Scientiae Mathematicae Japonicae Online, **Vol. 10**, (2004), 423–429

FIXED POINT THEOREMS FOR MULTIVALUED CONTRACTIONS OF HICKS TYPE IN PROBABILISTIC METRIC SPACES

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Received December 11, 2002; revised September 26, 2003

ABSTRACT. A probabilistic semi-metric space (S, F) is said to be *of class* $\mathcal{H}$ ([5]) if there exists a metric d on S such that, for $t > 0$,

$$d(p, q) < t \Leftrightarrow F_{pq}(t) > 1 - t.$$

We will prove that (S, F) is of class $\mathcal{H}$ iff the mapping $\mathbf{K}$, defined on $S \times S$ by $\mathbf{K}(p, q) = \sup\{t \geq 0 \mid t \leq 1 - F_{pq}(t)\}$ is a metric on S. Two fixed point theorems for multivalued contractions in probabilistic metric spaces are also proved. Incidentally, the equality of two well-known probabilistic metrics is obtained.

1 Preliminaries. In this section we recall some notions of probabilistic metric spaces theory that will be used in the sequel. For more details on this topic we refer the reader to the books [2] and [13].

The class of *distribution functions,* denoted by Δ_+, is the class of all functions $F : [0, \infty) \to [0, 1]$ with the properties:

a) $F(0) = 0$;
b) F is increasing;
c) F is left continuous on $(0, \infty)$.

A special element of Δ_+ is the function ε_0, defined by

$$\varepsilon_0(t) = \begin{cases} 0, & \text{if} \quad t = 0 \\ 1, & \text{if} \quad t > 0 \end{cases}.$$

If X is a nonempty set, a mapping $F : X \times X \longrightarrow \Delta_+$ is called a *probabilistic distance on* X and $F(x, y)$ is denoted by F_{xy}.

Let X be a nonempty set and F be a probabilistic distance. The pair (X, F) is called *a PSM space* if the following axioms $(PM0)$ and $(PM1)$ are satisfied:

$$(PM0) \quad : \quad F_{xy} = \varepsilon_0 \text{ iff } x = y$$
$$(PM1) \quad : \quad F_{xy} = F_{yx} \ \forall x, y \in X.$$

A *Menger space under the t-norm* T ([13]) is a triple (X, F, T) where (X, F) is a PSM space and the triangle inequality

$$(PM2_M) \quad F_{xy}(t + s) \geq T(F_{xz}(t), F_{zy}(s)), \ \forall x, y, z \in X, \forall t, s > 0$$

holds.

2000 *Mathematics Subject Classification.* 54E70, 54H25.
Key words and phrases. Menger space; Probabilistic contraction; Multivalued mapping.

It is well known ([4], [13]) that if (X, F, T) is a Menger space with $\sup_{a<1} T(a,a) = 1$ then the family $\{U_\delta\}_{\delta>0}$ where

$$U_\delta = \{(x,y) \in X \times X,\ F_{xy}(\delta) > 1 - \delta\}$$

is a base for a metrizable uniformity on X, called the *F-uniformity.* Note ([8]) that $\sup_{a<1} T(a,a) = 1$ is the weakest condition which ensures the existence of the F-uniformity in Menger spaces (X, F, T).

Actually, for the existence of the F-uniformity one can consider even a more general statement for the triangle inequality. This condition was suggested by *Hicks* and *Sharma* ([4]), who defined the notion of *H-space* as a PSM space $(S,\ F)$ that satisfies the triangle inequality

$$(PM3_H)\ \forall \varepsilon > 0\ \exists \delta > 0 : (F_{pr}(\delta) > 1 - \delta,\ F_{rq}(\delta) > 1 - \delta) \Rightarrow F_{pq}(\varepsilon) > 1 - \varepsilon$$

(every Menger space (X, F, T) with $\sup\limits_{a<1} T(a,a) = 1$ is an H-space).

The F-uniformity naturally determines a metrizable topology on X, called *the strong topology* or *the F-topology.* In this paper all topological notions refer to the F-topology.

We also recall that the *Łukasiewicz t-norm* T_L is defined by

$$T_L(a,b) = Max(a + b - 1, 0).$$

In the following we point out some known results regarding PSM spaces of class $\mathcal{H}$.

