FUZZY AR MODEL OF STOCK PRICE

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Abstract. The objective of economic analysis is to interpret the past, present or future economic state by analyzing economical data. In many cases an economic analysis is pursued based on the time-series data. Time-series analysis is a central tool to analyze time-series data.

Nevertheless, economic systems are a complex system resulted from human behaviors and related to many factors. When the system includes much uncertainty such as ones of human behaviors, it is better to employ methodologies of a fuzzy system in the analysis.

In this paper, we compare the characteristic between two fuzzy time-series models which are a fuzzy AR (autoregressive) model proposed by Ozawa et al and fuzzy autocorrelation model by Yabuuchi and Watada. Both models are built on the basis of the concepts of fuzzy systems. In the analysis of the Nikkei Stock Average, we compare the effectiveness and meaning between both the models.

1 Introduction It is necessary to employ some analysis and data that we interpret economic structure. Many econometric models including time-series model have been proposed for this purpose. In this paper, we proposed the fuzzy time-series model which is employed in analyzing an economic system.

The objective of the economic analysis is to understand the past and present states of the economic system more precisely based on the statistical data[6]. However, in the economic system, it is closely relating to many factors which bring out by the aggregation of many human behaviors[5, 8, 9].

Therefore, it is insufficient to interpret such an economic system only based on the results of conventional statistical method. So, it is desirable to apply the concept of the fuzzy system theory which can cope with the ambiguity of a structure when we analyze the economic system which has many vague factors.

We analyze Nikkei stock average employing the fuzzy autoregressive model which proposed by Ozawa et al [3] and fuzzy autocorrelation model which is proposed in this paper. Furthermore, it is expectable that we can forecast a future trend by sequential prediction which fits the present condition.

2 Modelling In this section, we explain the fuzzy autoregressive model which is proposed by Ozawa et al and the fuzzy autocorrelation model which is proposed in this paper. Though a fuzzy autoregressive model employs a trapezoidal fuzzy number in [3], we employ a triangular fuzzy number in our model. Therefore, we explain a fuzzy autoregressive model in terms of triangular fuzzy numbers.

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2.1 Fuzzy Autoregressive Model Let all fuzzy time-series data $\mathbf{Z}_t$ defined by a triangular fuzzy number. Triangular fuzzy numbers are defined by three parameters, and denoted as $\mathbf{Z}_t = (\alpha_t, \beta_t, \delta_t), (\alpha_t \leq \beta_t \leq \delta_t)$. The inclusion relation of triangular fuzzy numbers is defined as the following inequalities:

$$\mathbf{Z}_t \subseteq \mathbf{Z}_s \Leftrightarrow \{\alpha_t \geq \alpha_s,\ \delta_t \leq \delta_s \tag{1}$$

In the same way, arithmetic operations are defined as follows:

$$\mathbf{Z}_t + \mathbf{Z}_s = (\alpha_t + \alpha_s, \beta_t + \beta_s, \delta_t + \delta_s) \tag{2}$$

$$\mathbf{Z}_t - \mathbf{Z}_s = (\alpha_t - \alpha_s, \beta_t - \beta_s, \delta_t - \delta_s) \tag{3}$$

$$p \cdot \mathbf{Z}_t = \begin{cases} (p \times \alpha_t, p \times \beta_t, p \times \delta_t),\ p \geq 0 \\ (p \times \delta_t, p \times \beta_t, p \times \alpha_t),\ p < 0 \end{cases} \tag{4}$$

where p is a real crisp number.

A fuzzy autoregressive model is defined with the following equations.

$$\tilde{\mathbf{Z}}_t = \phi_1 \mathbf{Z}_{t-1} + \cdots + \phi_p \mathbf{Z}_{t-p} + \mathbf{u} \tag{5}$$

$$\mathbf{Z}_t \subseteq \tilde{\mathbf{Z}}_t,\ \tilde{\mathbf{Z}}_t = (\tilde{\alpha}_t, \tilde{\beta}_t, \tilde{\delta}_t) \tag{6}$$

It is clear that the following relations hold from Equations (1) and (6).

