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APPLICATION OF MODIFIED S-CURVE MEMBERSHIP FUNCTION IN DECISION MAKING PROCESS USING FLP APPROACH

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Abstract. In this paper, the modified S-curve membership function methodology is used in a real life industrial problem of mix product selection. This problem occurs in production planning management where by a decision maker plays an important role in making decision in a fuzzy environment. As an analyst, we try to find a good enough solution for the decision maker to make a final decision. An industrial application of FLP through the S-curve membership function has been investigated using a set of real life data collected from a Chocolate Manufacturing Company. The problem of fuzzy product mix selection has been defined. The objective of this paper is to find an optimal units of products with higher level of satisfaction with vagueness as a key factor. This problem has been considered because all the coefficient such as technical and resource variables are uncertain. This is considered as one of sufficiently large problem involving 29 constraints and 8 variables. Since there are several decisions that were to be taken, a table for optimal units of products respect to vagueness and degree of satisfaction has been defined to identify the solution with higher level of units of products and with a higher degree of satisfaction . It is to be noted that higher units of products need not lead to higher degree of satisfaction. Optimal units of products and satisfactory level have been computed using FLP approach. The fuzzy outcome shows that higher units of products need not lead to higher degree of satisfaction. The findings of this work indicates that the optimal decision is depend on vagueness factor in the fuzzy system of mix product selection problem. Further more the high level of units of products obtained when the vagueness in the system is low.

Introduction

A non linear membership function, referred to as the "Modified flexible S-curve membership function" has been used in problems involving fuzzy linear programming. The S-function (Kuz'min, 1981) and (Watada, 1997) can be applied and tested for its suitability through an applied problem. In this example, the S-function was applied to reach a decision when all two coefficients, such as technical coefficients and resources, of mix product selection (FPS) were fuzzy. The solution thus obtained is suitable to be given to decision maker and implementer for final implementation. The problem illustrated in this paper is only one of three cases of FPS problems which occur in real life applications. The above case of FPS problem is considered on a real life situation in the case of Chocolate Manufacturing. The data for this problem are taken from the data-bank of Chocoman Inc, USA (Tabucanon, 1996). Chocoman produces varieties of chocolate bars, candy and wafer using a number of raw materials and processes. The objective is to use the modified S-function as a methodology for obtaining an optimal units of products through fuzzy linear programming (FLP) approach.

Key words and phrases. Uncertainty, Fuzzy Constraint, Vagueness, Degree of Satisfaction and Decision Maker.

Approach and Methods

The methodology for this fuzzy linear programming (FLP) has references to Carlsson and Korhonen (1986), Bellman and Zadeh (1970), Chanas (1983), Delgado, Verdegay and Vila (1989), Dubois and Prade (1980), Hersh and Caramazza (1976), Jiuping (2000), Kickert (1978), Klir and Yuan (1995), Luhandjula (1986), Maleki, Tata, and Mashinchi (2000), Negoita (1981), Negoita and Ralescu (1977), Negoita and Sularia (1976). The approach proposed here is based on an interaction with the decision maker, the implementer and the analyst in order to find a compromised satisfactory solution for a fuzzy linear programming (FLP) problem. In a decision process using FLP model, source resource variables may be fuzzy, instead of precisely given numbers as in crisp linear programming (CLP) model. For example, machine hours, labor force, material needed and so on in a manufacturing center, are always imprecise, because of incomplete information and uncertainty in various potential suppliers and environments. Therefore, they should be considered as fuzzy resources, and the FLP problem should be solved by using fuzzy set theory (Orlovsky, 1980), (Rommenlfanger, 1996), (Ross, 1995), (Rubin and Narasimhan, 1984) and (Sengupta, Pal and Chakraborty, 2001).

A general model of crisp linear programming is formulated as :

$$Max \;\; z = cx \qquad \text{Standard formulation}$$

$$\text{Subject to} \quad Ax \leq b$$

$$x \leq 0 \tag{1}$$

where c and x are n dimensional vectors, b is an m dimensional vector, and A is $m\ x\ n$ matrix.