Definition 1.1. ([3], [5]) Let (S, F) be a PSM space. We say that the mapping $d : S \times S \to [0, \infty)$ *is compatible with* F (or *F-compatible*) if the following relation holds

$$d(p,q) < t \Leftrightarrow F_{pq}(t) > 1 - t,$$

where $t > 0$. We say that *the probabilistic semi-metric space* (S, F) *is of class* $\mathcal{H}$ if there exists a metric on S compatible with F.

Theorem 1.2. ([5]) *Let* (X, F) *be a PSM space with the property :*

$$(H) \quad (F_{xy}(\varepsilon) > 1 - \varepsilon,\ F_{yz}(\delta) > 1 - \delta) \Rightarrow F_{xz}(\varepsilon + \delta) > 1 - (\varepsilon + \delta).$$

If d *is the mapping defined on* $X \times X$ *by*

$$d(x,y) = \begin{cases} 0, \text{ if } y \in N_x(\varepsilon, \varepsilon) & \forall \varepsilon > 0 \\ \sup\{\varepsilon : y \notin N_x(\varepsilon, \varepsilon), & 0 < \varepsilon < 1\} \end{cases}$$

where $N_x(\varepsilon, \lambda) = \{y \mid F_{xy}(\varepsilon) > 1 - \lambda\}$, *then*

(i) *d is a metric on S which is compatible with F;*

(ii) *Menger spaces (S, F, T) with $T \geq T_L$ verify the condition (H).*

2 Main results.

In the first part of our paper we focus on the notion of PSM space of class $\mathcal{H}$.

Proposition 2.1. *If (S, F) is a PSM space and d_1, d_2 are F-compatible mappings then $d_1 = d_2$.*

Proof. Let $p, q \in S$ be fixed. We have $d_1(p,q) < \varepsilon \Leftrightarrow F_{pq}(\varepsilon) > 1 - \varepsilon \Leftrightarrow d_2(p,q) < \varepsilon$. Thus, for every $\varepsilon > 0$, $d_1(p,q) < \varepsilon \Leftrightarrow d_2(p,q) < \varepsilon$. This shows us that $d_1(p,q) = d_2(p,q)$.

Corollary 2.2. *Let (S, F) be a probabilistic semi-metric space of class $\mathcal{H}$ and ρ be an F-compatible mapping. Then ρ is a metric on S.*

Indeed, there exists an F-compatible metric d on S and, by Proposition 2.1, $\rho = d$.

Corollary 2.3. ([15, Theorem 2.1]) *Let (S, F, T) be a Menger space with $T \geq T_L$. Then the mapping β defined on $S \times S$ by $\beta(p, q) = d_L(F_{pq}, \varepsilon_0)$, where d_L is the modified Lévy distance* (see [13]), *is a metric on S* (named *the metric of Schweizer* & *Sklar).*

Indeed, from Theorem 1.2 it follows that (S, F) is of class $\mathcal{H}$. On the other hand, it is well known ([13], [15]) that β is F-compatible and we can apply Corollary 2.2.

The following corollary is a converse of Theorem 1.2 ii).

Corollary 2.4. *If every Menger space (S, F, T) under the t-norm T is of class $\mathcal{H}$ then $T \geq T_L$.*

Proof. We will show that if T is a t-norm with the property that there exist $u, v \in [0, 1]$ such that $T(u, v) < T_L(u, v)$, then there exists a Menger space under the t-norm T which is not of class $\mathcal{H}$.

Let T, u and v be like above. From [18, Th.1] it follows that we can find a Menger space (S, F, T), positive numbers t, s and $p, q, r \in S$ such that $F_{pq}(t) = u, F_{qr}(s) = v$ and $F_{pr}(t + s) = T(u, v)$. We will prove that the mapping β defined in Corollary 2.3 is not a metric on S.

Indeed, we have $\beta(p, q) = 1 - u$ and $\beta(q, r) = 1 - v$, wherefrom $\beta(p, q)+ \beta(q, r) = 2 - (u + v)$. On the other hand, $\beta(p, r) = 1 - T(u, v) > 1 - T_L(u, v) \geq 2 - u - v$, so β does not verify the triangle inequality.

