FUND ALLOCATION METHOD BASED ON A BLOCK OF SHARES

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Abstract. In a real investment, stocks are dealt with based on a block of shares. A block of shares is a minimum unit for trading stocks. However, a conventional portfolio selection problem does not consider about a block of shares. If we deal with stocks according to a block of shares, real allocations of funds to each stock should differ among the cases of different amounts of money. Furthermore, a decision maker should be unable to buy less than one block even if the investing ratio for some stock is much smaller.

The objective of this paper is to build a portfolio selection model in consideration of the amount of investing funds and a block of shares. Our model is formulated as an integer quadratic programming problem.

In general, an integer nonlinear programming problem is difficult to solve for all but the smallest cases. So we also propose the efficiently approximate model employing a Meta-controlled Boltzmann machine.

1 Introduction In 1952, H. Markowitz [1] proposed a method to allocate an amount of funds to plural stocks for investment. The method was named a portfolio selection problem. Based on time-series data of return rate, it theoretically decides the best investing rate to each of stocks, which minimizes the risk, i.e. the variance of the profits in keeping the least expected return rate that a decision maker desires. It is characteristic that the model can reduce the risk by means of allocating the amount of funds to many stocks. The model is excellently concise for real problems.

Since then, researches have been pursued on various aspects of the model, such as realizing efficient calculation [2, 5, 6, 7].

In a real investment, stocks are dealt with based on a block of shares [3]. A block of shares is a minimum unit for trading stocks. For example, in a Japan market, except for some cases, dealings are basically conducted according to a block of shares, as shown in Table 1.

Table 1: Basic block of shares in Japan markets

Face value (yen)	The block of shares
50	1000
500	100
50,000	1

By the way, the portfolio selection problem proposed by H. Markowitz is formulated based on the premise that the amount of funds can be divided into infinitely small amount without limit. If a decision maker invests the amount of funds according to investing ratio, it is obvious that a block of shares can not be maintained in dealing. If we deal with a

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stock in a block of shares, real allocations to each stock should differ among the cases where the funds are 1 billion yen and 500 million yen. Furthermore, it is possible that a decision maker may be unable to buy less than one block even if the investing ratio for an objective stock is much smaller.

In consideration of the above point, the objective of this paper is to build a portfolio selection model in consideration of the amount of investing funds and a block of shares. This model defines the investing ratio in terms of a stock unit price, a block of shares, the number of dealing block and the amount of investing funds. Therefore, our model can not obtain a solution with an investment ratio but with the number of dealing blocks. It is possible to obtain a solution more realistically by this formulation than the conventional model.

In general, an integer nonlinear programming problem will be very difficult to solve for all but small cases. This paper proposes an efficiently approximate approach by a Meta-controlled Boltzmann machine. The Meta-controlled Boltzmann machine is a neural network model which is proposed by T. Watanabe and J. Watada et al [6]. This model deletes the units of lower layer which are not selected in the Meta-controlling layer in its execution. Then the lower layer is restructured by using the selected units. Because of this feature, a Meta-controlled Boltzmann machine converges more efficiently than a conventional Boltzmann machine.

In this paper, as a numerical example, we will compare a solution between the Meta-controlled Boltzmann machine and LINGO 8.0. LINGO 8.0 is a commercial software package in order to solve mathematical programming problems. We employ problems with 15 stocks and 225 stocks in order to show that we can solve the portfolio selection model in consideration of amount of funds and a block of shares by a Meta-controlled Boltzmann machine. The problems with 15 stocks and 225 stocks are respectively named a 15-stock problem and 225-stock problem here after. Moreover, we compare computing time employing a 225-stock problem in order to show that we can obtain the solution of a large scale integer quadratic programming problem more efficiently by a Meta-controlled Boltzmann machine than by LINGO 8.0. We will discuss the results in three cases of funds 1 billion yen, 500 million yen and 100 million yen.

2 Fund allocation based on a block of shares In this section we build a portfolio model in consideration of the amount of funds and a block of shares. First, we explain a portfolio selection problem which is proposed by H. Markowitz. Next, we build our model.

2.1 Markowitz model A mean-variance approach to a portfolio selection problem was originally proposed by H. Markowitz [1]. Generally, a decision maker is more satisfied of that a risk is smaller and that the expected return rate is larger. From this point of view, the portfolio selection problem is formulated as the following two objective quadratic programming problem.

