, \Psi)$ is a (α, β) - semi-continuous mapping from a $\alpha-$semi connected space (X, Γ) onto (Y, Ψ), then (Y, Ψ) is a $\beta-$semi connected space.*

Proof: Suppose that (Y, Ψ) is not a $\beta-$semi connected space and let A, B be a β separation of Y such that $Y = A \cup B$. Then using Definition 8, we have that $(\beta - sCl(A)) \cap B = A \cap (\beta - sCl(B)) = \emptyset$. It follows that A and B are β-semi open and β-semi closed sets in Y; it follows from the hypothesis that $f^{-1}(A) \cup f^{-1}(B) = X$, $f^{-1}(A)$ and $f^{-1}(B)$ are α-semi open an α-semi closed in X. Therefore we obtain that X is not α-semi connected, contradiction.

Theorem 11 *Let $f\colon (X, \Gamma) \longmapsto (Y, \Psi)$ be a (α, β) - semi-continuous and open mapping and $A \subset X$ be an open set. If A is a $\alpha-$semi connected set, then $f(A)$ is a $\beta-$semi connected set.*

Proof: Since A is α-semi connected and open in X, then by Theorem 7, (A, Γ_A) is also α semi connected. But $f_{/A} : (A, \Gamma_A) \to (f(A), \Psi_{f(A)})$ is an onto and (α, β) - semi-continuous mapping. Now using Theorem 10, the result follows.

Definition 12 [7]*Let (X, Γ) and (Y, Φ) be two topological spaces and α, β be operators associated with Γ, Φ respectively. We say that a map $f : X \to Y$ is (α, β) relatively continuous at $x \in X$ if given an open set $V \in \Phi$ containing $f(x)$,the set $\alpha(f^{-1}(V))$ is an open subset in the subspace $f^{-1}(\beta(V))$.If this condition is satisfied for each $x \in X$, then f is said to be (α, β) relatively continuous.*

Remark 6 *The above definition generalizes the definition of relatively continuous map given by Levine in* [5], *when we choose the operator α to be the identity operator and β the closure operator.*

Definition 13 [7]. *Let (X,Γ) and (Y,Φ) be two topological spaces and α, β be operators associated with Γ, Φ respectively. We say that a map $f : X \to Y$ is (α,β) weakly continuous at $x \in X$ if given any open set $V \in \Phi$ containing $f(x)$, $\alpha(\ f^{-1}(V)) \subseteq int(f^{-1}(\beta(V)))$. If this condition is satisfied at each $x \in X$, then f is said to be (α,β) weakly continuous.*

Remark 7 *The above definition generalizes the definition of weakly continuous map given by Levine in* [5], *when we choose the operator α to be the identity operator and β the closure operator.*

Remark 8 *If f is a constant map, then f is (α,β) weakly continuous for any operator α and β that satisfies the condition $\alpha(\emptyset) = \beta(\emptyset) = \emptyset$. In the case that the operator α satisfies the condition $\alpha(\emptyset) \neq \emptyset$, then any constant map is not (α,β) weakly continuous for any operator β.*

Theorem 12 *If $f : X \to Y$ is (α, id) weakly continuous. Then f is (α,β) relatively continuous for any operator β associated with Φ, where α is a monotone operator.*

Proof: For any $V \in \Phi$, $int(f^{-1}(V)) \subseteq \alpha(int(f^{-1}(V))) \subseteq \alpha(f^{-1}(V)) \subseteq .int(f^{-1}(V))$. This implies that $\alpha(int(f^{-1}(V))) = int(f^{-1}(V))$ so we obtain that $\alpha(int(f^{-1}(V)))$ is an open set in $f^{-1}(V)$.By definition $f^{-1}(V) \subseteq f^{-1}(\beta(V))$ for any operator β, therefore $\alpha(int(f^{-1}(V)))$ is open in $f^{-1}(\beta(V))$.

Definition 14 *Let (X,Γ) be a topological space. A pair of operators α and β associated with $\Gamma$are mutually dual if $\alpha(V) \cap \beta(V) = V$ for every $V \in \Gamma$.*

Theorem 13 *If $f : X \to Y$ is (α,β) and (α,β^*) relatively continuous, where $\beta$and β^* are mutually dual. Then f is (α, id) weakly continuous.*

Proof: By hypothesis $\alpha(\ f^{-1}(V))$ is an open subset in the subspace $f^{-1}(\beta(V))$ and $f^{-1}(\beta^*(V))$, therefore $\alpha(\ f^{-1}(V))$ is an open subset in the subspace $f^{-1}(\beta(V)) \cap f^{-1}(\beta^*(V))$, but $f^{-1}(\beta(V)) \cap f^{-1}(\beta^*(V)) = f^{-1}(V)$ so f is (α, id) weakly continuous.

Example 9 *Let $X = R$ with the usual topology. $Y = \{a,b\}$ with the discrete topology. Define*

$f : X \to Y$ as follows

$$f(x) = \begin{cases} a & \text{if } x \in (-\infty, o] \\ b & \text{if } x \in (o, +\infty) \end{cases}$$

Taken α to be the closure operator on R and β the closure operator on Y. Then f is not (α,β) relatively continuous, since $\alpha(\ f^{-1}(\{a\}))$ =$(-\infty, o]$ which is an open set in the subspace $f^{-1}(\beta(\{a\}))$ =$(-\infty, o]$, but $\alpha(f^{-1}(\{b\})) = [0, +\infty)$ which is not an open set in $f^{-1}(\beta(\{b\})) = (0, +\infty)$. .$f$ is not (α,β) weakly continuous, since $\alpha(\ f^{-1}(\{a\}))$ =$(-\infty, o]$ is not contained in the set $int(f^{-1}(\beta(\{a\})))$= $(-\infty, 0)$.

If, in the above example, we use the identity operator instead of the operator α, then we obtain, that f is (id,β) relatively continuous but f is not (id,β) weakly continuous. If we take $V = \{a\}$, then there does not exist an open neighborhood U of 0 such $f(id(U)) \subseteq \beta(V)$.

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