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MAXIMAL AND OBSTINATE HYPER K-IDEALS

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Abstract.In this note first we define the notions of (weak) maximal hyper K-ideal, (weak) obstinate hyper K-ideals of types 1-4 and give some examples of these notions. Then we state and prove some theorems which determine the relationship between these notions and the other hyper K-ideals under some suitable conditions.

1 Introduction The hyperoperation structure theory was introduced by F. Marty [7] in 1934. Imai and Iseki [6] in 1966 introduced the notion of a BCK-algebra. Recently [4] Borzooei, Jun and Zahedi et.al. applied the hyperstructure to BCK-algebras and introduced the concept of hyper K-algebra which is a generalization of BCK-algebra. Now, in this note we define and study (weak) maximal and (weak) obstinate hyper K-ideals. Then we obtain some related results which have been mentioned in the abstract. In particular by given some examples we show that the converse of some of these theorems does not hold.

2 Preliminaries

Definition 2.1. [4] Let H be a nonempty set and " $\circ$ " be a *hyperoperation* on H, that is " $\circ$ " is a function from $H \times H$ to $\mathcal{P}^*(H) = \mathcal{P}(H)\backslash\{\emptyset\}$. Then H is called a *hyper K-algebra* if it contains a constant "0" and satisfies the following axioms:
(HK1) $(x \circ z) \circ (y \circ z) < x \circ y$
(HK2) $(x \circ y) \circ z = (x \circ z) \circ y$
(HK3) $x < x$
(HK4) $x < y,\, y < x \Rightarrow x = y$
(HK5) $0 < x$,
for all $x, y, z \in H$, where $x < y$ is defined by $0 \in x \circ y$ and for every $A, B \subseteq H$, $A < B$ is defined by $\exists a \in A$, $\exists b \in B$ such that $a < b$.

Note that if $A, B \subseteq H$, then by $A \circ B$ we mean the subset $\bigcup\limits_{\substack{a\in A\\ b\in B}} a \circ b$ of H.

Example 2.2. [4] Define the hyperoperation " $\circ$ " on $H = [0, +\infty)$ as follows:

$$x \circ y = \begin{cases} [0, x] & \text{if } x \leq y \\ (0, y] & \text{if } x > y \neq 0 \\ \{x\} & \text{if } y = 0 \end{cases}$$

for all $x, y \in H$. Then $(H, \circ, 0)$ is a hyper K-algebra.

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Theorem 2.3. [4] Let $(H, \circ, 0)$ be a hyper K-algebra. Then for all $x, y, z \in H$ and for all nonempty subsets A, B and C of H the following relations hold:
(i) $x \circ y < z \Leftrightarrow x \circ z < y$, (ii) $(x \circ z) \circ (x \circ y) < y \circ z$,
(iii) $x \circ (x \circ y) < y$, (iv) $x \circ y < x$,
(v) $A \subseteq B \Rightarrow A < B$, (vi) $x \in x \circ 0$,
(vii) $(A \circ C) \circ (A \circ B) < B \circ C$, (viii) $(A \circ C) \circ (B \circ C) < A \circ B$,
(ix) $A \circ B < C \Leftrightarrow A \circ C < B$.

Definition 2.4. [4] Let I be a nonempty subset of a hyper K-algebra $(H, \circ, 0)$ and $0 \in I$. Then,
(i) I is called a *weak hyper K-ideal* of H if $x \circ y \subseteq I$ and $y \in I$ imply that $x \in I$, for all $x, y \in H$.
(ii) I is called a *hyper K-ideal* of H if $x \circ y < I$ and $y \in I$ imply that $x \in I$, for all $x, y \in H$.

Theorem 2.5. [4] Any hyper K-ideal of a hyper K-algebra H, is a weak hyper K-ideal.