Corollary 2.5. *(S, F) is of class $\mathcal{H}$ iff the mapping $\mathbf{K}$, defined on $S \times S$ by $\mathbf{K}(p, q) = \sup\{t \geq 0 \mid t \leq 1 - F_{pq}(t)\}$ is a metric on S* (named *the Ky-Fan metric*).

Proof. It is known (see e.g. [5], [12]) that the mapping $\mathbf{K}$ is F-compatible.

Remark 2.6. Let (S, F, T) be a Menger space with $T \geq T_L$. Since $\mathbf{K}$ and β are F-compatible, we deduce that the metric $\mathbf{K}$ of *Ky-Fan* coincides with the metric of *Schweizer* & *Sklar.*

In the second part of our paper we make some considerations on the probabilistic contractions in PM spaces of class $\mathcal{H}$.

In the following $P(S)$ denotes the class of all nonempty subsets of S. If (S, F) is an H-space, by $P_{cl}(S)$ we will denote the class of all nonempty closed (in the F-topology) subsets of S.

As *Hicks* remarked in [5] and [6], the most usual topological properties of a PM space (S, F) of class $\mathcal{H}$ and some contractivity conditions can be translated in the metric space $(S, \mathbf{K})$. As a consequence, many fixed point theorems for probabilistic contractions in PM spaces of class $\mathcal{H}$ can be obtained from their metric counterpart in $(S, \mathbf{K})$.

However, a PM space (S, F) of class $\mathcal{H}$ has a richer structure than its "metric analogue" $(S, \mathbf{K})$, given by the probabilistic metric F. This fact allows us to weaken some conditions on the mapping by imposing, as a counterbalance, some conditions on the probabilistic metric. The other important difference between probabilistic metric spaces and metric spaces is given by the "triangle inequality" and we could try to work in PM spaces (S, F) which are not of class $\mathcal{H}$. Theorem 2.10 and Theorem 2.12 below improve a result obtained by *Hicks'* scheme in the two above mentioned directions.

The following Theorem 2.7, concerning $\overline{H}$-contractions defined (see[6, Def.1]) by

$$(\overline{H})\ \ \forall p \in f(x) \exists q \in f(y) : \quad u > 0, F_{xy}(u) > 1 - u \Longrightarrow F_{pq}(ku) > 1 - ku$$

can be obtained, using Hicks' procedure, from [6, Lemma 2] and the well-known *Avramescu-Markin-Nadler theorem* (see e.g.[9]):

" *If* (X, d) *is a complete metric space, then the mapping* $f : X \to P_{b,cl}(X)$ *with the property*

$$\exists k \in (0,1): \ \mathbf{H}(fx, fy) \leq kd(x,y), \ \forall x, y \in X$$

where $\mathbf{H}(A,B) = max\{\sup_{a\in A}\inf_{b\in B} d(a,b), \sup_{b\in B}\inf_{a\in A} d(a,b)\}$ *is the Hausdorff-Pompeiu metric, has a fixed point.*"

Theorem 2.7. *Let* (S,F) *be a complete PM space of class* $\mathcal{H}$ *and* $f : S \to P_{cl}(S)$ *(the family of all nonempty closed subsets of* S*) be an* $\overline{H}$*-contraction. Then there exists* $x \in S$ *such that* $x \in f(x)$.

Let us remark that, because $F_{xy}(t) > 1 - t$ for all $t > 1$, the contractivity relation $(\overline{H})$ imposes to verify that $\forall x \in S, \forall p \in f(x), \exists q \in f(y) : F_{pq}(s) > 1 - s, \forall s \in (k,1)$. We will see that this condition can be replaced by a more convenient one, namely "$F_{pq}(1) > 0$ for some $p \in S$ and $q \in f(p)$ ".

It is also worth noting that if d is F-compatible then $d(p,q) \leq 1 \ \forall p, q \in S$, for $F_{pq}(t) > 1 - t \ \ \forall t > 1$. Therefore, if (S,F) is of class $\mathcal{H}$ then every nonempty subset of X is **K**-bounded.