Formulation 1

$$\text{minimize} \quad \sum_{i=1}^{m}\sum_{j=1}^{m}\sigma_{ij}x_i x_j, \tag{1}$$

$$\text{maximize} \quad \sum_{i=1}^{m}\mu_i x_i, \tag{2}$$

$$\text{subject to} \quad \sum_{i=1}^{m} x_i = 1, \tag{3}$$

$$x_i \geq 0 \quad (i = 1, 2, \ldots, m), \tag{4}$$

where σ_{ij} denotes a covariance between stocks i and j, μ_i an expected return rate of stock i. x_i an investing rate to stock i, respectively.

However, it is difficult to efficiently obtain the optimal solution of FORMULATION 1 because of the two objective problem. Therefore, H. Markowitz assumed that almost all the decision maker should be a risk aversion person. According to this assumption, H. Markowitz has formulated a portfolio selection problem as the following quadratic programming problem under the restriction that the expected return rate is made sure to be more than amount R.

Formulation 2

$$\text{minimize} \quad \sum_{i=1}^{m}\sum_{j=1}^{m} \sigma_{ij} x_i x_j \tag{5}$$

$$\text{subject to} \quad \sum_{i=1}^{m} \mu_i x_i \geq R \tag{6}$$

$$\sum_{i=1}^{m} x_i = 1 \tag{7}$$

$$x_i \geq 0 \quad (i = 1, 2, \ldots, m) \tag{8}$$

where R denotes an acceptable least rate of the expected return, other notations illustrated in Formulation 1.

The objective of this model is to minimize its risk in allocating the amount of funds to many stocks. In a real investment problem, there is a limitation in the amount of funds which a decision maker can manage. Since each of stocks is dealt in a block of shares, it is ordinarily impossible to spend 100% of the provided fund in buying or selling. However, the portfolio selection problem proposed by H. Markowitz is formulated based on assumption that each of stocks can be dealt in an infinitely small unit. The objective of this paper is to propose the portfolio selection model in consideration of a block of shares.

2.2 Formulation of our model Our model is formulated on the basis of the Markowitz model in Formulation 2. In our model, we express an investment ratio of a Markowitz model using a unit price of stock, a block of shares, the number of dealing blocks and the amount of funds which a decision maker can manage. Our model can be rewritten as the following integer quadratic programming model.

Formulation 3

$$\text{minimize} \quad \sum_{i=1}^{m}\sum_{j=1}^{m} \sigma_{ij} \frac{n_i p_i q_i}{B} \frac{n_j p_j q_j}{B} \tag{9}$$

$$\text{subject to} \quad \sum_{i=1}^{m} \mu_i \frac{n_i p_i q_i}{B} \geq R \tag{10}$$

$$\sum_{i=1}^{m} n_i p_i q_i \leq B \tag{11}$$

$$\frac{n_i p_i q_i}{B} \geq 0 \quad (i = 1, 2, \ldots, m) \tag{12}$$

Step 1.	Set each parameter to its initial value.
Step 2.	Input the values of K_u and K_l.
Step 3.	Execute the Meta-controlling layer.
Step 4.	If the output value of a unit in the Meta-controlling layer is 1, add some amount of value to the corresponding unit in the lower layer. Execute the lower layer.
Step 5.	After executing the lower layer the constant number of times, decreases the temperature.
Step 6.	If the output value is sufficiently large, add a certain amount of value to the corresponding unit in the Meta-controlling layer.
Step 7.	Iterate from Step 4 to Step 6 until the temperature reaches the restructuring temperature.
Step 8.	Restructure the lower layer using the selected units of the Meta-controlling layer.
Step 9.	Execute the lower layer until reaching at the termination.

Figure 1: Algorithm of Meta-controlled Boltzmann machine

where, n_i denotes the number of dealing blocks of stock i, p_i a unit price of stock i, q_i a block of shares of stock i and B an amount of funds, respectively. Other notations are given previously.

The characteristic of our model can obtain a solution not with an investment ratio but with the number of dealing blocks.

3 Meta-controlled Boltzmann machine for fund allocation based on a block of shares A Meta-controlled Boltzmann machine [6] employs a Hopfield network [9] as a Meta-controlling layer and a Boltzmann machine [8] as a lower layer. The Meta-controlling layer supervises the subordinate lower layer to obtain the best portfolio within the optimal combination of invested stocks and the lower layer decides the optimal investment rate over the limited number of stocks supervised by the Meta-controlling layer. This model deletes units of the lower layer which are not selected in the Meta-controlling layer in its execution. Then the lower layer is restructured by using the selected units. Executing the Meta-controlled Boltzmann machine according to the above mentioned algorithm, the Meta-controlled Boltzmann machine converges more efficiently than a conventional Boltzmann machine.