Definition 2.6. [4] Let H be a hyper K-algebra. An element $a \in H$ is called a *left (resp. right) scalar* if $|a \circ x| = 1$ (resp. $|x \circ a| = 1$) for all $x \in H$. If $a \in H$ is both left and right scalar, we say that a is a *scalar element.*

Definition 2.7.[3] Let I be a nonempty subset of a hyper K-algebra $(H, \circ, 0)$ such that $0 \in I$. Then I is called a positive implicative hyper K-ideal of

(i) type 1, if for all $x, y, z \in H$, $(x \circ y) \circ z \subseteq I$ and $y \circ z \subseteq I$ imply that $x \circ z \subseteq I$,

(ii) type 2, if for all $x, y, z \in H$, $(x \circ y) \circ z < I$ and $y \circ z \subseteq I$ imply that $x \circ z \subseteq I$,

(iii) type 3, if for all $x, y, z \in H$, $(x \circ y) \circ z < I$ and $y \circ z < I$ imply that $x \circ z \subseteq I$,

(iv) type 4, if for all $x, y, z \in H$, $(x \circ y) \circ z \subseteq I$ and $y \circ z < I$ imply that $x \circ z \subseteq I$,

(v) type 5, if for all $x, y, z \in H$, $(x \circ y) \circ z \subseteq I$ and $y \circ z \subseteq I$ imply that $x \circ z < I$,

(vi) type 6, if for all $x, y, z \in H$, $(x \circ y) \circ z < I$ and $y \circ z < I$ imply that $x \circ z < I$,

(vii) type 7, if for all $x, y, z \in H$, $(x \circ y) \circ z \subseteq I$ and $y \circ z < I$ imply that $x \circ z < I$,

(viii) type 8, if for all $x, y, z \in H$, $(x \circ y) \circ z < I$ and $y \circ z \subseteq I$ imply that $x \circ z < I$.

Definition 2.8. [4] Let H be a hyper K-algebra. If there exists an element $e \in H$ such that $x < e$ for all $x \in H$, then H is called a *bounded hyper K-algebra* and e is said to be the *unit* of H.

Definition 2.9. [1] Let H be a hyper K-algebra. Then a nonempty subset I of H is called:

(a) a *weak implicative hyper K-ideal* if it satisfies:
(i) $0 \in I$
(ii) $(x \circ z) \circ (y \circ x) \subseteq I$ and $z \in I$ imply $x \in I$, for all $x, y, z \in H$,

(b) an *implicative hyper K-ideal* if it satisfies:
(i) $0 \in I$
(ii) $(x \circ z) \circ (y \circ x) < I$ and $z \in I$ imply $x \in I$, for all $x, y, z \in H$.

Definition 2.10. [3] (i) A non-empty subset I of H is called s-reflexive, if for any $x, y \in H$, $(x \circ y) \bigcap I \neq \emptyset$, implies $x \circ y \subseteq I$.
(ii) A non empty subset I of H is called reflexive, if for any $x \in H$, $x \circ x \subseteq I$.

(iii) If I is a hyper K-ideal of H and it is s-reflexive (reflexive), then I is called an s-reflexive (a reflexive) hyper K-ideal of H.

Definition 2.11. [4] Let H_1 and H_2 are two hyper K-algebras. A mapping $f : H_1 \to H_2$ is said to be a homomorphism if :

(i) $f(0) = 0$

(ii) $f(x \circ y) = f(x) \circ f(y)$, for all $x\, y \in H_1$.

If f is $1-1$ (onto) we say that f is a monomorphism (epimorphism). And if f is both $1-1$ and onto, we say that f is an isomorphism.

Definition 2.12. [2] Let $\sim$ be an equivalence relation on H and $A, B \subseteq H$. Then

(i) $A \sim B$ if there exist $a \in A$ and $b \in B$ such that $a \sim b$,

(ii) $A \approx B$ if for any $a \in A$ there exists $b \in B$ such that $a \sim b$, and for any $b \in B$ there exists $a \in A$ such that $a \sim b$,

(iii) $A \simeq B$ if for all $a \in A$ and for all $b \in B$ we have $a \sim b$,

(iv) $\sim$ is called regular to the right if $a \sim b$ implies that $a \circ c \approx b \circ c$, for any $a, b, c \in H$,

(v) $\sim$ is called strongly regular to the right if $a \sim b$ implies that $a \circ c \simeq b \circ c$, for any $a, b, c \in H$,

(vi) $\sim$ is called good, if $a \circ b \sim \{0\}$ and $b \circ a \sim \{0\}$ implies that $a \sim b$, for all $a, b \in H$.

Similarly we can define the regularity (strong regularity) of an equivalence equivalence to the left. A regular equivalence (strongly regular) to the right and to the left is called regular (strongly regular).

From now on $\sim$ is a good regular relation, and for any x in H by C_x we mean that equivalence class of x under $\sim$, also $I = C_0$.

