PRE-CONVERGENCE OF FILTERS AND $p\gamma$-CONTINUITY*

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Abstract. In this paper, we introduce a new class of sets, called $p\gamma$-sets, and the notion of $p\gamma$-continuity and investigate some properties and characterizations in terms of pre-convergence of filters. In particular, $p\gamma$-sets are used to extend known results for preopen sets and precontinuity.

1. INTRODUCTION.

Let X, Y and Z be topological spaces on which no separation axioms are assumed unless explicit stated. Let S be a subset of X. The closure (resp. interior) of S will be denoted by clS (resp. $intS$). A subset S of X is called *preopen* set [1] (resp. *semi-open* set [3], *α-set* [4]) if $S \subset int(cl(S))$ (resp. $S \subset cl(int(S))$, $S \subset int(cl(int(S)))$). The complement of a preopen set (resp. semi-open set, α-set) is called *preclosed* set (resp. *semi-closed* set, *α-closed* set). The family of all preopen sets (resp. semi-open sets, α-sets) in X will be denoted by $PO(X)$ (resp. $SO(X)$, $\alpha(X)$). A function $f : X \to Y$ is called *precontinuous* [5] (resp. *semi-continuous* [3], *α-continuous* [4]) if $f^{-1}(V) \in PO(X)$ (resp. $f^{-1}(V) \in SO(X)$, $f^{-1}(V) \in \alpha(X)$) for each open set V of Y. A function $f : X \to Y$ is called *preirresolute* [8] (resp. *α-irresolute* [8]) if $f^{-1}(V) \in PO(X)$ (resp. $f^{-1}(V) \in \alpha(X)$) for each preopen set (resp. α-set) V of Y. A function $f : X \to Y$ is called *preopen* [5] (resp. *α-open* [4]) if for every open set (resp. α-set) U in X, $f(U)$ is a preopen set (resp. α-set) in Y.

A subset $M(x)$ of a space X is called a *semi-neighborhood* (resp. *pre-neighborhood* [5]) of a point $x \in X$ if there exists a semi-open (resp. preopen) set S such that $x \in S \subset M(x)$. In [2], R. M. Latif introduced the notion of semi-convergence of filters. And he investigated some characterizations related to semi-open continuous functions. Now we recall the concept of semi-convergence of filters. Let $S(x) = \{A \in SO(X) : x \in A\}$ and let $S_x = \{A \subset X : \text{there exists } \mu \subset S(x) \text{ such that } \mu \text{ is finite and } \cap\mu \subset A\}$. Then S_x is called the *semi-neighborhood filter* at x. For any filter $\mathcal{F}$ on X, we say that $\mathcal{F}$ *semi-converges* to x if and only if $\mathcal{F}$ is finer than the semi-neighborhood filter at x.

2. $p\gamma$-sets.

In this section, we introduce the concepts of pre-convergence of filters, $p\gamma$-set, $p\gamma$-interior and $p\gamma$-closure of a set and $p\gamma$-compact. And we characterize them in terms of pre-convergence of filters.

Definition 2.1. Let (X, τ) be a topological space, $x \in X$, $P(x) = \{A \in PO(X) : x \in A\}$ and let $P_x = \{A \subset X : \text{there exists } \mu \subset P(x) \text{ such that } \mu \text{ is finite and } \cap\mu \subset A\}$. Then P_x is

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called the *pre-neighborhood filter* at x. For any filter $\mathcal{F}$ on X, we say that $\mathcal{F}$ *pre-converges* to x if and only if $\mathcal{F}$ is finer than the pre-neighborhood filter at x.

Definition 2.2. Let (X, τ) be a topological space. A subset A of X is called a *$p\gamma$-set* in X if for each x in A, there exists an element $U \in P_x$ such that $x \in U \subset A$. And a subset $M(x)$ of X is called a *$p\gamma$-neighborhood* of x in X if there exists a $p\gamma$-set S such that $x \in S \subset M(x)$.

The family of all $p\gamma$-sets in X will be denoted by $p\gamma(X)$. In particular, the family of all $p\gamma$-sets induced by the topology τ will be denoted by $p\gamma_\tau$.

