A BEST-CHOICE PROBLEM FOR A PRODUCTION SYSTEM WHICH DETERIORATES AT A DISORDER TIME

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Abstract. A production system is working in the GOOD state and there is a constant probability that it falls into the BAD state (and remains there) at a disorder moment. A decision-maker observes the output X_t of the system at each time $t = 1, 2, \ldots, n$, and decide either CONTINUE (*i.e.* reject X_t and observe X_{t+1}), or STOP (*i.e.* accept and receive X_t). The objective is to maximize the expected net value of the X_τ at the stopping time τ he decides during the given finite period of time n. Recall is not allowed (*i.e.* the observation once rejected cannot be recalled later.) For uniformly distributed observations we derive the Optimality Equation and show that the optimal policy is not necessarily of a control-limit type. Also an example in which the optimal policy is of a control-limit type is shown. An analytical solution for the infinite-horizon version by introducing discount rate over time, where a functional equation must be solved, is as yet unknown.

1 Problem and the Optimality Equation. The output of a production system is being observed one by one sequentially over the time. The system can be in either a GOOD or a BAD state. The true state of the system is unknown and only be inferred from the quality of the output. Let $f_0(f_1)$ be the pdf of quality X of the output in the GOOD(BAD) state. We assume that f_0 stochastically dominates f_1.

At each period $k, k = 1, 2, \ldots, n$, the controller observes the output X_k and must choose one of the two decisions; STOP(= accepts the output value X_k) or CONTINUE (= reject the X_k and then observe the next X_{k+1}). After the decision CONTINUE has been selected, state transitions occur which is descrived by a Markov chain with the transition matrix

$$\begin{array}{cc} & \begin{array}{cc} \text{GOOD} & \text{BAD} \end{array} \\ \begin{array}{c} \text{GOOD} \\ \text{BAD} \end{array} & \begin{bmatrix} a & \bar{a} \\ 0 & 1 \end{bmatrix} \end{array} \tag{1.1}$$

where $a = 1 - \bar{a} \in (0, 1)$ is the transition probability that a sysytem in state 0(= GOOD) during one period, remains in state 0 during the next one piriod. If the state enters state 1(= BAD), it remains in state 1 thereafter.

The objective is to maximize the expected net value of the output X_τ at the stopping time τ. The time horizon is n. If the controller fails to stop until the n-th output, he must stop at the n-th. Recall is not allowed (*i.e.* the observation once rejected cannot be recalled later).

For uniformly distributed observations we develop the routine method of deriving the optimal policy by dynamic programming, and we show that (1) Under some condition the expected net value obtained by following the optimal policy becomes larger as the

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controller's opinion that the unknown true state of the system is GOOD increases. (2) The optimal policy is not necessarily of a control-limit type (*i.e.* STOP if and only if the observation $X_\tau$exceeds some "control-limit" which is a function of the time remaining and the posterior knowledge currently possesed).

The model discussed in this paper belong to the area of the disorder problem in the partially observable Markov process combined with best-choice problems in the area of the so-called secretary problems. The model has many applications and has atracted the attention of many authors. Fundamental facts and important results are contained in the works of [1, 2, 4, 5, 6, 9]. Closely related works to this study are Gilbert and Mosteller [2; Section 3], Grosfeld-Nir [3] and Sakaguchi [8].

Let $\pi(\bar{\pi}) \in [0,1]$ be the probability that the system is in state 0(1) at the begining of a period, just prior to the decision. If the current information on the unknown true state at period t is $<\pi, \bar{\pi}>$, the action CONTINUE is chosen, and an r.v. X_t is observed, then the posterior information about the true state at the begining of period $t+1$ is $<\pi(X_t), \bar{\pi}(X_t)>$, where

$$\pi(x) = a\pi f_0(x) / \left(\pi f_0(x) + \bar{\pi} f_1(x)\right) \tag{1.2}$$

via Bayes' formula and state-transition by matrix (1.1). The decision in the $(t+1)-$st period is made based on $<\pi(X_t), \bar{\pi}(X_t)>$. Note that since $\pi(x) = a\left\{1 + (\bar{\pi}/\pi) f_1(x)/f_0(x)\right\}^{-1}$.