Since we are living in an uncertain environment, the coefficients of objective function (c), the technical coefficients of matrix (A) and the resource variables (b) are fuzzy. Therefore it can be represented by fuzzy numbers, and hence the problem can be solved by FLP approach.

The fuzzy linear programming problem is formulated as :

$$Max \;\; z = \tilde{c}x \qquad \text{Fuzzy formulation}$$

$$\text{Subject to} \quad \tilde{A}x \lesssim \tilde{b}$$

$$x \geq 0 \tag{2}$$

where x is the vector of decision variables ; $\tilde{A}, \tilde{b}$ and $\tilde{c}$ are fuzzy quantities ; the operations of addition and multiplication by a real number of fuzzy quantities are defined by Zades's extension principle (Zadeh, 1975) ; the inequality relation $\lesssim$ is given by a certain fuzzy relation and the objective function, z, is to be maximized in the sense of a given crisp LP problem. Carlsson and Korhonen (1986) approach is considered to solve FLP problem (2) which is fully trade-off, meaning that the solution will be with certain degree of satisfaction.

First of all, formulate the membership functions for the fuzzy parameters of $\tilde{c}$, $\tilde{A}$ and $\tilde{b}$. Here a non-linear membership function such as logistic function is employed. The membership functions are represented by $\mu_{a_{ij}}$, μ_{b_i} and μ_{c_j}, where a_{ij} are the technical coefficients of matrix A for $i = 1, \ldots, m$ and $j = 1, \ldots, n, b_i$ are the resource variables for $i = 1, \ldots, m$ and c_j are the coefficients of objective function z for $j = 1, \ldots, n$.

Next, through the appropriate transformation with the assumption of trade-off between fuzzy numbers of $\tilde{a}_{ij}$, $\tilde{b}_i$ and $\tilde{c}_j$ $\forall\ i$ and j, an expression for $\tilde{a}_{ij}$, $\tilde{b}_i$ and $\tilde{c}_j$ will be obtained. After trade-off between $\tilde{a}_{ij}$, $\tilde{b}_i$ and $\tilde{c}_j$ the solution will always exist at (Carlsson and Korhonen, 1986) :

$$\mu = \mu_{c_j} = \mu_{a_{ij}} = \mu_{b_i} \quad \text{for all} \quad i = 1, \ldots, m \quad \text{and} \quad j = 1, \ldots, n \tag{3}$$

Therefore, we can obtain:

$$c = g_c(\mu), \qquad A = g_A(\mu) \quad \text{and} \quad b = g_b(\mu) \tag{4}$$

where $\mu \in [0, 1]$ and g_c, g_A and g_b are inverse functions (Carlsson and Korhonen,1986) of μ_c, μ_A and μ_b respectively. Equation (2) becomes

$$\text{Max} \quad z = [g_c(\mu)]x$$

$$\text{Subject to} \qquad [g_A(\mu)]x \leq g_b(\mu)$$

$$x \geq 0 \tag{5}$$

The FLP problem, formulated in equation (1) can be written as :

$$\text{Max} \quad z = \sum_{j=1}^{8} x_j$$

$$\text{Subject to} \qquad \sum_{j=1}^{29} \tilde{a}_{ij} x_j \leq \tilde{b}_i$$

$$\text{where} \qquad x_j \geq 0, \quad j = 1, 2, 3, \ldots, 8. \tag{6}$$

where $\tilde{a}_{ij}$ and $\tilde{b}_i$ are fuzzzy parameters.

First of all, construct the membership functions for the fuzzy parameters of $\tilde{A}$ and $\tilde{b}$. Here a non-linear membership function such as S-curve function (Bells, 1999) is employed. The membership functions are represented by $\mu_{a_{ij}}$, and μ_{b_i}, where a_{ij} are the technical coefficients of matrix A for $i = 1, \ldots, 29$ and $j = 1, \ldots, 8, b_i$ are the resource variables for $i = 1, \ldots, 29$.