Definition 2.8. Let S be a nonempty set and F be a probabilistic distance on S. The mapping $f : S \to P(S)$ is called a *multivalued weak-Hicks contraction* (shortly *w*-$\overline{H}$ *contraction)* if there exists $k \in (0,1)$ such that the following implication holds for all $x, y \in S$:

$$(w\text{-}\overline{H}) \ \ \forall \ p \in f(x) \ \exists q \in f(y): \ u \in (0,1), \, F_{xy}(u) > 1 - u \Longrightarrow F_{pq}(ku) > 1 - ku.$$

Note that $(w\text{-}\overline{H})$ implies $\mathbf{H}(f(x), f(y)) \leq k\mathbf{K}(x,y), \forall x, y \in S : \mathbf{K}(x,y) < 1$.

Indeed, let $x, y \in S$ be such that $\mathbf{K}(x,y) < 1$. Choose an arbitrary ε, $\mathbf{K}(x,y) < \varepsilon < 1$. Then $F_{xy}(\varepsilon) > 1 - \varepsilon > 0$, so for every $p \in f(x)$ there exists $q \in f(y)$ such that

$$F_{pq}(k\varepsilon) > 1 - k\varepsilon.$$

This means that $\mathbf{K}(p,q) \leq k\varepsilon$, wherefrom we deduce that

$$\sup_{p\in f(x)} \inf_{q\in f(y)} \mathbf{K}(p,q) \leq k\varepsilon$$

and, by symmetry,

$$\sup_{p\in f(y)} \inf_{q\in f(x)} \mathbf{K}(p,q) \leq k\varepsilon.$$

It follows that

$$\mathbf{H}(f(x), f(y)) \leq k\varepsilon$$

therefore

$$\mathbf{H}(f(x), f(y)) \leq k\mathbf{K}(x,y).$$

So, $\mathbf{K}(x,y) < 1 \Longrightarrow \mathbf{H}(f(x), f(y)) \leq k\mathbf{K}(x,y)$.

In the proof of Theorem 2.10 we need the following

Lemma 2.9. ([6,Theorem 4]) *Let (X, d) be a complete metric space. Suppose that $T : (X, d) \to (P_{b,cl}(X), \mathbf{H})$ is continuous. Then there exists $p \in X$ with $p \in T(p)$ iff there exists a sequence $\{x_n\}_{n\in N}$ in X such that*

$$x_{n+1} \in T(x_n) \text{ and } \sum_{n+1}^{\infty} d(x_n, x_{n+1}) < \infty.$$

Theorem 2.10. *Let (S, F) be a complete PM space of class $\mathcal{H}$ and $f : S \to P_{cl}(S)$ be a w-$\overline{H}$ contraction. If there exist $p \in S$ and $q \in f(p)$ with $F_{pq}(1) > 0$ then there exists $x \in S$ such that $x \in f(x)$.*

Proof. Since (S, F) is of class $\mathcal{H}$, $\mathbf{K}$ is a metric on S and $\mathbf{K}(p, q) < t \iff F_{pq}(t) > 1 - t$. Therefore (S, F) is complete iff $(S, \mathbf{K})$ is complete and a set $A \subset M$ is closed in the F-topology iff it is closed in the $\mathbf{K}$-topology. Since $\mathbf{K}$ is a bounded metric, it follows that $P_{b,cl}(S) = P_{cl}(S)$.

Next, if $\varepsilon > 0$ is given, by choosing $\delta = \min\{\varepsilon/k, 1\}$, from the remark after Definition 2.8 it follows that $\mathbf{K}(x, y) < \delta \Longrightarrow \mathbf{H}(f(x), f(y)) < \varepsilon$. Therefore f is continuous.

Let us now construct a sequence $\{p_n\}_{n\in N}$ like in the statement of the lemma.