$$\pi \left\{ \begin{array}{l} = 0 \\ \in (0,1) \\ = 1 \end{array} \right\} \Rightarrow \pi(x) \left\{ \begin{array}{l} \equiv 0 \\ \in [0,a] \\ \equiv a \end{array} \right\}, \quad \forall x. \tag{1.2a}$$

$$\pi < \pi' \Rightarrow \pi(x) \leq \pi'(x), \quad \forall x. \tag{1.2b}$$

Define state (k, π) to mean that (1^0) we have not yet stopped the process, (2^0) the current information about the unknown true state of the system is $<\pi, \bar{\pi}>$, (3^0) there remain k periods until horizon comes, and (4^0) we choose the decision CONTINUE. We denote by $v_k(\pi)$ the expected value that will be obtained if all decisions in and after state (k, π) are made optimally. Then we have the Optimality Equation(OE)

$$v_k(\pi) = \mathrm{E}\left[X \vee v_{k-1}(\pi(X)) \mid \pi\right] \qquad (k = 1, 2, \ldots, n; \quad 0 \leq \pi \leq 1; \quad v_0(\pi) \equiv 0) \tag{1.3}$$

where $x \vee y = \max(x, y)$, and $\mathrm{E}[g(x)|\pi]$ means $\int g(x) \left(\pi f_0(x) + \bar{\pi} f_1(x)\right) dx$ for any function $g(x)$. The optimal decision in state (k, π) and after observing the first r.v. X is ; STOP(CONTINUE) if $X > (<) v_{k-1}(\pi(X))$.

From (1.2a) we evidently have

$$v_k(0) = \mathrm{E}_1\left[X \vee v_{k-1}(0)\right] \quad \text{and} \quad v_k(1) = \mathrm{E}_0\left[X \vee v_{k-1}(a)\right], \tag{1.4}$$

where $\mathrm{E}_i (i = 0, 1)$ means expectation taken under the pdf f_i.

First we prove the following result.

Theorem 1 (i) $v_k(\pi) < v_{k+1}(\pi), \quad \forall \pi \in [0,1]$.
(ii) *If* $\pi(x)$ *is a non-decreasing function of* x, *then* $v_k(\pi),\ k \geq 1$, *is non-decreasing in* π.

Proof. Induction is used.
(i); $\quad v_1(\pi) = \mathrm{E}[X \vee 0 \mid \pi] = \pi \mathrm{E}_0 X + \bar{\pi} \mathrm{E}_1 X > 0 = v_0(\pi)$.

Soppose that

$$v_0(\pi) < v_1(\pi) < \cdots < v_k(\pi), \quad \forall \pi \in [0,1].$$

Then

$$v_{k+1}(\pi) = \mathrm{E}[X \vee v_k(\pi(X)) \mid \pi] > \mathrm{E}[X \vee v_{k-1}(\pi(X)) \mid \pi] = v_k(\pi).$$

(ii); $\quad v_1(\pi) = \pi \mathrm{E}_0 X + \bar{\pi} \mathrm{E}_1 X = \mathrm{E}_1 X + \pi(\mathrm{E}_0 X - \mathrm{E}_1 X)$

is non-decreasing in π, since $\mathrm{E}_0(X) \geq \mathrm{E}_1(X)$.

Soppose that $v_1(\pi), v_2(\pi), \cdots, v_k(\pi)$ are non-decreasing in π. Then for $\pi < \pi'$.

$$v_{k+1}(\pi') = \mathrm{E}[X \vee v_k(\pi'(X)) \mid \pi'] \geq \mathrm{E}[X \vee v_k(\pi(X)) \mid \pi'],$$

since $\pi(x) \leq \pi'(x), \ \forall x$, and hence $v_k(\pi(x)) \leq v_k(\pi'(x))$ by induction hypothesis.