The membership function for μ_{b_i} and the fuzzy interval, b_i^a to b_i^b, for $\tilde{b}_i$ is given in Figure 1.

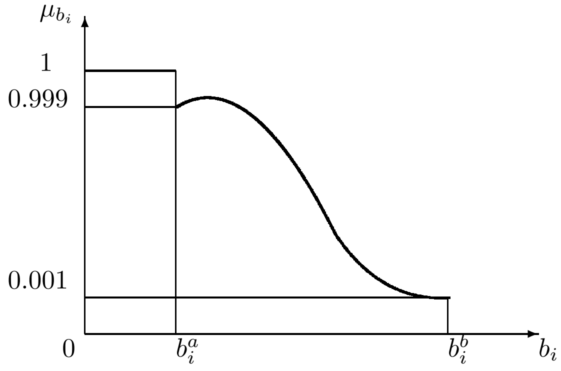

Figure 1- Membership Function μ_{b_i} and Fuzzy Interval for b_i

Similarly we can formulate membership function for fuzzy technical coefficients and it's derivations (Pandian, 2002 and 2004(a), 2004(b)).

Fuzzy Resource Variable $\tilde{b}_i$

For an interval $b_i^a < b_i < b_i^b$,

$$\mu_{b_i} = \frac{B}{1 + C_e^{\alpha\left(\frac{b_i - b_i^a}{b_i^b - b_i^a}\right)}}$$

Taking $\log_e$ both sides

$$e^{\alpha\left(\frac{b_i - b_i^a}{b_i^b - b_i^a}\right)} = \frac{1}{C}\left(\frac{B}{\mu_{b_i}} - 1\right)$$

Hence

$$\alpha\left(\frac{b_i - b_i^a}{b_i^b - b_i^a}\right) = \text{In}\frac{1}{C}\left(\frac{B}{\mu_{b_i}} - 1\right)$$

(7) $$b_i = b_i^a + \left(\frac{b_i^b - b_i^a}{\alpha}\right)\text{In}\frac{1}{C}\left(\frac{B}{\mu_{b_i}} - 1\right)$$

Since b_i is the fuzzy resource variable in equation (7), it is denoted by $\tilde{b}_i$. Therefore

(8) $$\tilde{b}_i = b_i^a + \left(\frac{b_i^b - b_i^a}{\alpha}\right)\text{In}\frac{1}{C}\left(\frac{B}{\mu_{b_i}} - 1\right)$$

The membership function for μ_{b_i} and the fuzzy interval, b_i^a to b_i^b for $\tilde{b}_i$ is given in Figure 1.

Due to limitations in resources for manufacturing a product and the need to satisfy certain conditions in manufacturing and demand, a problem of fuzziness occurs in production planning systems. This problem occurs also in chocolate manufacturing when deciding a mixed selection of raw materials to produce varieties of products. This is referred here to as the Product- mix Selection (Tabucanon, 1996).

The Fuzzy Product - mix Selection (FPS) is stated as :
There are n products to be manufactured by mixing m raw materials with different proportion and by using k varieties of processing. There are limitations in resources of raw materials. There are also some constraints imposed by marketing department such as product - mix requirement, main product line requirement and lower and upper limit of demand for each product. All the above requirements and conditions are fuzzy. It is necessary to obtain maximum units of products with certain degree of satisfaction by using fuzzy linear programming approach.
Since the technical coefficients and resource variables are fuzzy therefore the outcome of the units of products will be fuzzy.

Fuzzy Constraints
The product demand, material and facility available are as illustrated in Table 1 and 2 respectively. Table 3 and 4 give the mixing proportions and facility usage required for manufacturing each product.