Remark 2.3. From the definitions of pre-neighborhood filter and $p\gamma$-set, we can say every preopen set is a $p\gamma$-set, but the converse is always not true.

Example 2.4. Let X be the set of real numbers with the cofinite topology. For each $x \in Q$, since both Q and $Q^c \cup \{x\}$ are preopen sets containing x, $\{x\}$ is an element of P_x. Thus $\{x\}$ is a $p\gamma$-set but not preopen.

Remark 2.5. In a topological space (X, τ), we get the following relations:

$\tau \subset \alpha$ (X) $\subset$ PO(X) $\subset p\gamma$ (X).

Theorem 2.6. Let (X, τ) be a topological space. A subset U of X is a $p\gamma$-set in X if and only if whenever a filter $\mathcal{F}$ pre-converges to x and $x \in U$, $U \in \mathcal{F}$.

Proof. $\Leftarrow$) For each $x \in U$, the pre-neighborhood filter P_x pre-converges to x. From the condition, we get $U \in P_x$. Thus U is a $p\gamma$-set.
$\Rightarrow$) It follows from the definitions of $p\gamma$-sets and pre-convergence.

Theorem 2.7. Let (X, τ) be a topological space. The intersection of finitely many preopen subsets in X is a $p\gamma$-set.

Proof. Let U_1 and U_2 be preopen sets in X and suppose $U_1 \cap U_2 \neq \emptyset$. For each $x \in U_1 \cap U_2$, we get $U_1 \cap U_2 \in P_x$. Hence $U_1 \cap U_2$ is a $p\gamma$-set.

Theorem 2.8. Let (X, τ) be a topological space. Then the family $p\gamma(X)$ of all $p\gamma$-sets in X is a topology on X.

Proof. Since $\emptyset$ and X are preopen, they are also $p\gamma$-sets in X.
Let $A, B \in p\gamma(X)$, $x \in A \cap B$ and let $\mathcal{F}$ be a filter pre-converging to x. From Theorem 2.6, it follows $A, B \in \mathcal{F}$ and since $\mathcal{F}$ is a filter, we have $A \cap B \in \mathcal{F}$. It follows that $A \cap B$ is a $p\gamma$-set from Theorem 2.6.
For each $\alpha \in I$, $A_\alpha \in p\gamma(X)$ and $U = \cup A_\alpha$. Let $x \in U$ and $\mathcal{F}$ be a filter pre-converging to x. Then there exists an A_α such that $x \in A_\alpha$ and since A_α is a $p\gamma$-set, we can say $A_\alpha \in \mathcal{F}$. Also U is an element of the filter $\mathcal{F}$, and thus $U = \cup A_\alpha$ is a $p\gamma$-set.

Definition 2.9. Let (X, τ) be a topological space and $A \subset X$. The *$p\gamma$-interior* of A in X, denoted by $int_p(A)$, is the union of all $p\gamma$-sets contained in A.

Theorem 2.10. Let (X, τ) be a topological space and $A \subset X$.
(a) $int_p(A) = \{x \in A : A \in P_x\}$.

(b) A is a $p\gamma$-set if and only if $A = int_p(A)$.

Proof. (a) For each $x \in int_p(A)$, there exists a $p\gamma$-set U such that $x \in U \subset A$. From the definition of $p\gamma$-set, the subset U is in the pre-neighborhood filter P_x. Thus we get $A \in P_x$. Conversely, let $x \in A$ and $A \in P_x$. Then by Definition 2.1, there exist $U_1, \cdots, U_n \in P(x)$ such that $U = U_1 \cap \cdots \cap U_n \subset A$. By Theorem 2.7, U is a $p\gamma$-set and $x \in U \subset A$. Thus we get $x \in int_p(A)$.
(b) Obvious.

In a topological space (X, τ), the family of all $p\gamma$-sets induced by the topology τ will be denoted by $(X, p\gamma_\tau)$. A subset B of X is called a *$p\gamma$-closed set* if the complement of B is a $p\gamma$-set.