Therefore it follows that

$$\begin{aligned} & v_{k+1}(\pi') - v_{k+1}(\pi) \geq \mathrm{E}[X \vee v_k(\pi(X)) \mid \pi'] - \mathrm{E}[X \vee v_k(\pi(X)) \mid \pi] \\ = \ & [(\pi' \mathrm{E}_0 + \bar{\pi}' \mathrm{E}_1) - (\pi \mathrm{E}_0 + \bar{\pi} \mathrm{E}_1)]\,[X \vee v_k(\pi(X))] = (\pi' - \pi)(\mathrm{E}_0 - \mathrm{E}_1)[X \vee v_k(\pi(X))]. \end{aligned}$$

Now since $\pi(x)$ is non-decreasing in x by the assumed condition in (ii), and hence $v_k(\pi(x))$ is non-decreasing in x by induction hypothesis, and since f_0 stochastically dominates f_1, we obtain $(\mathrm{E}_0 - \mathrm{E}_1)\,[X \vee v_k(\pi(X))] \geq 0$. This implies

$$\pi' > \pi \Rightarrow v_{k+1}(\pi') - v_{k-1}(\pi) \geq 0$$

which completes the induction arguments. $\square$

Remark 1. The condition that f_0 stochastically dominates f_1 dose not imply that $\pi(x) = \frac{a\pi f_0(x)}{\pi f_0(x) + \bar{\pi} f_1(x)}$ is non-decreasing in x. An example is as follows:

Let, for $p \in (0,1)$,

$$\begin{aligned} & f_0(x) = I \quad (0 \leq x \leq 1), \\ & f_1(x) = (p^{-1} + p^{-2})(p - x) I(0 \leq x \leq p) + (1/\bar{p})\,(x - p) I(p < x \leq 1). \end{aligned} \tag{1.5}$$

It is easy to see that f_0 dominates f_1. We have in this case

$$\pi(x) = \begin{cases} \frac{a\pi}{\pi+\bar{\pi}(p^{-1}+p^{-2})(p-x)}, & \text{if } x < p \\ \frac{a\pi}{\pi+(\bar{\pi}/\bar{p})(x-p)}, & \text{if } x > p \end{cases}$$

which is decreasing in $p \le x \le 1$. See Figures 1 and 2.

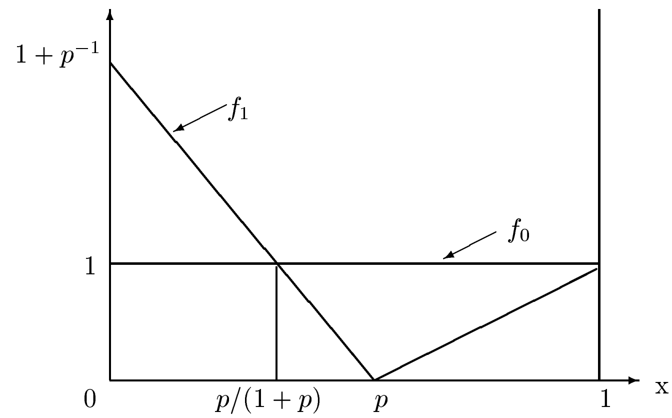


Figure 1: The two pdfs given by (1.5)

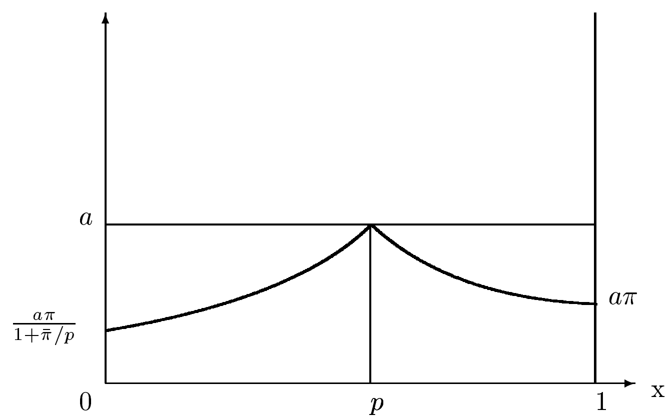


Figure 2: Posterior probability $\pi(x)$ for the pdf s (1.5).