Table 1 : Demand of Product

Product	Fuzzy Interval ($x10^3$ units)
Milk chocolate, 250 g	[500,625)
Milk chocolate, 100 g	[800,1000)
Crunchy chocolate, 250 g	[400,500)
Crunchy chocolate, 100 g	[600,750)
Chocolate with nuts, 250g	[300,375)
Chocolate with nuts,100 g	[500,625)
Chocolate candy	[200,250)
Wafer	[400,500)

Table 2 : Raw Material and Facility Availability

Raw Material/Facility (units)	Fuzzy Interval
Coco (kg)	[75000,125000)
Milk (kg)	[90000,150000)
Nuts (kg)	[45000,75000)
Confectionery sugar (kg)	[150000,250000)
Flour (kg)	[15000,25000)
Aluminum foil (ft^2)	[375000,625000)
Paper (ft^2)	[375000,625000)
Plastic (ft^2)	[375000,625000)
Cooking (ton-hours)	[750,1250)
Mixing (ton-hours)	[150,250)
Forming (ton-hours)	[1125,1875)
Grinding (ton-hours)	[150,250)
Wafer making (ton-hours)	[75,125)
Cutting (hours)	[300,500)
Packaging 1 (hours)	[300,500)
Packaging 2 (hours)	[900,1500)
Labor (hours)	[750,1250)

Table 3 : Mixing Proportions (Fuzzy)

	Product Types - Fuzzy Interval							
Materials required (per 1000 units)	MC 250	MC 100	CC 250	CC 100	CN 250	CN 100	CANDY	WAFER
Cocoa (kg)	[66,109)	[26,44)	[56,94)	[22,37)	[37,62)	[15,25)	[45,75)	[9,21)
Milk (kg)	[47,78)	[19,31)	[37,62)	[15,25)	[37,62)	[15,25)	[22,37)	[9,21)
Nuts (kg)	0	0	[28,47)	[11,19)	[56,94)	[22,37)	0	0
Cons.sugar (kg)	[75,125)	[30,50)	[66,109)	[26,44)	[56,94)	[22,37)	[157,262)	[18,30)
Flour (kg)	0	0	0	0	0	0	0	[54,90)
Alum.foil (ft^2)	[375,625)	0	[375,625)	0	0	0	0	[187,312)
Paper(ft^2)	[337,562)	0	[337,563)	0	[337,562)	0	0	0
Plastic (ft^2)	[45,75)	[95,150)	[45,75)	[90,150)	[45,75)	[90,150)	[1200,2000)	[187,312)

Table 4 : Facility Usage (Fuzzy)

	Product Types - Fuzzy Interval							
Facility usage required (per 1000 units)	MC 250	MC 100	CC 250	CC 100	CN 250	CN 100	CANDY	WAFER
Cooking (ton-hours)	[0.4,0.6)	[0.1,0.2)	[0.3,0.5)	[0.1,0.2)	[0.3,0.4)	[0.1,0.2)	[0.4,0.7)	[0.1,0.12)
Mixing (ton-hours)	0	0	[0.1,0.2)	[0.04,0.07)	[0.2,0.3)	[0.07,0.12)	0	0
Forming (ton-hours)	[0.6,0.9)	[0.2,0.4)	[0.6,0.9)	[0.2,0.4)	[0.6,0.9)	[0.2,0.4)	[0.7,1.1)	[0.3,0.4)
Grinding (ton-hours)	0	0	[0.2,0.3)	[0.07,0.12)	0	0	0	0
Wafer making (ton-hours)	0	0	0	0	0	0	0	[0.2,0.4)
Cutting(hours)	[0.07,0.12)	[0.07,0.12)	[0.07,0.12)	[0.07,0.12)	[0.07,0.12)	[0.07,0.12)	[0.15,0.25)	0
Packaging 1 (hours)	[0.2,0.3)	0	[0.2,0.3)	0	[0.2,0.3)	0	0	0
Packaging 2 (hours)	[0.04,0.06)	[0.2,0.4)	[0.04,0.06)	[0.2,0.4)	[0.04,0.06)	[0.2,0.4)	[1.9,3.1)	[0.1,0.2)
Labor(hours)	[0.2,0.4)	[0.2,0.4)	[0.2,0.4)	[0.2,0.4)	[0.2,0.4)	[0.2,0.4)	[1.9,3.1)	[1.9,3.1)

There are two sets of fuzzy constraints such as raw material availability and facility capacity constraints. These constraints are inevitable for each material and facility that are based on the material consumption, facility usage and the resource availability.
The following nomenclature is maintained in solving the FLP of Chocoman Inc.