We write the contractivity relation as

$$(k - \overline{C})\ \forall p \in f(x) \exists q \in f(y) : \quad u \in (0, 1),\ \mathbf{K}(x, y) < u \Longrightarrow \mathbf{K}(p, q) < ku$$

and define $p_0 = p$ and $p_1 = q$. Since $F_{pq}(1) > 0$, it follows that there exists $\delta \in (0, 1)$ such that $\mathbf{K}(x, y) < \delta$ (if we suppose that $F_{p_0p_1}(\delta) \le 1 - \delta$ for all $\delta \in (0, 1)$ then by the left continuity of $F_{p_0p_1}$ we deduce that $F_{p_0p_1}(1) \le 0$, which is a contradiction).

Next, using (k-$\overline{C}$) we can find $p_2 \in f(p_1)$ such that $\mathbf{K}(p_1, p_2) < k\delta < k$, and, by induction, p_n such that $p_n \in f(p_{n-1})$ and $\mathbf{K}(p_{n-1}, p_n) < k^{n-1}$ for all $n \ge 2$. Thus $\{p_n\}$ satisfies $p_n \in f(p_{n-1})$ for all n and $\sum_{n=1}^{\infty} \mathbf{K}(p_n, p_{n+1}) < \sum_{n=1}^{\infty} k^n < \infty$.

Therefore, we can apply Lemma 2.9 to obtain a fixed point of f.

In the following we will show that in the above theorem one can consider spaces which are not of class $\mathcal{H}$.

Definition 2.11. Let b be the class of all strictly increasing sequences $(b_n)_{n\in\mathbf{N}}$, with $\lim_{n\to\infty} b_n = 1$. If $(b_n) \in b$, by a (b_n)-*t-norm of Hadžić type* we will understand a t-norm T with $T(b_n, b_n) = b_n\ \forall n$.

Let $(b_n) \in b$. We say that the PSM space (S, F) *is a (b_n)-probabilistic metric structure* (shortly (b_n)-*probabilistic structure*) if the following relation takes place:

$$(PM3_b)\ \ n \in N,\ F_{pq}(s) \ge b_n,\ F_{qr}(t) \ge b_n \Rightarrow F_{pr}(s + t) \ge b_n.$$

It is easy to verify that every (b_n)-probabilistic metric structure is an H-space. Indeed, if $\varepsilon = \varepsilon_1 + \varepsilon_2$ ($\varepsilon_1, \varepsilon_2 > 0$) is given, by choosing m such that $b_m > 1-\varepsilon$ and $\delta = \min\{\varepsilon_1, \varepsilon_2, 1-b_m\}$ we have: $F_{pq}(\delta) > 1-\delta$, $F_{qr}(\delta) > 1-\delta \Rightarrow F_{pq}(\delta) > b_m$, $F_{qr}(\delta) > b_m \Longrightarrow F_{pq}(\varepsilon_1) > b_m$, $F_{qr}(\varepsilon_2) > b_m \Rightarrow F_{pr}(\varepsilon) > b_m \Rightarrow F_{pr}(\varepsilon) > 1 - \varepsilon$.

If T is a (b_n)-t-norm of Hadžić-type and (S, F, T) is a Menger space, then $F_{pq}(s) \ge b_n$, $F_{qr}(t) \ge b_n \Rightarrow F_{pr}(s + t) \ge b_n$, therefore every Menger space under a (b_n)-t-norm of Hadžić-type is a (b_n)-probabilistic metric structure.

On the other hand, it can easily be seen that (S, F) is of class $\mathcal{H}$ iff the condition

$$(H) \quad F_{xy}(\varepsilon) > 1 - \varepsilon,\ F_{yz}(\delta) > 1 - \delta \Rightarrow F_{xz}(\varepsilon + \delta) > 1 - (\varepsilon + \delta)$$

holds.

Therefore (see [2, Remark 1.69 and Proposition 1.70]), a (b_n)-probabilistic structure generally is not a PM space of class $\mathcal{H}$.

Theorem 2.12. *Let $f : S \to P_{cl}(S)$ be a $(w\text{-}\overline{H})$ contraction on a complete (b_n)- probabilistic structure (S, F). If $F_{pq}(1) > 0$ for some $p \in S$ and $q \in f(p)$ then there exists $x \in S$ such that $x \in f(x)$.*

Proof. As in the proof of Theorem 2.10, let us put $p_0 = p$ and $p_1 = q$. Since $F_{pq}(1) > 0$, it follows that there exists $\delta \in (0,1)$ such that $F_{p_0p_1}(\delta) > 1 - \delta$.