In this section, we illustrate the energy functions of a Meta-controlled Boltzmann machine to solve a portfolio selection problem formulated as the Formulation 3. The energy functions are written as follows:

Meta-controlling layer

$$E_u = \frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{m}\sigma_{ij}s_is_j - \frac{K_u}{2}\sum_{i=1}^{m}\mu_i s_i, \tag{13}$$

Lower layer

$$\begin{aligned} E_l &= \frac{1}{2}\left\{\sum_{i=1}^{m}\sum_{j=1}^{m}\sigma_{ij}\frac{n_ip_iq_i}{B}\frac{n_jp_jq_j}{B} + \sum_{i=1}^{m}\sum_{j=1}^{m}\frac{n_ip_iq_i}{B}\frac{n_jp_jq_j}{B}\right\} \\ &- \sum_{i=1}^{m}\frac{n_ip_iq_i}{B} - \frac{K_l}{2}\sum_{i=1}^{m}\mu_i\frac{n_ip_iq_i}{B}, \end{aligned} \tag{14}$$

Table 2: Price of each stock for 15-stock problem

	Unit price (yen)	Stock price per block(yen)
Toray Industries	520	520,000
Obayashi Corp	375	375,000
Snow Brand Milk	446	446,000
Teijin	494	494,000
Kao	1,010	1,010,000
Asahi Glass	935	935,000
Daikin Industries	638	638,000
Hitachi	770	770,000
Honda Motor	1,190	1,190,000
Dai Nippon Printing	1,400	1,400,000
Taisho Marine & Fire	540	540,000
Tobu Rail	429	429,000
All Nippon Airways	775	775,000
Osaka Gas	250	250,000
Shochiku	1,150	1,150,000

where K_u, K_l are weight of the expected return rate for each layer and s_i is output value of unit i of the Meta-controlling layer. Other notations are defined previously.

Output value, s_i, of the Meta-controlling layer takes 0 or 1. If corresponding stock i is selected, the output value of unit i is 1.

In the Meta-controlling layer if K_u is set to a larger value, the selected number of stocks will increase. In the lower layer if K_l is set to a smaller value, we can obtain a solution sufficiently approximate to the minimum risk solution.

Therefore, as we change each parameter K, we will obtain various investing pattern according to the aspiration level of a decision maker's in the same as in a fuzzy portfolio selection model which is proposed by J. Watada et al [7].

Algorithm of a Meta-controlled Boltzmann machine is shown in Figure 1.

Owing to restructuring the lower layer, we can shorten drastically its computing time to reach at the termination.

4 Numerical example In this section, we employ numerical examples to show the effectiveness of our method.

First, we will show that we can solve Formulation 3 by employing a Meta-controlled Boltzmann machine employing 15 stocks in a Japan market. Second, we compare computing time between the Meta-controlled Boltzmann machine and LINGO 8.0 employing 225 stocks in a Japan market.

4.1 Comparison of results of 15-stock problem The nominated stocks for investing are 15 stocks including Toray Industries, Obayashi Corp, Snow Brand Milk, Teijin, Kao, Asahi Glass, Daikin Industries, Hitachi, Honda Motor, Dai Nippon Printing, Taisho Marine & Fire, Tobu Rail, All Nippon Airways (ANA), Osaka Gas and Shochiku. The price of each stock is illustrated in Table 2. The simulation conditions are shown in Table 3. 1 billion yen, 500 million yen and 100 million yen are examined for an amount of funds for investing.

Also we calculate Formulation 3 by LINGO 8.0 in order to compare with the result of the Meta-controlled Boltzmann machine. Acceptable least rate R of the expected return is set to 0.015.

Table 3: Parameters of Meta-controlled Boltzmann machine for 15-stock problem

K_u	1.0
K_l	0.0, 0.3, 0.5
Start temperature	100
Restructuring temperature	0.0001
Termination temperature	0.00001
Executing number of the meta-controlling layer 1	60
Executing number of the meta-controlling layer 2	60
Executing number of the lower layer	10000

Results of the Meta-controlled Boltzmann machine are shown in Tables 4 to 6 and results of Formulation 3 by LINGO 8.0 are shown in Table 7.