2 Finite Horizon with Uniformly Distributed Observations.

We first study a special case where the pdf s are both uniform and f_0 stochastically dominates f_1. That is, let

(2.1) $$f_0(x) = I\ (0 \le x \le 1) \ \text{ and } \ f_1(x) = p^{-1} I\ (0 \le x \le p),$$

where $p \in (0,1)$, and $I(\mathrm{e})$ is the indicater function of the event e.

From (1.1) and (1.2) it is clear that

(2.2) $$\pi(x) = \hat{\pi} I \ ((0 \le x \le p) + aI \ ((p < x \le 1),$$

where $\hat{\pi} \equiv ap\pi/(p\pi + \bar{\pi})$.

If $p < x \le 1$, the system is known to be in good state, and so the strategy thereafter is forced to follow the optimal policy starting from state with $\pi = a$. The OE (1.3) becomes

(2.3) $$v_k(\pi) = \left[\int_0^p \{x \vee v_{k-1}(\hat{\pi})\} + \int_p^1 \{x \vee v_{k-1}(a)\}\right] (\pi f_0(x) + \bar{\pi} f_1(x)) dx$$

$$= (\pi + \bar{\pi}/p) \int_0^p \{x \vee v_{k-1}(\hat{\pi})\} dx + \pi \int_p^1 \{x \vee v_{k-1}(a)\} dx \quad ;$$

and, for $\pi = 0$,

$$v_k(0) = p^{-1} \int_0^p (x \vee v_{k-1}(0)) dx \qquad \text{(by using (1.4))}$$

which is rewritten as

(2.4) $$v_k(0) = pU_k$$

where $\{U_k\}$ satisfies the recurrsion $U_k = \frac{1}{2}(1 + U_{k-1}^2)$, $(k = 1, 2, \ldots, U_0 \equiv 0)$.

Also for $\pi = 1$, (1.4) gives

$$v_k(1) = \int_0^1 (x \vee v_{k-1}(a)) dx = \frac{1}{2}\left[1 + (v_{k-1}(a))^2\right],$$

since $v_{k-1}(a) \le v_{k-1}(1) < 1$.

From the OE (2.3) we obtain

Theorem 2. *For the best-choice problem with two uniform pdf with a disorder moment, described by* (2.1) $\sim$ (2.3), *the optimal decision in state* (k, π) *and after observing a r. v.* X, *is*

Stop if $X \in B_k(\pi)$, *and Continue, if otherwise,*

where

(2.5) $$B_k(\pi) = \begin{cases} (v_{k-1}(\hat{\pi}), 1), & \textit{in Case 1} \ \ (\textit{i.e.}\ v_{k-1}(a) < p) \\ (v_{k-1}(\hat{\pi}), p) \cup (v_{k-1}(a), 1), & \textit{in Case 2} \ \ (\textit{i.e.}\ v_{k-1}(\hat{\pi}) < p < v_{k-1}(a)) \\ (v_{k-1}(a), 1), & \textit{in Case 3} \ \ (\textit{i.e.}\ p < v_{k-1}(\hat{\pi})). \end{cases}$$

We show that $B_2(\pi)$ can be a disconnected set.

Since we have

$$v_1(\pi) = \pi \mathrm{E}_0 X + \bar{\pi} \mathrm{E}_1 X = \frac{1}{2}(\pi + \bar{\pi} p) = \frac{1}{2}(p + \bar{p}\pi),$$

$$\frac{1}{2}(p+\bar{p}\pi) < p < \frac{1}{2}(p+\bar{p}a) \Longleftrightarrow p \in \left(0, \frac{a}{1+a}\right), \quad \textit{indep. of } \pi$$

and

$$\frac{1}{2}(p+\bar{p}a) < p \Longleftrightarrow p \in \left(\frac{a}{1+a}, 1\right),$$