The decision variables for the FPSP are defined as :

x_1 = milk chocolate of 250 g to be produced (in 10^3)

x_2 = milk chocolate of 100g to be produced (in 10^3)

x_3 = crunchy chocolate of 250g to be produced (in 10^3)

x_4 = crunchy chocolate of 100g to be produced (in 10^3)

x_5 = chocolate with nuts of 250g to be produced (in10^3)

x_6 = chocolate with nuts of 100g to be produced (in10^3)

x_7 = chocolate candy to be produced (in 10^3 packs)

x_8 = chocolate wafer to be produced (in 10^3 packs)

The following constraints were established by the sales department of Chocoman:
Product mix requirements. Large -sized products (250g) of each type should not exceed 60% (non fuzzy value) of the small-sized product (100g)

$$x_1 \leq 0.6x_2 \tag{9}$$

$$x_3 \leq 0.6x_4 \tag{10}$$

$$x_5 \leq 0.6x_6 \tag{11}$$

Main product line requirement. The total sales from candy and wafer products should not exceed 15% (non fuzzy value) of the total revenues from the chocolate bar products.

Results

The FPS problem is solved by using MATLAB and its tool box of Linear Programming (LP). The vagueness is given by α, and μ is the degree of satisfaction. The LP tool box has two inputs namely α and μ in addition to the fuzzy parameters. There is one output z^*, the optimal units of products.
The given values of various parameters of Chocolate Manufacturing are fed to the tool box. The solution can be tabulated and presented as 2 and 3 dimensional graphs.

Table 5- Optimal Units of Products and Degree of Satisfaction

No	Degree of Satisfaction (μ)	Optimal Units of Products (z^*)
1	0.0010	2755.4
2	0.0509	2837.8
3	0.1008	2852.9
4	0.1507	2862.3
5	0.2006	2869.4
6	0.2505	2875.3
7	0.3004	2880.4
8	0.3503	2885.0
9	0.4002	2889.4
10	0.4501	2893.5
11	0.5000	2897.6
12	0.5499	2901.7
13	0.5998	2905.8
14	0.6497	2910.2
15	0.6996	2914.8
16	0.7495	2919.8
17	0.7994	2925.6
18	0.8493	2932.6
19	0.8992	2941.8
20	0.9491	2956.6
21	0.9990	3034.9

From Table 5 and Figure 2, it's noticed that higher degree of satisfaction gives higher units of products. But the realistic solution for the above problem exist at 50% of degree of satisfaction, that is 2897 units. From Figure 2 it's concluded that the fuzzy outcome of the objective function, z^* is a an increasing function (Zimmermman, 1985) .

Units of Products z^* for Various Vagueness Values, α
Figure 3, displays the objective values plot for various values of a from 1 to 39. The graph shows the nature of variations of z^* with respect to μ.

The realistic solution with an uncertainties in fuzzy parameters of technical coefficients and resource variables exists at $\mu = 50\%$. Hence the result for 50% degree of satisfaction for $1 \leq \alpha \leq 39$ and the corresponding values for z^* are tabulated in Table 6.