Denote $H/I = \{C_x : x \in H\}$ where $I = C_0$ and define

$$* : H/I \times H/I \to H/I$$

$$(C_x, C_y) \mapsto \{C_t \mid t \in x \circ y\}$$

Now we define the relation $<$ on H/I by $C_x < C_y$ if and only if $C_0 \in C_x * C_y$. Hence we have

$$x < y \Leftrightarrow 0 \in x \circ y \Longrightarrow C_0 \in C_x * C_y \Leftrightarrow C_x < C_y$$

Theorem 2.13. [2] Let $I = C_0$. Then $(H/I, *, C_0)$ is a hyper K-algebra.

3 Main Results

From now on $(H, \circ, 0)$ is a hyper K-algebra.

Definition 3.1. Let M be a proper (weak) hyper K-ideal of H. Then M is called a (weak) maximal hyper K-ideal of H if $M \subseteq I \subseteq H$ for some (weak) hyper K-ideal I of H, then $M = I$ or $I = H$.

Example 3.2. Let $H = \{0, 1, 2\}$. Then the following table shows a hyper K-algebra structure on H.

$\circ$	0	1	2
0	$\{0\}$	$\{0\}$	$\{0\}$
1	$\{1\}$	$\{0\}$	$\{0\}$
2	$\{2\}$	$\{1, 2\}$	$\{0\}$

We can check that $I = \{0\}$ is a maximal hyper K-ideal of H.

Proposition 3.3. Each maximal hyper K-ideal is a weak maximal hyper K-ideal.

Proof. It is easy.□

Example 3.4. Let $H = \{0, 1, 2\}$. Then the following table shows a hyper K-algebra structure on H.

$\circ$	0	1	2
0	$\{0, 1, 2\}$	$\{0, 1, 2\}$	$\{0, 1, 2\}$
1	$\{1, 2\}$	$\{0, 1, 2\}$	$\{0, 1\}$
2	$\{2\}$	$\{2\}$	$\{0, 1, 2\}$

We can check that $I = \{0, 2\}$ is a weak maximal hyper K-ideal of H, while it is not a maximal hyper K-ideal of H.

Theorem 3.5. Let H be a bounded hyper K-algebra and $|H| \geq 2$. Then H has a (weak) maximal hyper K-ideal.

Proof. The proof follows from Zorn's Lemma.□

As an immediate consequence of the above theorem we have:

Theorem 3.6. Let H be a bounded hyper K-algebra and $|H| \geq 2$, also let I be a proper subset of H. Then

(i) if I is a (weak) implicative hyper K-ideal, then there is a maximal (weak) implicative hyper K-ideal containing I.

(ii) if I is a positive implicative hyper K-ideal of type i, where $1 \leq i \leq 8$, then there is a maximal positive implicative hyper K-ideal of type i containing I where $1 \leq i \leq 8$.

Theorem 3.7. Let M be a proper hyper K-ideal of H. Then M is a maximal hyper K-ideal if and only if for any hyper K-ideal I of H we have $I \subseteq M$ or $< I \cup M >= H$, where by $< I \cup M >$ we mean the hyper K-ideal generated by $I \cup M$.

Proof. Let for all hyper K-ideal of H we have $I \subseteq M$ or $< I \cup M >= H$ we show that M is a maximal hyper K-ideal of H. Let $M \subseteq I \subseteq H$. If $I \subseteq M$, then $I = M$, and so we are done. Otherwise $< I \cup M >= H$, since $M \subseteq I$ thus $M \cup I = I$. Therefore $I =< I \cup M >= H$, thus M is a maximal hyper K-ideal. The converse is trivial.□

Theorem 3.8. Let $f : H_1 \to H_2$ be an epimorphism of hyper K-algebras . Then

(i) If M is a (weak) maximal hyper K-ideal of H_1 such that $\ker f \subseteq M$, then $f(M)$ is a (weak) maximal hyper K-ideal of H_2,

(ii) If M is a (weak) maximal hyper K-ideal of H_2, then $f^{-1}(M)$ is a (weak) maximal hyper K-ideal of H_1 containing $kerf$,

(iii) The map $M \mapsto f(M)$ is a one-one corresponding between the (weak) maximal hyper K-ideals of H_1 containing $\ker f$ and the (weak) maximal hyper K-ideals of H_2.