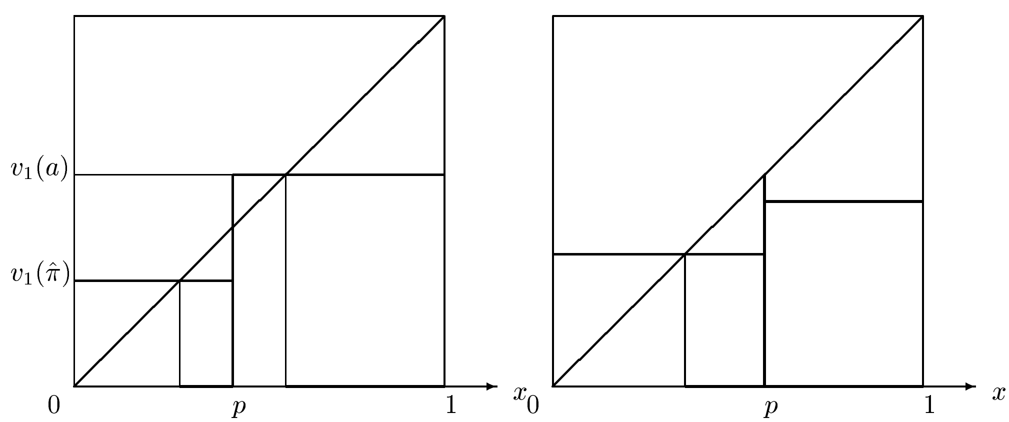
the optimal stopping region $B_2(\pi)$ is by (2.5)

$$B_2(\pi) = \left(\frac{1}{2}(p+\bar{p}\hat{\pi}), p\right) \cup \left(\frac{1}{2}(p+\bar{p}a), 1\right), \tag{2.6}$$

if $0 < p < a/(1+a)$, *i.e.*, Case 2;

$$= \left(\frac{1}{2}(p+\bar{p}\hat{\pi}), 1\right), \tag{2.7}$$

if $a/(1+a) < p < 1$, *i.e.*, Case 1. See Figure 3.



Case 2, *i.e.* $0 < p < \frac{a}{1+a}$. **Case 1,** *i.e.* $\frac{a}{1+a} < p < 1$

Figure 3. Optimal stopping region in state $(2, \pi)$

3 Finite Horizon with Uniformly Distributed Observations—Continued.

Let, for $p \in (0, \frac{1}{2})$,

$$f_0(x) = I\ (p \leq x \leq 1+p), \quad \text{and} \quad f_1(x) = I\ (0 \leq x \leq 1). \tag{3.1}$$

From (1.1)-(1.2) it is clear that

$$\pi(x) = a\pi I\ (p \leq x \leq 1) + aI\ (1 \leq x \leq 1+p). \tag{3.2}$$

Note that if $x \in \left\{ \begin{array}{c} (0,p) \\ (1,1+p) \end{array} \right\}$ the system is known to be in $\left\{ \begin{array}{c} \text{bad} \\ \text{good} \end{array} \right\}$ state and so the strategy thereafter is forced to follow the optimal policy starting from state with $\pi = \left\{ \begin{array}{c} 0 \\ 1 \end{array} \right\}$.

Thus the OE(1.3) becomes

(3.3)

$$v_k(\pi) = \left[\int_0^p (x \vee v_{k-1}(0)) + \int_p^1 (x \vee v_{k-1}(a\pi)) + \int_0^{1+p} (x \vee v_{k-1}(a))\right] (\pi f_0(x) + \bar{\pi} f_1(x))ds$$

$$= \bar{\pi} p U_{k-1} + \int_p^1 (x \vee v_{k-1}(a\pi))dx + \pi \int_1^{1+p} (x \vee v_{k-1}(a))dx$$

$$(k = 1, 2, \ldots, n; \quad 0 \le \pi \le 1; \quad v_0(\pi) \equiv 0, \ v_k(0) = U_k)$$

since the first integral in $[\ \cdots\ ]$ is equal to

$$\bar{\pi} \int_0^p (x \vee v_{k-1}(0))(\pi f_0(x) + \bar{\pi} f_1(x))dx$$

$$= \bar{\pi} \int_0^p (x \vee U_{k-1})dx = \bar{\pi} p U_{k-1}.$$

Here the sequence $\{U_k\}$ is the one which appeared in Section 2.

Also for $\pi = 1$, (3.3) gives

$$v_k(1) = \int_p^{1+p} (x \vee v_{k-1}(a))dx = v_{k-1}(a) + \frac{1}{2}(1 + p - v_{k-1}(a))^2 \tag{3.4}$$

since $p < \frac{1}{2} \le U_{k-1} = v_{k-1}(0) \le v_{k-1}(a) < 1 + p$.