$$\alpha_t \geq \tilde{\alpha}_t, \quad \delta_t \leq \tilde{\delta}_t$$

Namely, a value in terms of estimated the fuzzy time-series model includes all the fuzzy data of the original series[1]. The autoregressive parameters $\phi_1, \phi_2, \cdots, \phi_p$ are a real value, and show the degree that the fuzzy time-series data depend on the past. An error term is the constant term which is characteristic of the model, and shows the part of the fuzzy data which does not depend on past data. This is also defined as a triangular fuzzy number.

$$\mathbf{u} = (u_\alpha, u_\beta, u_\delta)$$

A fuzzy autoregressive model results in the linear programming which minimizes ambiguity of the model according to the inclusion condition (6) as follows:

$$\begin{array}{ll} \text{minimize} & \displaystyle\sum_{t=p+1}^{n} (\tilde{\delta}_t - \tilde{\alpha}_t) \\ \text{subject to} & \alpha_t \geq \tilde{\alpha}_t,\ \delta_t \leq \tilde{\delta}_t \\ & \qquad (t = p+1, p+2, \cdots, n) \\ & u_\alpha \leq u_\delta \end{array}$$

2.2 Fuzzy Autocorrelation Model In the fuzzy autocorrelation model, time-series data z_t are transformed into a fuzzy number to express the possibility of data. The following fuzzy equation shows the case where only one time point before and after time point t is taken into consideration in building a fuzzy number [2, 7].

$$\mathbf{Y}_t = (Y_t^L, Y_t^C, Y_t^U) = (\min(z_{t-1}, z_t, z_{t+1}), z_t, \max(z_{t-1}, z_t, z_{t+1}))$$

Next, we employ a calculus of finite differences to filter out the time-series data of trend. It enables us first-order difference-equation to write the following:

$$\mathbf{T}_t = (T_t^L, T_t^C, T_t^U) = (\min(\mathbf{Y}_t - \mathbf{Y}_{t-1}), Y_t^C - Y_{t-1}^C, \max(\mathbf{Y}_t - \mathbf{Y}_{t-1})) \tag{7}$$

Generally, if we take finite differences then we reduce the trend variation, and only an irregular pattern is included in the difference series. However, when we calculate the fuzzy operation, the ambiguity may become large and the value of an autocorrelation coefficient may also take 1 or more and -1 or less value. In order to solve this problem in the case of the fuzzy operation, we adjust the width of a fuzzy number using α-cut when we calculate the difference series. An α-cut level h is decided from the value of the autocorrelation. When we calculate the fuzzy autocorrelation, we employ usual fuzzy operation under condition that the fuzzy autocorrelation of lag 0 is set $\rho_0 = \lambda_0/\lambda_0$ =(1,1,1). It results in the following linear programming to decide the value at the α-cut level. When we set α-cut level to 1, the ambiguity of fuzzy autocorrelation is made the smallest, but we can not obtain the fuzzy autocorrelation which reflects the possibility of the system. So, we maximize the width of autocorrelation. However, the size of width is decided automatically as the value of autocorrelation should be include in [-1,1].

$$\begin{array}{ll} \text{maximize} & \sum(\rho_i^U - \rho_i^L) \\ \text{subject to} & \rho_i^U \leq 1 \\ & \rho_i^L \geq -1 \\ & \rho_i^L \leq \rho_i^C \leq \rho_i^U \\ & \quad (i = 1, 2, \cdots, p) \end{array}$$

We can define the fuzzy covariance and the fuzzy autocorrelation as follows:

$$\Lambda_k \equiv Cov[\mathbf{T}_t \mathbf{T}_{t-k}] = E[\mathbf{T}_t \mathbf{T}_{t-k}] = [\lambda_k^L, \lambda_k^C, \lambda_k^U]$$
$$\mathbf{r}_k = \Lambda_k/\Lambda_0 = [\rho_k^L, \rho_k^C, \rho_k^U]$$

We adjust the ambiguity of the difference series employing the α-cut level h which is obtained by solving the above linear programming. Using fuzzy autocorrelation coefficient which is calculated by employing α-cut level h, we redefine Yule-Walker equations as in linear programming, and calculate the partial autocorrelation.

We calculate the following autoregressive process.

$$\mathbf{T}_t = \Phi_1 \mathbf{T}_{t-1} + \Phi_2 \mathbf{T}_{t-2} + \cdots + \Phi_p \mathbf{T}_{t-p}$$

where $\Phi = [\phi^L, \phi^C, \phi^U]$ is a fuzzy partial autoregressive coefficient.