First, we compare the result between the Meta-controlled Boltzmann machine in the case of $K_l = 0.0$ and LINGO 8.0. Comparing the expected return rate, the risk and the total investing funds, we can obtain sufficiently good result by employing the Meta-controlled Boltzmann machine. Although some pattern of the portfolio differs between them, it has pattern which does not purchase stocks which the value of n_i is smaller in LINGO8.0. Moreover, in Tables 4 to 6, as we change the value of K_l, we can obtain the various expected return rate and risk which satisfy the decision maker's aspiration level. When the value of K_l is set to a larger value, it shows that Dai Nippon Printing was removed from the portfolio and Daikin Industries and Honda Motor are newly included in the portfolio. Moreover, the amount of dealings of Snow Brand Milk is decreasing larger. Therefore, Daikin Industries and Honda Motor are high-risk high-return stocks. On the other hand, Dai Nippon Printing and Snow Brand Milk are low-risk low-return stocks. Also the amount of funds can be distributed efficiently over stocks. As these results, we can obtain the sufficiently good approximate solution of Formulation 3 by employing the Meta-controlled Boltzmann machine.

4.2 Comparison of results of 225-stock problem In this simulation, we mainly compare computing time employing large scale data of 225 Japanese stocks. The employed data is daily stock price data which is chosen from Nikkei 225 on the 60th from June 2 to August 26, 1997.

The simulation conditions are shown in Table 8. An acceptable least rate of the expected return R for LINGO 8.0 is set to 0.001. Since, as for this data period, the Japan market is in depressed economical state and high expected return rate cannot be expected, we set R to a rather low value. Moreover, 1 billion yen, 500 million yen, and 100 million yen are an amount of funds which a decision maker can manage is as.

Results of Meta-controlled Boltzmann machine are shown in Tables 9 to 11 and solutions of Formulation 3 by LINGO 8.0 are shown in Table 12. We compare the result between the Meta-controlled Boltzmann machine in the case of $K_l = 0.0$ and LINGO 8.0. The case where the amount of funds is 1 billion yen, we can obtain the solution which the risk is almost not different and the expected return rate is higher by employing the Meta-controlled Boltzmann machine. The case where an amount of funds is 500 million yen and 100 million yen, we can obtain the sufficiently large expected return rate, although it is a little inferior in the risk. Generally, some of nonlinear programming problem can not guarantee the uniqueness and optimality of a obtained solution. Therefore, it is sometimes

Table 4: Results of 15-stock problem by Meta-controlled Boltzmann machine($K_l = 0.0$)

	Amount of investing funds					
	1 billion yen		500 million yen		100 million yen	
	Number of dealing blocks	Allocated funds(yen)	Number of dealing blocks	Allocated funds(yen)	Number of dealing blocks	Allocated funds(yen)
Toray Industries	0	-	0	-	0	-
Obayashi Corp	0	-	0	-	0	-
Snow Brand Milk	727	324,424,000	377	168,142,000	57	25,422,000
Teijin	0	-	0	-	0	-
Kao	87	87,870,000	43	43,430,000	8	8,080,000
Asahi Glass	0	-	0	-	0	-
Daikin Industries	0	-	0	-	0	-
Hitachi	138	106,260,000	61	46,970,000	17	13,090,000
Honda Motor	0	-	0	-	0	-
Dai Nippon Printing	90	126,000,000	52	72,800,000	3	4,200,000
Taisho Marine & Fire	0	-	0	-	0	-
Tobu Rail	7	3,003,000	4	1,716,000	12	5,148,000
All Nippon Airways	137	106,175,000	61	47,275,000	28	21,700,000
Osaka Gas	532	133,000,000	268	67,000,000	31	2,170,000
Shochiku	95	109,250,000	43	49,450,000	10	11,150,000
Total		995,800,000		496,783,000		96,890,000
Expected Return Rate	0.01610		0.01605		0.01558	
Risk	0.000676		0.000674		0.000716	

Table 5: Results of 15-stock problem by Meta-controlled Boltzmann machine($K_l = 0.3$)