Hence we obtain :

Theorem 3. *For the best-choice problem with two pdfs and a disorder moment described by* (3.1) $\sim$ (3.3), *the optimal decision in state* (k, π) *and after observing a r.v.* X, *is :*

Stop, if $X \in B_k(\pi)$, *and Continue, if otherwise,*

where

(3.5)

$$B_k(\pi) = \begin{cases} (v_{k-1}(a\pi), 1+p), & \textit{in Case 1 i.e., } v_{k-1}(a) < 1, \\ (v_{k-1}(a\pi), 1) \cup (v_{k-1}(a), 1+p), & \textit{in Case 2 i.e., } v_{k-1}(a\pi) < 1 < v_{k-1}(a), \\ (v_{k-1}(a\pi), 1+p), & \textit{in Case 3 i.e., } 1 < v_{k-1}(a\pi). \end{cases}$$

We obtain from (1.5)

$$v_1(\pi) = \frac{1}{2} + p\pi, \qquad \text{(Stop is optimal in state } (1, \pi))$$

and therefore state $(2,\pi)$ is in Case 1. The optimal decision in state $(2,\pi)$, and observing the r.v. X is: Stop, if $X \in B_2(\pi) = (\frac{1}{2} + pa\pi, 1+p)$, and Continue, if otherwise. So, from (3.3) $\sim$ (3.5), we have

$$v_2(\pi) = \frac{1}{2}p\bar{\pi} + \int_p^1 \{x \vee \left(\frac{1}{2} + pa\pi\right)\}dx + \pi \int_1^{1+p} xdx \tag{3.6}$$

$$= \frac{5}{8} + \left\{\frac{1}{2}(1+p) + a\left(\frac{1}{2} - p\right)\right\} p\pi + \frac{1}{2}(ap\pi)^2.$$

Note that $v_2(\pi)$ is convexly increasing in $\pi \in [0,a]$, with values

$$v_2(0) = \frac{5}{8} \quad \text{and} \quad v_2(a) = \frac{5}{8} + \frac{1}{2}(a+a^2)p + \frac{1}{2}a\bar{a}(1-a-a^2)p^2.$$

From Theorem 3 and (3.4)-(3.5), it follows that

$$v_{k-1}(a) \geq v_{k-1}(a\pi) \geq v_{k-1}(0) \xrightarrow[(k\to\infty)]{} 1.$$

Starting at any given $\pi \in (0,1]$ and large k, the posterior $\pi(\cdot)$ and $v_{k-1}(\pi(\cdot))$ both become smaller as the process goes on. The optimal strategy behaves as in Case 3, when k remains large, and passing Case 2, reaches Case 1 at last, as k decreases.

For example, from (3.5)-(3.6) we have in state $(3,\pi)$,

$$v_2(a\pi) < 1 < v_2(a), \quad i.e., \quad \text{Case 2},$$

$$\Longleftrightarrow \left\{\frac{1}{2}(1+p) + a\left(\frac{1}{2} - p\right)\right\} ap\pi + \frac{1}{2}a^4p^2\pi^2 < \frac{3}{8} < \frac{1}{2}(a+a^2)p + \frac{1}{2}(a - 2a^2 + a^4)p^2,$$

and this double inequalities hold true, when $a = 1-0$, $p \to \frac{1}{2} - 0$ and $\pi^2 + 3\pi - 3 < 0$, $(i.e., 0 < \pi < \frac{1}{2}(\sqrt{21}-3) \fallingdotseq 0.7913)$. So for these triples of a, p and π, $B_3(\pi)$ is a disconnected set.

4 An Example where the Optimal Policy is a Control-limit Type.

Next we consider the case where the pdfs are both power densities and f_0 stochastically dominates f_1. That is, let

$$f_0(x) = 2xI(0 \leq x \leq 1) \quad \text{and} \quad f_1(x) = I(0 \leq x \leq 1), \tag{4.1}$$

and therefore

$$\pi(x) = \frac{2a\pi x}{2\pi x + \bar{\pi}}, \quad \text{for } 0 \leq x \leq 1, \tag{4.2}$$

which is concavely increasing in $0 \leq x \leq 1$, with values 0 at $x = 0$, and $\frac{2a\pi}{1+\pi}$ at $x = 1$.