Proof. It is not difficult.□

Definition 3.9. Let I be a (weak) hyper K-ideal of H. Then I is called a (weak) obstinate hyper K-ideal of

(i) *type* 1, if for any $x, y \in H$, $x, y \notin I$ imply that $x \circ y \subseteq I$, and $y \circ x \subseteq I$.

(ii) *type* 2, if for any $x, y \in H$, $x, y \notin I$ imply that $x \circ y < I$, and $y \circ x \subseteq I$.

(iii) *type* 3, if for any $x, y \in H$, $x, y \notin I$ imply that $x \circ y \subseteq I$, and $y \circ x < I$.

(iv) *type* 4, if for any $x, y \in H$, $x, y \notin I$ imply that $x \circ y < I$, and $y \circ x < I$.

Example 3.10. Let $H = \{0, 1, 2, 3\}$. Then the following table shows a hyper K-algebra structure on H

$\circ$	0	1	2	3
0	$\{0\}$	$\{0\}$	$\{0\}$	$\{0\}$
1	$\{1\}$	$\{0\}$	$\{0\}$	$\{0\}$
2	$\{2\}$	$\{2\}$	$\{0\}$	$\{0\}$
3	$\{3\}$	$\{2\}$	$\{1\}$	$\{0, 1\}$

Then $I = \{0, 1\}$ is obstinate hyper K-ideal of type 1.

Proposition 3.11. (1) Each (weak) obstinate hyper K-ideal of type 3 or 2 is of type 4.
(2) Each (weak) obstinate hyper K-ideal of type 1 is of types 2 and 3.

Proof. It is easy.□

Example 3.12. Let $H = \{0, 1, 2\}$. Then the following table shows a hyper K-algebra structure on H.

$\circ$	0	1	2
0	$\{0, 1\}$	$\{0\}$	$\{0, 1\}$
1	$\{1\}$	$\{0, 1\}$	$\{1\}$
2	$\{1, 2\}$	$\{0, 1, 2\}$	$\{0, 2\}$

We can check that $I = \{0, 2\}$ is an obstinate hyper K-ideal of type 4 but it is not of type 2 (3), since $1 \circ 1 \nsubseteq I$.

Problem 3.13. Is there an example of obstinate hyper K-ideal of type 2 or 3 in which it is not of type 1?

Theorem 3.14. Let H be a hyper K-algebra, with order more than 2. Then the hyper K-ideal $\{0\}$ is not an obstinate hyper K-ideal of types 4.

Proof. Let $I = \{0\}$ be an obstinate hyper K-ideal of type 4. Then $x \circ y < I$ and $y \circ x < I$ for all $x, y \notin I$. Thus we must have $0 \in x \circ y$ and $0 \in y \circ x$, therefore $x = y$ for all $x, y \neq 0$, which is a contradiction.□

Theorem 3.15. In a hyper K-algebra of order 3, any hyper K-ideal I with two elements is an obstinate hyper K-ideal of type 4.

Proof. Let I be a hyper K-ideal with two element in hyper K-algebra of order 3. Then there exists just one element x in which $x \notin I$. But $0 \in x \circ x$, thus $x \circ x < I$.□

Theorem 3.16. Let $I \subseteq H$ be a proper s-reflexive and an obstinate hyper K-ideal of type 1, and H' be a hyper K-algebra with two scalar elements 0 and a. Then there exists $f \in Hom(H, H')$ such that $\ker f = I$.

Proof. Define f as follows:

$$f(x) = \begin{cases} 0 & \text{if } x \in I \\ a & \text{if } x \notin I \end{cases}$$

Now we prove that $f \in Hom(H, H')$

(1) If $x, y \in I$ since $x \circ y < x$ and $x \in I$ we get that $x \circ y < I$. Thus there exists a $t \in x \circ y$ and $r \in I$ such that $t < r$, hence $t \circ r < I$. Since $r \in I$ we can get that $t \in I$, therefore $x \circ y \subseteq I$, because I is an s-reflexive hyper K-ideal of H. So $f(t) = 0 = 0 \circ 0 = f(x) \circ f(y)$, for all $t \in x \circ y$. Thus $f(x \circ y) = f(x) \circ f(y)$.

(2) If $x, y \notin I$, then $x \circ y \subseteq I$, $y \circ x \subseteq I$, since I is an obstinate hyper K-ideal. Then $f(t) = 0 = a \circ a = f(x) \circ f(y)$, for all $t \in x \circ y$. Therefore $f(x \circ y) = f(x) \circ f(y)$.