Table 6- Vagueness a and Objective Value z^* for $\mu = 50\%$

Vagueness α	Units of Products z^*
1	3034.5
3	3033.2
5	3027.6
7	3006.5
9	2968.4
11	2933.0
13	2906.4
15	2886.5
17	2871.2
19	2859.1
21	2849.3
23	2841.2
25	2834.4
27	2828.6
29	2823.5
31	2819.2
33	2815.3
35	2811.9
37	2808.7
39	2805.8

The fuzzy outcome of the units of products are decreases as vagueness a increases in the fuzzy parameters of technical coefficients and resource variables. This is clearly shown in Table 6. Table 6 is very important to the decision maker in picking up the α so that the outcome will be at good enough satisfactory level.

The 3 dimensional plot for μ, a and z^* is given in Figure 4.
The outcome in the Figure 4 shows that when the vagueness in the increases results in less units of products. Also it is found that the S-curve membership function with various values of α provides a possible solution with certain degree of satisfaction.

Furthermore the relationship between z^*, μ and α is given in Table 7. This Table is very useful for the decision maker to find the units of products at any given value of α with degree of satisfaction μ. From Table 7 it is observed that at any particular degree of satisfaction μ the optimal units of products z^* decreases as the vagueness a increases between 1 and 39. Similarly at any particular value of vagueness the optimal units of products are increases as the degree of satisfaction increases.

Table 8 is the outcome of diagonal values of z^* respect to α and μ from Figure 4 and Table 7. The findings of this outcome shows that :

(i) When vagueness is low at $\alpha = 1, 3$ and 5 then optimal units of products z^* is achieved at lower level of degree of satisfaction, that is at $\mu = 0.1\%$, $\mu = 5\%$ and $\mu = 10\%$.

(ii) When vagueness is high at $\alpha = 35, 36$ and 37 then optimal units of products z^* is achieved at higher level of degree of satisfaction, that is at $\mu = 89.9\%$, $\mu = 94.9\%$ and $\mu = 99.92\%$.

Table 7(a) - Optimal Units of Products z^*

z^*	Vagueness α			
μ	1	3	5	7
0.0010	2755.4	2755.4	2755.4	2755.4
0.0509	3026.5	3007.7	2963.2	2915.0
0.1008	3030.9	3020.9	2989.6	2942.4
0.1505	3032.4	3025.8	3002.4	2958.4
0.2006	3033.3	3028.4	3010.1	2969.9
0.2505	3033.6	3030.0	3015.3	2978.8
0.3004	3033.9	3031.1	3019.1	2986.0
0.3503	3034.1	3031.8	3022.0	2992.2
0.4002	3034.2	3032.4	3024.3	2997.6
0.4501	3034.4	3032.9	3026.1	3002.3
0.5000	3034.5	3033.2	3027.6	3006.5
0.5499	3034.6	3033.6	3028.9	3010.4
0.5998	3034.6	3033.8	3029.9	3013.9
0.6497	3034.7	3034.0	3030.9	3017.2
0.6996	3034.8	3034.2	3031.7	3020.2
0.7495	3034.8	3034.4	3032.4	3023.0
0.7994	3034.8	3034.5	3033.0	3025.7
0.8493	3034.8	3034.6	3033.6	3028.2
0.8992	3034.9	3034.7	3034.1	3030.5
0.9491	3034.9	3034.8	3034.5	3032.8
0.9990	3034.9	3034.9	3034.9	3034.9

Table 7(b) - Optimal Units of Products z^*

z^*	Vagueness α			
μ	9	11	13	15
0.0010	2755.4	2755.4	2755.4	2755.4
0.0509	2881.2	2858.6	2842.9	2831.3
0.1008	2904.0	2877.6	2859.0	2845.2
0.1505	2918.0	2889.3	2869.0	2853.9
0.2006	2928.5	2898.1	2876.5	2860.5
0.2505	2937.1	2905.4	2882.7	2865.9
0.3004	2944.5	2911.7	2888.1	2870.6
0.3503	2951.1	2917.5	2893.0	2874.9
0.4002	2957.2	2922.9	2897.6	2878.9
0.4501	2962.9	928.0	2906.0	2882.7
0.5000	2968.4	2933.0	2906.4	2886.5
0.5499	2973.8	2937.9	2910.7	2890.3
0.5998	2979.2	2943.0	2915.1	2894.1
0.6497	2984.5	2948.2	2919.6	2898.1
0.6996	2990.0	2953.8	2924.5	2902.4
0.7495	2995.8	2959.8	2929.9	2907.1
0.7994	3001.8	2966.6	2936.0	2912.4
0.8493	3008.4	2974.7	2943.3	2918.9
0.8992	3015.8	2985.0	2953.0	2927.5
0.9491	3024.4	3000.1	2968.4	2941.3
0.9990	3034.9	3034.9	3034.9	3034.9