Definition 2.11. Let (X, τ) be a topological space and $A \subset X$. A point x in X is called a *$p\gamma$-cluster point* of A if $(U - \{x\}) \cap A \neq \emptyset$, for every $p\gamma$-set U containing x. The set $d_p(A)$ of all $p\gamma$-cluster points of A is called the *$p\gamma$-derived set* of A.

Theorem 2.12. Let (X, τ) be a topological space and $A \subset X$. A point x in X is a $p\gamma$-cluster point of A if and only if $(U - \{x\}) \cap A \neq \emptyset$, for all $U \in P_x$.

Proof. Obvious.

Definition 2.13. Let (X, τ) be a topological space and $A \subset X$. The *$p\gamma$-closure* of A in X, denoted by $cl_p(A)$, is the intersection of all $p\gamma$-closed sets containing A.

We can get the following theorem from Definition 2.9 and Definition 2.13.
Theorem 2.14. Let (X, τ) be a topological space and $A \subset X$.
(1) $A \subset cl_p(A)$.
(2) A is $p\gamma$-closed if and only if $A = cl_p A$.
(3) $int_p(A) = X - cl_p(X - A)$.
(4) $cl_p(A) = X - int_p(X - A)$.

Theorem 2.15. Let (X, τ) be a topological space and $A \subset X$.
Then $cl_p(A) = A \cup d_p(A)$.

Proof. It follows from Definition 2.13.

Theorem 2.16. Let (X, τ) be a topological space and $A \subset X$.

$$cl_p(A) = \{x \in X : A \cap U \neq \emptyset \text{ for all } U \in P_x\}.$$

Proof. It follows from Theorem 2.12 and Theorem 2.15.

Theorem 2.17. Let (X, τ) be a topological space. A subset A in X is $p\gamma$-closed if and only if whenever a filter $\mathcal{F}$ on X pre-converges to x and $A \in \mathcal{F}$, $x \in A$.

Proof. $\Rightarrow$) Let A be a $p\gamma$-set, $A \in \mathcal{F}$ and let $\mathcal{F}$ be a filter finer than P_x. Then by Theorem 2.16, we have $x \in cl_p(A)$. And it follows $x \in A$ from Theorem 2.14(2).
$\Leftarrow$) Let $x \in cl_p(A)$. Then we can say that $A \cap U \neq \emptyset$ for all $U \in P_x$. Let $\mathcal{F}$ be a filter generated by the filter base $\{A \cap U : U \in P_x\}$. Then $\mathcal{F}$ pre-converges to x and $A \in \mathcal{F}$, and

so by the condition $x \in A$ and we have $cl_p(A) = A$.

Definition 2.18. Let (X, τ) be a topological space, $x \in X$ and let $\mathcal{F}$ be a filter on X. We say that $\mathcal{F}$ has a *$p\gamma$-cluster point* x if each $F \in \mathcal{F}$ meets each $U \in P_x$.

Theorem 2.19. Let (X, τ) be a topological space, $x \in X$ and let $\mathcal{F}$ be a filter on X. Then $\mathcal{F}$ has a $p\gamma$-cluster point x if and only if $x \in \cap\{cl_pF : F \in \mathcal{F}\}$.

Proof. It is obvious from Theorem 2.16 and Definition 2.18.

Theorem 2.20. Let (X, τ) be a topological space, and let $\mathcal{F}$ be a filter on X. $\mathcal{F}$ has x as a $p\gamma$-cluster point if and only if there is a filter $\mathcal{G}$ finer than $\mathcal{F}$ which pre-converges to x.

Proof. $\Rightarrow$) Let x be a $p\gamma$-cluster point of $\mathcal{F}$. Then from Definition 2.18, the collection $G = \{U \cap F : U \in P_x, F \in \mathcal{F}\}$ is a filter base for a filter $\mathcal{G}$ which is finer than $\mathcal{F}$ and pre-converges to x.
$\Leftarrow$) It is obvious from the definition of pre-convergence of filters.

In a topological space X, a family $\mathcal{C}$ of $p\gamma$-sets in X will be called a *$p\gamma$-cover* of X if $\mathcal{C}$ covers X.