As mentioned above, the next value of observation value exceeds observed value at present by the size of the value of autocorrelation, or it is less. For this reason, autocorrelation is important for the time-series analysis. So, we build the model which illustrates ambiguity of the system shown by fuzzy autocorrelation. The reason for the autocorrelation is also fuzzy autocorrelation, Yule-Walker equations can be also calculated by the fuzzy equation in the same way.

$$\mathbf{R}_t = \Phi_1 \mathbf{r}_{t-1} + \Phi_2 \mathbf{r}_{t-2} + \cdots + \Phi_p \mathbf{r}_{t-p} \tag{8}$$

Φ in Equation (8) is an unknown coefficient. We are building the model in terms of fuzzy autocorrelation which can describe the ambiguity of the system. However, when ambiguity of a model is large, the relation between a model and a system becomes ambiguous. Therefore, the possibility of the system can not be described properly. So, in order to obtain the fuzzy partial autocorrelation coefficient whose ambiguity of a time-series model should be

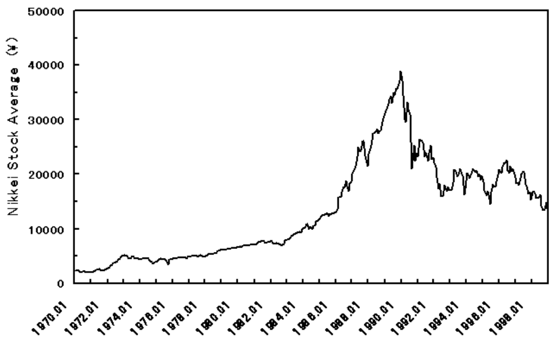

Figure 1: Nikkei Stock Average in 1970 to 1998

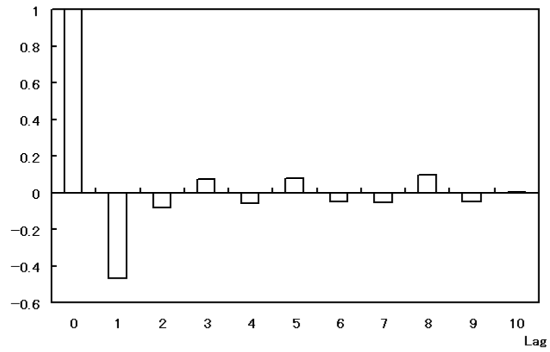

Figure 2: Autocorrelation of Nikkei Stock Average

minimized, we come down to the following linear programming:

$$\begin{array}{ll} \text{minimize} & \sum(\rho_t^U - \rho_t^L) \\ \text{subject to} & R_t^U \geq \rho_t^U \\ & R_t^C = \rho_t^C \\ & R_t^L \leq \rho_t^L \\ & \rho_t^L \leq \rho_t^C \leq \rho_t^U \\ & \qquad (t = 1, 2, \cdots, p) \end{array}$$

As mentioned above $\mathbf{R}$ is obtained by the fuzzy operation employing fuzzy autocorrelation $\mathbf{r}$ and fuzzy partial autocorrelation Φ. R^L, R^C and R^U represent the lower limit, the center, and the upper limit of $\mathbf{R}$, respectively.

A fuzzy autocorrelation model expresses the possibility that the change of the system is realized in data, different from the conventional statistical method. We are building the model which can show an ambiguous portion called a possibility that it has not expressed clearly by the conventional statistics technique.

3 Numerical Example In this section, we employ Nikkei stock average which indicates the trend of the whole stock market as an index of Japanese stock market[4]. We use the monthly data from 1970 to 1998.

We show the sample autocorrelation coefficient at each time lag (Figure 2) in order to determine the order. Figure 2 shows the negative correlation in lags 1 and 2 where the sign