The OE(1.3) becomes

$$v_k(\pi) = \int_0^1 (2\pi x + \bar{\pi})\left\{x \vee v_{k-1}\left(\frac{2a\pi x}{2\pi x + \bar{\pi}}\right)\right\}dx \tag{4.3}$$

$$(k = 1, 2, \ldots, n;\ 0 \leq \pi \leq 1;\ v_0(\pi) \equiv 0)$$

Theorem 4. *For the best-choice problem with two power pdfs with disorder moment descrived by* (4.1) $\sim$ (4.3), *the optimal policy is of a control-limit type.*

Proof. Eq.(4.3) gives

$$v_1(\pi) = \int_0^1 (2\pi x^2 + \bar{\pi}x)dx = \frac{1}{2} + \frac{1}{6}\pi$$

and

$$\begin{aligned} v_2(\pi) &= \int_0^1 (2\pi x + \bar{\pi}) \left\{ x \vee v_1\left(\frac{2a\pi x}{2\pi x + \bar{\pi}}\right) \right\} dx \\ &= \left[\int_0^{b_1} v_1\left(\frac{2a\pi x}{2\pi x + \bar{\pi}}\right) + \int_{b_1}^1 x \right] (2\pi x + \bar{\pi})dx, \end{aligned}$$

where $b_1 \in (0,1)$ is a unique root of the quadratic equation

$$v_1\left(\frac{2a\pi x}{2\pi x + \bar{\pi}}\right) = \frac{1}{2} + \frac{a}{6} \cdot \frac{2\pi x}{2\pi x + \bar{\pi}} = x.$$

See Figure 4.

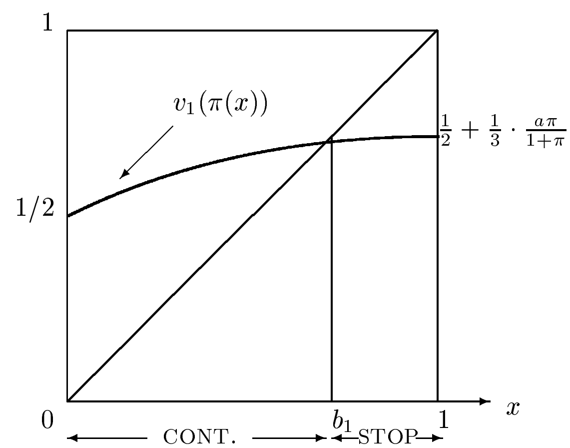


Figure 4. Optimal decision in state $(2, \pi)$.

From (1.2a), (4.1) and Theorem 1(ii), $v_{k-1}(\pi(x))$ is non-decreasing in x, *i.e.*,

$$x < x' \Rightarrow \pi(x) < \pi(x') \Rightarrow v_{k-1}(\pi(x)) \le v_{k-1}(\pi(x')).$$

Moreover we have, from Theorem 1 (i),

$$v_{k-1}(\pi(x)) < v_k(\pi(x)), \quad \forall x.$$

Therefore $B_k(\pi) = (b_k, 1)$, in state (k, π) where $b_k \in (0,1)$ is a unique root of the equation $v_{k-1}(\pi(x)) = x$. That is, the optimal policy is of a control-limit type. $\square$

5 Final Remark.

Remark 1 is given in the previous Section 1.

Remark 2. By introducing discount rate $\alpha \in (0,1)$ over time, we can consider the infinite-horizon version. The Optimality Equation is functional equation

$$v(\pi) = \mathrm{E}[X \vee \alpha v(\pi(X)) \mid \pi].$$

The boundary conditions are: $v(0)$ is a unique root of the equation $\bar{\alpha} v(0) = \mathrm{E}_1[(X - \alpha v(0))^+]$, and $v(1)$ must satisfy the relation $v(1) = \mathrm{E}_0[X \vee \alpha v(a)]$.

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