Table 7(c) - Optimal Units of Products z^*

z^*	Vagueness α			
μ	17	19	21	23
0.0010	2755.4	2755.4	2755.4	2755.4
0.0509	2822.4	2815.4	2809.7	2805.0
0.1008	2834.7	2826.4	2819.7	2814.1
0.1507	2842.4	2833.3	2825.9	2819.8
0.2006	2848.2	2838.5	2830.6	2824.1
0.2505	2853.0	2842.8	2834.5	2827.6
0.3004	2857.2	2846.5	2837.9	2830.8
0.3503	2861.0	2849.9	2841.0	2833.6
0.4002	2864.5	2853.1	2843.9	2836.2
0.4501	2867.9	2856.1	2846.6	2838.7
0.5000	2871.2	2859.1	2849.3	2841.2
0.5499	2874.6	2862.1	2852.0	2843.7
0.5998	2878.0	2865.2	2854.8	2846.2
0.6497	2881.5	2868.3	2857.7	2848.8
0.6996	2885.3	2871.7	2860.7	2851.7
0.7495	2889.4	2875.5	2864.1	2854.7
0.7994	2894.2	2879.7	2868.0	2858.3
0.8493	2899.9	2884.9	2872.7	2862.6
0.8992	2907.6	2891.8	2878.9	2868.3
0.9491	2919.9	2902.8	2888.9	2877.4
0.9990	3034.9	3034.9	3034.9	3034.9

Table 7(d) - Optimal Units of Products z^*

z^*	Vagueness α			
μ	25	27	29	31
0.0010	2755.4	2755.4	2755.4	2755.4
0.0509	2801.0	2797.6	2794.7	2792.2
0.1008	2809.4	2805.4	2802.0	2799.0
0.1505	2814.7	2810.3	2806.5	2803.2
0.2006	2818.6	2814.0	2809.9	2806.4
0.2505	2821.9	2817.0	2812.8	2809.1
0.3004	2824.7	2819.6	2815.2	2811.4
0.3503	2827.3	2822.0	2817.5	2813.5
0.4002	2829.8	2824.3	2819.6	2815.4
0.4501	2832.1	2826.4	2821.6	2817.3
0.5000	2834.4	2828.6	2823.5	2819.2
0.5499	2836.7	2830.7	2825.5	2821.0
0.5998	2839.0	2832.8	2827.5	2822.9
0.6497	2841.4	835.1	2829.6	2824.8
0.6996	2844.0	2837.5	2831.8	2826.9
0.7495	2846.9	2840.1	2834.3	2829.2
0.7994	2850.1	2843.1	2837.1	2831.9
0.8493	2854.1	2846.8	2840.5	2835.1
0.8992	2859.3	2851.7	2845.1	2839.3
0.9491	2867.8	2859.5	2852.4	2846.2
0.9990	3034.9	3034.9	3034.9	3034.9