Definition 2.21. A topological space X is said to be *$p\gamma$-compact* if each $p\gamma$-cover of X has a finite subcover.

Theorem 2.22. For topological space (X, τ), the following are equivalent:
a) X is $p\gamma$-compact,
b) each family $\mathcal{C}$ of $p\gamma$-closed sets in X with the finite intersection property has nonempty intersection,
c) each filter in X has a $p\gamma$-cluster point.

Proof. a) $\Rightarrow$ b) and b) $\Rightarrow$ c) are clear.
c) $\Rightarrow$ a): Suppose $\mathcal{C}$ is a $p\gamma$-cover of X with no finite subcover. Then $X - (U_1 \cup \cdots \cup U_n) \neq \emptyset$ for each finite collection $\{U_1, \cdots, U_n\}$ from $\mathcal{C}$. Thus we get a filter base $\{X - (U_1 \cup \cdots \cup U_n) : \{U_1, \cdots, U_n\} \subset \mathcal{C}\}$, generating a filter $\mathcal{F}$. Then by c), $\mathcal{F}$ has a $p\gamma$-cluster point, say to x. Then x is contained in a member U of $\mathcal{C}$ and $U \in P_x$. For $X - U \in \mathcal{F}$, it is impossible the intersection of $(X - U)$ and U is not empty, thus we have a contradiction. Consequently $\mathcal{C}$ must have a finite subcover.

3. $p\gamma$-continuous and $p\gamma$-irresolute functions

In this section, we introduce the concepts of $p\gamma$-continuous, $p\gamma$-irresolute and $p\gamma$-open functions and characterize their properties in terms of pre-convergence of filters.

Definition 3.1. Let (X, τ) and (Y, μ) be topological spaces. A function $f : X \to Y$ is said to be *$p\gamma$-continuous* if the inverse image of each open set of Y is a $p\gamma$-set in X.

Since the family of all $p\gamma$-sets in a given topological space is a topology, we get the following equivalent statements.

Theorem 3.2. Let (X,τ) and (Y,μ) be topological spaces. If $f:(X,\tau)\to(Y,\mu)$ is a function, then the following statements are equivalent:
(1) f is $p\gamma$-continuous.
(2) The inverse image of each closed set in Y is $p\gamma$-closed.
(3) $cl_p(f^{-1}(B))\subset f^{-1}(cl(B))$ for every $B\subset Y$.
(4) $f(cl_p(A))\subset cl(f(A))$ for every $A\subset X$.
(5) $f^{-1}(int(B))\subset int_p(f^{-1}(B))$ for every $B\subset Y$.

Let (X,τ) and (Y,μ) be topological spaces. We say that a function $f:X\to Y$ is *$p\gamma$-continuous* at x if for each neighborhood U of $f(x)$ in Y, there exists a $p\gamma$-neighborhood V of x in X such that $f(V)\subset U$.

Theorem 3.3. Let $f:(X,\tau)\to(Y,\mu)$ be a function between topological spaces. Then the following are equivalent:
(1) f is $p\gamma$-continuous at x.
(2) If a filter $\mathcal{F}$ pre-converges to x, then $f(\mathcal{F})$ converges to $f(x)$.

Proof. (1) $\Rightarrow$ (2): Let U be a neighborhood of $f(x)$ in Y. Then by (1), $f^{-1}(U)$ is an element in P_x. Since $\mathcal{F}$ pre-converges to x and $f(\mathcal{F})$ is a filter, $U\in f(\mathcal{F})$. Consequently, $f(\mathcal{F})$ converges to $f(x)$.
(2) $\Rightarrow$ (1): Let U be a neighborhood of $f(x)$. Since P_x pre-converges to x, $f(P_x)$ converges to $f(x)$, and so $U\in f(P_x)$. Thus there is a $p\gamma$-neighborhood $V\in P_x$ such that $f(V)\subset U$.

Corollary 3.4. Let $f:(X,\tau)\to(Y,\mu)$ be a precontinuous function and $x\in X$. If a filter $\mathcal{F}$ pre-converges to x, then $f(\mathcal{F})$ converges to $f(x)$ in Y.

Proof. It is obvious by using Theorem 3.2 and Theorem 3.3.