(3) If $x \notin I, y \in I$, then $(x \circ y) \cap I = \emptyset$, since if there exists $t \in x \circ y \cap I$ then $(x \circ y) < I$ and $y \in I$ implies that $x \in I$, which is a contradiction. Then $f(t) = a = a \circ 0 = f(x) \circ f(y)$, for all $t \in x \circ y$. Thus $f(x \circ y) = f(x) \circ f(y)$.

(4) If $x \in I, y \notin I$ then $x \circ y < I$. Similar to (1) we can conclude that $x \circ y \subseteq I$. Therefore

$$f(t) = 0 = 0 \circ a = f(x) \circ f(y).$$

for all $t \in x \circ y$. Thus $f(x \circ y) = f(x) \circ f(y)$. Therefore we prove that $f \in Hom(H, H')$ and clearly $I = ker f$.□

Theorem 3.17. Let I be a proper hyper K-ideal of H. If for any hyper K-algebra H', there exists $f \in Hom(H, H')$ such that $\ker f = I$, then I is an obstinate hyper K-ideal of type 1.

Proof. Let $H' = \{0, 1\}$ be the following hyper K-algebra

$\circ$	0	1
0	$\{0\}$	$\{0\}$
1	$\{1\}$	$\{0\}$

Then by hypothesis there exists $f \in Hom(H, H')$ such that $ker f = I$ and $f^{-1}(1) = H \setminus I$. Thus for all $x, y \in H \setminus I$ we have $f(x) = f(y) = 1$. Therefore

$f(x \circ y) = f(x) \circ f(y) = 1 \circ 1 = 0$

$f(y \circ x) = f(y) \circ f(x) = 1 \circ 1 = 0.$

Hence $x \circ y \subseteq I$ and $y \circ x \subseteq I$, which means that I is an obstinate hyper K-ideal of type 1.□

Lemma 3.18. Let $I \subseteq H$ be an s-reflexive and positive implicative hyper K-ideal of type 4. Then $I_a = \{x \mid x \circ a \subseteq I\}$ is the least weak hyper K-ideal of H containing $I \cup \{a\}$.

Proof. By Theorem 4.2 of [3], I is a weak hyper K-ideal. Since I is s-reflexive then $a \circ a \subseteq I$ which means that $a \in I_a$. If $x \in I$, then since $x \circ a < x$ we can get that $x \circ a \subseteq I$, hence $x \in I_a$ for all $x \in I$. Now, let B be a weak hyper K-ideal containing I and a. Then for all $x \in I_a$ we have $x \circ a \subseteq I$. Thus $x \circ a \subseteq B$ and $a \in B$, therefore $x \in B$. This shows that $I_a \subseteq B$.□

Theorem 3.19. Let $I \subseteq H$ be a weak maximal, s-reflexive and positive implicative hyper K-ideal of type 4. Then I is a weak obstinate hyper K-ideal of type i for any $1 \leq i \leq 4$.

Proof. By considering Proposition 3.11 it is enough to show that I is a weak obstinate hyper K-ideal of type 1. To show this let $x \notin I$ and $y \notin I$. Since I is a positive implicative hyper K-ideal of type 4, then by Lemma 3.18, $I_y = \{u \in H \mid u \circ y \subseteq I\}$ is the least weak hyper K-ideal of H containing I and y. By maximality of I we get that $I_y = H$. Hence $x \in I_y$, therefore $x \circ y \subseteq I$. Similarly $y \circ x \subseteq I$, so I is a weak obstinate hyper K-ideal of type 1.□

Theorem 3.20. Let $I \subseteq H$ be an s-reflexive and obstinate hyper K-ideal of type 1. Then I is an implicative hyper K-ideal.

Proof. Let $x \circ (y \circ x) < I$ and on the contrary let $x \notin I$. We consider the following two cases for $y \circ x$:

(i) $(y \circ x) < I$ (ii) $(y \circ x) \nless I$

In the case (i), by s-reflexivity of I we get that $y \circ x \subseteq I$. On the other hand $x \circ (y \circ x) < I$, implies that there exists $t \in y \circ x$ such that $x \circ t < I$. Now $y \circ x \subseteq I$ implies that $t \in I$, therefore $x \in I$, which is a contradiction.