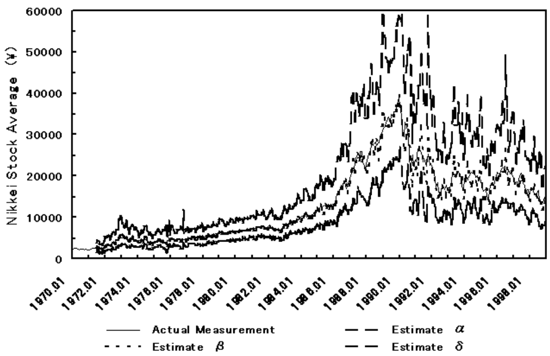

Figure 3: The conjectured result by Fuzzy Autoregressive Model

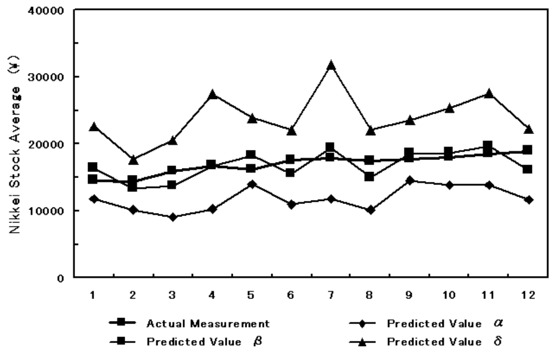

Figure 4: The prediction result in 1999 by Fuzzy Autoregressive Model

of the autocorrelation coefficient changes minus to plus. Because of this result, we analyze Nikkei stock average employing AR(2) model with the second-order.

Furthermore, because of existing seasonal variation in this data, we employ the calculation $\bigtriangledown^2 \bigtriangledown_{12} z_t$ which take the first-order seasonal difference of every 12 period after processing taking the second-order difference.

$$\bigtriangledown^2 \bigtriangledown_{12} \tilde{\mathbf{Z}}_t = \phi_1 \bigtriangledown^2 \bigtriangledown_{12} \mathbf{Z}_{t-1} + \phi_2 \bigtriangledown^2 \bigtriangledown_{12} \mathbf{Z}_{t-2} + \mathbf{u}$$

where the data z_t in analysis are statistical data and actual measurement. $\mathbf{u}$ is an error term of the model. In order that ambiguity of time-series system reflects on these data, we employ these data to fuzzy numbers to deal with these data.

3.1 Fuzzy Autoregressive Model We analyze this data employing the fuzzy autoregressive model with the triangular fuzzy number. The coefficients of this model are decided as follows:

$$\bigtriangledown^2 \bigtriangledown_{12} \tilde{\mathbf{Z}}_t = -0.749 \bigtriangledown^2 \bigtriangledown_{12} \mathbf{Z}_{t-1} - 0.348 \bigtriangledown^2 \bigtriangledown_{12} \mathbf{Z}_{t-2} + (-0.143, -0.013, 0.117)$$

The model which is obtained by the fuzzy autoregressive model has a negative coefficient the same as the result which is obtained by the autocorrelation. An original series and an estimated series are shown in the Figure 3.

Figure 3 shows that the estimated model has a large width of possibility. Therefore, the estimated model is a fuzzy model. Numerically, the width of the possibility of the model is 12,500 yen on the average, 41,340 yen in the maximum and 1,770 yen at the minimum. The ambiguity of this model is extremely large.

However, the central value of the estimated value shows the value which is almost the same as the original series. Because it showed a largely oscillating tendency in the past, this results in the large width of the model estimation. This can be understood from the section of the error term of the model as well.

A result of sequential prediction of this model is shown in the Figure 4.

3.2 Fuzzy Autocorrelation Model Next, we analyze Nikkei stock average employing the fuzzy autocorrelation model which is proposed in this paper. In this model, we set the α-cut level 0.088. The fuzzy autocorrelation showed minus correlation in lag 2 similar to the case of the autocorrelation.

In the estimated fuzzy autocorrelation model, the coefficient was determined as follows:

$$\begin{aligned} \triangledown^2 \triangledown_{12} \tilde{\mathbf{Z}}_t &= (-0.834, -0.642, -0.000) \triangledown^2 \triangledown_{12} \mathbf{Z}_{t-1} \\ &\quad +(-1.00, -0.380, -0.380) \triangledown^2 \triangledown_{12} \mathbf{Z}_{t-2} \end{aligned}$$

The model which is obtained by the fuzzy autocorrelation model has a negative coefficient the same as the result which is obtained by the fuzzy autoregressive model.

Original series and estimated series are shown in the Figure 6.

As shown in Figure 6, the estimated model has small width and has brought the small fuzziness. Numerically, the width of the possibility of the model is 2,500 yen on the average, 18,000 yen in the maximum and 100 yen at the minimum. In Figure 6, the width of a model is smaller than that in Figure 3.

A result obtained by sequential prediction of this model is shown in the Figure 7.