The following example shows that the converse of Corollary 3.4. may not be true.

Example 3.5. Consider the set R of real numbers with the cofinite topology. We define $f:R\to R$ by $f(x)=0$, if $x=0$ and otherwise, $f(x)=1$. Clearly a filter $\mathcal{F}$ pre-converges to x if and only if $\mathcal{F}$ is finer than $\dot{x}$. Thus $\dot{f}(x)\subset f(\mathcal{F})$, and $f(\mathcal{F})$ converges to $f(x)$. For an open set $U=R-\{1\}$, $f^{-1}(U)=\{0\}$. But $\{0\}$ is not preopen in R, and so f is not pre-continuous.

Definition 3.6. Let (X,τ) and (Y,μ) be topological spaces. A function $f:X\to Y$ is said to be *$p\gamma$-irresolute* if the inverse image of each $p\gamma$-set of Y is a $p\gamma$-set in X.

The following are obtained by Definition 3.6.
Theorem 3.7. Let $f:(X,\tau)\to(Y,\mu)$ be a function between topological spaces. Then the following are equivalent :
(1) f is $p\gamma$-irresolute.
(2) The inverse image of each $p\gamma$-closed set in Y is a $p\gamma$-closed set.
(3) $cl_{p\tau}(f^{-1}(V))\subset f^{-1}(cl_{p\mu}(V))$, for every $V\subset Y$.
(4) $f(cl_{p\tau}(U))\subset cl_{p\mu}(f(U))$, for every $U\subset X$.
(5) $f^{-1}(int_{p\mu}(B))\subset int_{p\tau}(f^{-1}(B))$, for every $B\subset Y$.

Theorem 3.8. Let $f : (X, \tau) \to (Y, \mu)$ be a function between topological spaces. Then the following are equivalent:
(1) f is $p\gamma$-irresolute.
(2) For $x \in X$ and for each $V \in P_{f(x)}$, there exists an element U in the pre-neighborhood filter P_x such that $f(U) \subset V$.
(3) For each $x \in X$, if a filter $\mathcal{F}$ pre-converges to x, then $f(\mathcal{F})$ pre-converges to $f(x)$ in Y.

Proof. (1) $\Rightarrow$ (2): It is obvious.
(2) $\Rightarrow$ (3): Let V be an element of the pre-neighborhood filter $P_{f(x)}$ and let $\mathcal{F}$ be a filter on X pre-converging to x. Then by 2), there exists an element U in the pre-neighborhood filter P_x such that $f(U) \subset V$. Since U is an element in P_x and $f(\mathcal{F})$ is a filter finer than $f(P_x)$, we can say $V \in f(\mathcal{F})$. Consequently $f(\mathcal{F})$ pre-converges to $f(x)$.
(3) $\Rightarrow$(1): Let V be a $p\gamma$-set in Y. Suppose $f^{-1}(V)$ is not empty. For each $x \in f^{-1}(V)$, since the pre-neighborhood filter P_x pre-converges to x, by 3) we can say $f(P_x)$ pre-converges to $f(x)$. And since V is a $p\gamma$-set containing $f(x)$ and $P_{f(x)} \subset f(P_x)$, $V \in f(P_x)$. Thus we can take some $p\gamma$-set U in P_x such that $f(U) \subset V$, and so $f^{-1}(V)$ is an element of P_x. Thus $f^{-1}(V)$ is a $p\gamma$-set in X by Theorem 2.10 (b).

Corollary 3.9. Let $f : (X, \tau) \to (Y, \mu)$ be a function. If f is preirresolute, then whenever a filter $\mathcal{F}$ pre-converges to x in X, $f(\mathcal{F})$ pre-converges to $f(x)$ in Y.

Proof. Let V be an element of the pre-neighborhood filter $P_{f(x)}$ and let $\mathcal{F}$ be a filter on X pre-converging to x. By Definition 2.1, there exist $U_1, \cdots, U_n \in P(f(x))$ such that $U{=}U_1 \cap \cdots \cap U_n \subset V$. From f is a preirresolute function, it follows $f^{-1}(V) \in P_x$. Since $f(\mathcal{F})$ is a filter finer than $f(P_x)$, $V \in f(\mathcal{F})$, so that $f(\mathcal{F})$ pre-converges to $f(x)$.