In case (ii), first we show that $y \in I$. On the contrary let $y \notin I$. Since I is an obstinate hyper K-ideal of type 1, we get that $y \circ x \subseteq I$. Hence $y \circ x < I$, which is a contradiction. Therefore $y \in I$. Now since $y \circ x < y$, we conclude that $y \circ x < I$ which is a contradiction.

So that we must have $x \in I$, which means that I is an implicative hyper K-ideal, by Theorem 4.12 of [1].□

Example 3.21. The following example shows that the converse of above theorem is not correct in general, let $H = \{0, 1, 2, 3\}$. Them the following table shows a hyper K-algebra structure on H.

$\circ$	0	1	2	3
0	$\{0\}$	$\{0\}$	$\{0\}$	$\{0\}$
1	$\{1\}$	$\{0\}$	$\{0\}$	$\{0,1\}$
2	$\{2\}$	$\{2\}$	$\{0\}$	$\{2\}$
3	$\{3\}$	$\{3\}$	$\{3\}$	$\{0\}$

It can be checked that $I = \{0, 1\}$ is an implicative and s-reflexive hyper K-ideal, while is not an obstinate hyper K-ideal of type 1 since $2, 3 \notin I$ and $2 \circ 3 \nsubseteq I$ and $3 \circ 2 \nsubseteq I$.

Theorem 3.22. Let $I = C_0$ and J be a hyper K-ideals of H and $I \subseteq J$. Then J/I is an obstinate hyper K-ideal of types 1-4 of H/I if and only if J is an obstinate hyper K-ideal of types 1-4 of H.

Proof. Without loss of generality we prove theorem for type 1. Let J be an obstinate hyper K-ideal of type 1 of H and $C_x, C_y \notin J/I$, we claim that $x, y \notin J$, in the contrary from $x, y \in J$ we conclude that $C_x, C_y \in J/I$. Since J is obstinate hyper K-ideal of type 1 then we have $x \circ y \subseteq J$ and $y \circ x \subseteq J$. Now consider $C_x * C_y = \{C_t \mid t \in x \circ y\}$, then for all $C_t \in C_x * C_y$ $C_t \in J/I$, so J/I is an obstinate hyper K-ideal of type 1 of H/I.

Conversely, let J/I be an obstinate hyper K-ideal of type 1 of H/I and $x, y \notin J$. Then $C_x \notin J/I$ and $C_y \notin J/I$ in otherwise if C_x be in J/I then we must have $C_x = C_t$ where $t \in J$. Then $x \sim t$ so $x \circ t \approx \{0\}$ thus $x \circ t < I$ then we conclude that $x \circ t < J$ by $t \in J$ we get that $x \in J$ which is contradiction. Therefore $C_x, C_y \notin J/I$. Since J/I is an obstinate hyper K-ideal of type 1 then $C_x * C_y \in J/I$ and $C_y * C_x \in J/I$ then we get that $x \circ y \subseteq J$ and $y \circ x \subseteq J$.□

References

[1] A. Borumand Saeid, R.A. Borzooei and M.M. Zahedi, *(Weak) Implicative hyper K-ideals*, Bull. Korean Math. Soc., **40** (2003), No. **1**, pp 123-137.

[2] A. Borumand Saeid and M.M. Zahedi, *Quotient hyper K-algebras*, Submitted.

[3] R.A. Borzooei, P. Corsini and M.M. Zahedi, *Some kinds of positive Implicative hyper K-ideals*, Journal of Discrete Mathematical Sciences and Cryptography, Delhi, Vol. 6, No 1. (2003), pp. 97-108.

[4] R.A. Borzooei, A. Hasankhani, M.M. Zahedi and Y.B. Jun, *On hyper K-algebras*, Math. Japon. Vol. **52**, No. **1** (2000), 113-121.

[5] R.A. Borzooei and M.M. Zahedi, *Positive Implicative hyper K-ideals,* Scientiae Mathematicae Japonicae, Vol. **53**, No. **3** (2001), 525-533.

[6] Y. Imai and K. Iseki, *On axiom systems of propositional calculi*, XIV Proc. Japan Academy, **42** (1966), 19-22.

[7] F. Marty, *Sur une generalization de la notion de groups*, 8th congress Math. Scandinaves, Stockholm, (1934), 45-49.

[8] M.M. Zahedi, R.A. Borzooei, Y.B. Jun and A. Hasankhani, *Some results on hyper K-algebra,* Scientiae Mathematicae Japonicae, Vol. **3**, No. **1** (2000), 53-59.

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