There is a point which is the predicted value which differs from the original series by the figure 7. Since the fuzzy autocorrelation model places the stress on the fluctuation of a system unlike the fuzzy autoregressive model which includes all possibility of the system, produced this model has large error.

4 Conclusion Let us compare the results between a fuzzy autoregressive model and a fuzzy autocorrelation model employing triangular fuzzy number which are illustrated in Figures 3 and 6. These figures show that the fuzzy autoregressive model is estimated as the model which has a large width of possibility and the fuzzy autocorrelation is estimated as the model which has a small width. An ambiguity of fuzzy autocorrelation model is little, and this model estimated the original series more correctly. However, the fuzzy autoregressive model includes all fluctuation of an original series by estimating its fluctuation with a little large value.

Let us compare the results of sequential prediction which are illustrated in Figures 4 and 7. Though the width of the possibility of fuzzy autoregressive model was larger, it should be predicted that estimated central value corresponds actual measurement. In the case of the fuzzy autocorrelation model, the estimated value is different from the original series at three points. Since fuzzy autocorrelation has described fluctuation of a system, this is considered to be the cause which an error produces.

The return and the amount of funds for an investor, a company and so on, are greatly influenced by Nikkei stock average. Therefore, if a prediction will be greatly different from a reality, it becomes a large damage, especially in the case of a company, it must reduce the business or lay off company employees. Sometimes, the Japanese economy may inactive, because of the influence of a wrong prediction. Therefore, the less such risk, the more favorable it becomes for the company and the investor when they consider about the future return, the future prospects, and so on. On the other hand, the monthly fluctuation of real

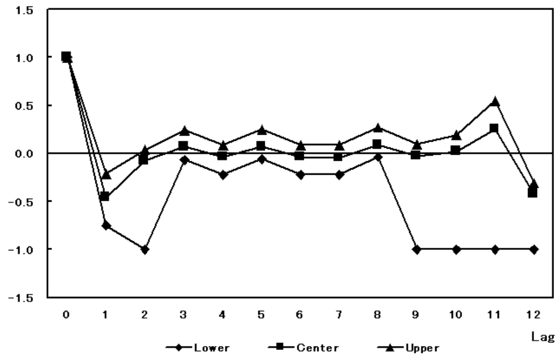

Figure 5: Fuzzy Autocorrelation

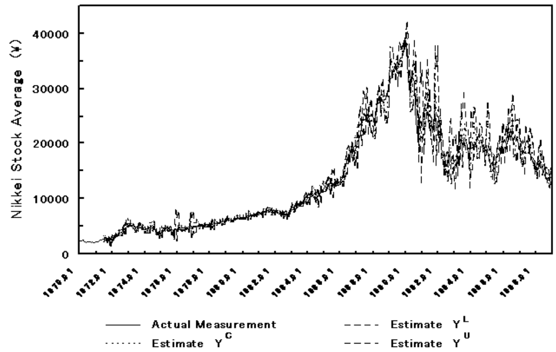

Figure 6: The conjectured result by Fuzzy Autocorrelation Model

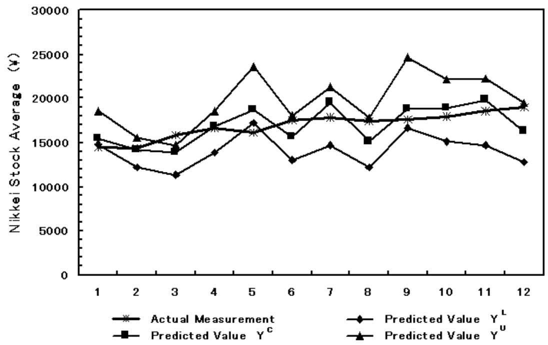

Figure 7: The prediction result in 1999 by Fuzzy Autocorrelation Model

Nikkei stock average is almost less than 10,000 yen. Therefore, for these reasons, we could regard a fuzzy autocorrelation model as the suited for estimation and prediction of Nikkei stock average.

In the case of the fuzzy autocorrelation model, we should not include all economic data within the model, but we should construct the model employing the fuzzy autocorrelation in order to include the possibility of the fluctuation of data. We could show the effectiveness of the economic analysis using Nikkei stock average by the fuzzy autocorrelation model, because the proposed model could illustrate the fluctuation of the system.

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