Remark. 3.10. We can get the following diagrams :

(1) continuity $\Longrightarrow$ α-continuity $\Longrightarrow$ pre-continuity $\Longrightarrow$ $p\gamma$-continuity.

(2) preirresolute $\Longrightarrow$ $p\gamma$-irresolute.

From Example 3.5, we can show the converse in (2) need not be true. And non of the implications in the diagram (1) above is reversible.

Theorem 3.11. a) The $p\gamma$-continuous image of a $p\gamma$-compact space is compact.
b) The $p\gamma$-irresolute image of a $p\gamma$-compact space is $p\gamma$-compact.

Proof. From the definitions of $p\gamma$-compactness and $p\gamma$-continuity, we get the results.

Definition 3.12. For two topological spaces (X, τ) and (Y, μ), a function $f : (X, \tau) \to (Y, \mu)$ is said to be *$p\gamma$-open* if for every open set G in X, $f(G)$ is a $p\gamma$-set in Y.

Theorem 3.13. Let $f : (X, \tau) \to (Y, \mu)$ be a function between topological spaces. Then f is $p\gamma$-open if and only if $int(f^{-1}(B)) \subset f^{-1}(int_{p\mu}(B))$ for each $B \subset Y$.

Proof. $\Rightarrow$) Let $B \subset Y$ and $x \in int(f^{-1}(B))$. Then $f(int(f^{-1}(B)))$ is a $p\gamma$-set containing $f(x)$. Since $f(int(f^{-1}(B))) \in P_{f(x)}$ and $P_{f(x)}$ is a filter, $B \in P_{f(x)}$. Thus $f(x) \in int_{p\mu}(B)$ and so $x \in f^{-1}(int_{p\mu}(B))$.

$\Leftarrow$) Let A be an open subset in X and $y \in f(A)$. Then $A \subset int(f^{-1}f(A)) \subset f^{-1}(int_{p\mu}(f(A)))$. Let x be an element in A such that $f(x) = y$. Then $x \in f^{-1}(int_{p\mu}(f(A)))$, and $y \in int_{p\mu}(f(A))$. Thus from Theorem 2.10(b), we can say that $f(A)$ is a $p\gamma-$set.

Remark. 3.14. If a function is preopen, then the function is also $p\gamma$-open. But the converse may not hold. Consider the set R of real numbers with cofinite topology and a function $f : R \to R$ defined by $f(x) = 0$ for all $x \in R$. Then f is $p\gamma$-open. But for any open set G, $f(G) = \{0\}$ and $\{0\}$ is not a preopen set, thus f is not preopen.

Theorem 3.15. Let $f : (X, \tau) \to (Y, \mu)$ be a function between topological spaces. Then f is $p\gamma$-open if and only if for each $x \in X$ and for each neighborhood G of x, $f(G)$ is also an element of pre-neighborhood filter $P_{f(x)}$ in Y.

Proof. $\Rightarrow$) Let G be a neighborhood of x, then there exists an open set U such that $x \in U \subset G$. Since f is $p\gamma$-open, $f(x) \in f(U) = int_{p\mu}(f(U))$, and so $f(U) \in P_{f(x)}$. Thus we have $f(G) \in P_{f(x)}$.
$\Leftarrow$) Let $B \subset Y$ and $x \in int(f^{-1}(B))$, then since $f^{-1}(B)$ is a neighborhood of x, $f^{-1}(B) \in P_x$. By the condition, we have $f(f^{-1}(B)) \in P_{f(x)}$. And since $P_{f(x)}$ is a filter, B is also an element of $P_{f(x)}$. By Theorem 2.10, it follows $f(x) \in int_{p\mu}(B)$. Thus by Theorem 3.13, the function f is $p\gamma$-open.

Remark. 3.16. We get the following relations:

open function $\Longrightarrow$ α-open function $\Longrightarrow$ preopen function $\Longrightarrow$ $p\gamma$-open function.

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