Table 7(e) - Optimal Units of Products z^*

z^*	Vagueness α			
μ	33	35	37	39
0.0010	2755.4	2755.4	2755.3	2755.1
0.0509	2790.0	2788.0	2786.1	2784.3
0.1008	2796.4	2794.0	2791.8	2789.8
0.1505	2800.3	2797.8	2795.4	2793.1
0.2006	2803.3	2800.6	2798.1	2795.7
0.2505	2805.8	2802.9	2800.3	2797.8
0.3004	2808.0	2805.0	2802.2	2799.6
0.3503	2810.0	2806.8	2804.0	2801.3
0.4002	2811.8	2808.6	2805.6	2802.8
0.4501	2813.6	2810.3	2807.2	2804.3
0.5000	2815.3	2811.9	2808.7	2805.8
0.5499	2817.0	2813.5	2810.3	2807.3
0.5998	2818.8	2815.2	2811.9	2808.8
0.6497	2820.6	2816.9	2813.5	2810.3
0.6996	2822.6	2818.8	2815.3	2812.0
0.7495	2824.8	2820.8	2817.2	2813.8
0.7994	2827.3	2823.2	2819.4	2815.9
0.8493	2830.3	2826.0	2822.1	2818.5
0.8992	2834.3	2829.8	2825.7	2821.9
0.9491	2840.7	2835.8	2831.4	2827.3
0.9990	3034.9	3034.9	3034.9	3034.9

μ = Degree of Satisfaction, z^* = Units of Products, α = Vagueness

Table 8 - Z^* Respect to α and μ

Degree (μ) Of Satisfaction	Vagueness (α)	Optimal Units Of Products Z^*
0.0010	1	2755.4
0.0509	3	3007.7
0.1008	5	2989.6
0.1507	7	2958.4
0.2006	9	2937.1
0.2505	11	2928.5
0.3004	13	2888.1
0.3503	SC	2885.0
0.4002	15	2878.9
0.4501	17	2867.9
0.5000	19	2859.1
0.5499	21	2852.0
0.5998	23	2846.2
0.6497	25	2841.4
0.6996	27	2837.5
0.7495	29	2834.3
0.7994	31	2831.9
0.8493	33	2830.3
0.8992	35	2829.8
0.9491	37	2831.4
0.9990	39	3034.9

SC : S-curve $\alpha = 13.81350956$

Fuzzy α Selection and Decision Making

In order the decision maker to obtain the best outcome for the units of products z^*, the analyst has design Table 9. From Table 9 the decision maker can select the value for vagueness α according to his or her preferences. The fuzzy range for z^* is classified in three groups, that is low, medium and high. It is possible that the fuzzy groups can be change if the input data for technical coefficients and resource variables changes. The fuzzy groups also can be called as fuzzy band. The decision can be made by the decision maker in picking up the good enough outcome for z^* and provides the solution for the implementation.

Table 9 - Fuzzy Band for Units of Products z^*

Fuzzy Band z^*	Low	Medium	High
Units of Products	2750–2850	2851–2950	2951-3050
Vagueness	$27 < \alpha \leq 39$	$13 < \alpha \leq 27$	$1 < \alpha \leq 13$

Discussion

The finding shows that the minimum units of products is 2755.4 and maximum is 3034.9. It can seen that when the vagueness α is in between 0 and 1 the maximum units of z^* 3034.9 is achieved at smaller value of m. Similarly when a is greater then 39 the minimum value for z^* 2755.4 is achieved at larger value of m. Since the solution for the fuzzy mix product selection is satisfactory optimal solution with degree of satisfaction therefore it is important to select the vagueness a in between minimum and maximum value of z^*. The well distributed value for z^* falls in the group of medium fuzzy band.

Conclusion

The objective of this research work in finding the maximum units of products for the fuzzy mix products selection problem is achieved. The newly constructed modified S-Curve membership function as a methodology for this work has solved the above problem successfully. The decision making process and the implementation will be easier if the decision make and the implementer can work together with the analyst to achieve the best outcome with respect to degree of satisfaction. There are two more cases to be considered in the future work whereby the technical coefficients are non fuzzy and resource variables are non fuzzy. There is a possibility to design the self organizing of fuzzy system for the mix products selection problem in order to find the satisfactory solution. The decision maker, the analyst and the implementer can incorporate their knowledge and experience to obtain the best outcome.

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