Next, using $(w\text{-}\overline{H})$ we find $p_2 \in f(p_1)$ such that $F_{p_1p_2}(k\delta) > 1 - k\delta$. It follows that $F_{p_1p_2}(k) > 1 - k$ and, by induction, we can obtain p_n such that $p_n \in f(p_{n-1})$ and $F_{p_{n-1}p_n}(k^{n-1}) > 1 - k^{n-1}$ for all $n \geq 2$.

We will show that $\{p_n\}_{n\in N}$ is a Cauchy sequence.

Let $\varepsilon > 0$ be given. Choose $m \in N$ such that $b_m > 1 - \varepsilon$ and $n_1 \in N$ such that $1 - k^{n-1} \geq b_m \; \forall n \geq n_1$.

Then, for all $n \geq n_1$ we have $F_{p_np_{n+1}}(k^n) \geq 1 - k^n \geq b_m$, wherefrom we obtain successively: $F_{p_np_{n+2}}(k^n + k^{n+1}) \geq b_m$, $F_{p_np_{n+3}}(k^n + k^{n+1} + k^{n+2}) \geq b_m$, and, by induction,

$$F_{p_np_{n+j}}\Big(\sum_{i=n}^{n+j-1} k^i\Big) \geq b_m \text{ for all } n \geq n_1 \text{ and } j \in N.$$

Since the series $\sum_{n=1}^{\infty} k^n$ is convergent, there exists $n_2 \in N$ such that $\sum_{n=n_2}^{\infty} k^n < \varepsilon$.

Then, for all $n \geq \max\{n_1, n_2\}$ and all $j \in N$ we have

$$F_{p_np_{n+j}}(\varepsilon) \geq F_{p_np_{n+j}}\Big(\sum_{i=n}^{n+j-1} k^i\Big) \geq b_m > 1 - \varepsilon.$$

So, $\{p_n\}$ is a Cauchy sequence. Since S is complete, there exists $x \in S$, $x = \lim_{n\longrightarrow\infty} p_n$.

We prove that $x \in f(x)$ $(= \overline{f(x)})$. We have to prove that for every $\varepsilon > 0$ there exists y $(= y(\varepsilon)) \in f(x)$ such that $F_{xy}(\varepsilon) > 1 - \varepsilon$.

Let $\varepsilon > 0$ be given. Since (S, F) is an H-space, we can find $\delta > 0$ such that $F_{pr}(\delta) > 1 - \delta, F_{rq}(\delta) > 1 - \delta \Rightarrow F_{pq}(\varepsilon) > 1 - \varepsilon$ and $\delta_1 > 0$ such that $F_{pr}(\delta_1) > 1 - \delta_1, F_{rq}(\delta_1) > 1 - \delta_1 \Rightarrow F_{pq}(\delta) > 1 - \delta$.

From $p_n \longrightarrow x$ it follows that there exists $n_1 = n_1(\delta_1) \in N$ with

$$F_{xp_n}(\delta_1) > 1 - \delta_1 \; \forall n \geq n_1.$$

Also, there exists $n_2 = n_2(\delta_1) \in N$ with

$$F_{p_np_{n+1}}(\delta_1) > 1 - \delta_1 \; \forall n \geq n_2.$$

Let $t_0 \in (0,1)$ be such that $kt_0 < \delta$. From $p_n \longrightarrow x$ it follows that there exists $s(= s(\delta)) \in N$, $s \geq \max\{n_1, n_2\}$ such that

$$F_{p_sx}(t_0) > 1 - t_0.$$

By $(w\text{-}\overline{H})$ we deduce that there exists $y \in f(x)$ such that

$$F_{p_{s+1}y}(\delta) > 1 - \delta.$$

Since $F_{xp_s}(\delta_1) > 1 - \delta_1$ and $F_{p_s p_{s+1}}(\delta_1) > 1 - \delta_1$, we also have $F_{xp_{s+1}}(\delta) > 1 - \delta$, therefore $F_{xy}(\varepsilon) > 1 - \varepsilon$.

The theorem is proved.

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