	Amount of investing funds					
	1 billion yen		500 million yen		100 million yen	
	Number of dealing blocks	Allocated funds(yen)	Number of dealing blocks	Allocated funds(yen)	Number of dealing blocks	Allocated funds(yen)
Toray Industries	0	-	0	-	0	-
Obayashi Corp	0	-	0	-	0	-
Snow Brand Milk	543	242,178,000	268	119,528,000	35	15,610,000
Teijin	0	-	0	-	0	-
Kao	104	105,040,000	52	52,520,000	8	8,080,000
Asahi Glass	0	-	0	-	0	-
Daikin Industries	0	-	0	-	11	7,018,000
Hitachi	133	102,410,000	56	43,120,000	14	10,780,000
Honda Motor	0	-	0	-	8	9,520,000
Dai Nippon Printing	157	219,800,000	84	117,600,000	0	-
Taisho Marine & Fire	0	-	0	-	0	-
Tobu Rail	363	155,727,000	169	72,501,000	20	8,580,000
All Nippon Airways	3	2,325,000	6	4,650,000	18	13,950,000
Osaka Gas	166	41,500,000	100	25,000,000	44	11,000,000
Shochiku	113	129,950,000	43	64,400,000	11	12,650,000
Total		998,930,000		499,319,000		97,188,000
Expected Return Rate	0.01730		0.01725		0.01663	
Risk	0.000853		0.000846		0.000809	

Table 6: Results of 15-tock problem by Meta-controlled Boltzmann machine ($K_l = 0.5$)

	Amount of investing funds					
	1 billion yen		500 million yen		100 million yen	
	Number of dealing blocks	Allocated funds(yen)	Number of dealing blocks	Allocated funds(yen)	Number of dealing blocks	Allocated funds(yen)
Toray Industries	0	-	0	-	0	-
Obayashi Corp	0	-	0	-	0	-
Snow Brand Milk	178	79,388,000	90	40,140,000	20	8,920,000
Teijin	0	-	0	-	0	-
Kao	90	90,900,000	43	43,430,000	8	8,080,000
Asahi Glass	0	-	0	-	0	-
Daikin Industries	278	177,364,000	146	93,148,000	23	14,674,000
Hitachi	152	117,040,000	76	58,520,000	19	14,630,000
Honda Motor	79	94,010,000	39	46,410,000	6	7,140,000
Dai Nippon Printing	0	-	0	-	0	-
Taisho Marine & Fire	0	-	0	-	0	-
Tobu Rail	577	247,533,000	295	126,555,000	55	23,595,000
All Nippon Airways	36	27,900,000	9	6,975,000	4	3,100,000
Osaka Gas	1	250,000	6	1,500,000	7	1,750,000
Shochiku	144	165,600,000	72	82,800,000	12	13,800,000
Total		999,985,000		499,478,000		95,689,000
Expected Return Rate	0.01860		0.01864		0.01755	
Risk	0.001344		0.001373		0.001136	

Table 7: Results of 15-stock problem by LINGO 8.0

	Amount of investing funds					
	1 billion yen		500 million yen		100 million yen	
	Number of dealing blocks	Allocated funds(yen)	Number of dealing blocks	Allocated funds(yen)	Number of dealing blocks	Allocated funds(yen)
Toray Industries	32	16,640,000	16	8,320,000	4	2,080,000
Obayashi Corp	0	-	0	-	0	-
Snow Brand Milk	685	305,510,000	342	152,532,000	68	30,328,000
Teijin	0	-	0	-	0	-
Kao	84	84,840,000	42	42,420,000	8	8,080,000
Asahi Glass	25	23,375,000	13	12,155,000	2	1,870,000
Daikin Industries	0	-	0	-	0	-
Hitachi	81	62,370,000	40	30,800,000	8	6,160,000
Honda Motor	40	47,600,000	20	23,800,000	3	3,570,000
Dai Nippon Printing	67	93,800,000	33	46,200,000	6	8,400,000
Taisho Marine & Fire	0	-	0	-	0	-
Tobu Rail	0	-	0	-	0	-
All Nippon Airways	142	110,050,000	71	55,025,000	14	10,850,000
Osaka Gas	565	141,250,000	282	70,500,000	56	14,000,000
Shochiku	97	111,550,000	48	55,200,000	9	10,350,000
Total		996,985,000		496,952,000		95,688,000
Expected Return Rate	0.01609		0.01603		0.01534	
Risk	0.000662		0.000658		0.000612	

Table 8: Simulation conditions for 225-stock problem

(1) CPU & OS

CPU	Pentium 4 2.4GHz
OS	Windows XP

(2) Parameters for Meta-controlled Boltzmann machine

K_u	10.0
K_l	0.0, 0.3, 0.5
Start temperature	100
Restructuring temperature	0.00007
Termination temperature	0.00001
Executing times of Meta-controlling layer 1	675
Executing number of Meta-controlling layer 2	675
Executing times of Lower layer	10000

Table 9: Results of 225-stock problem by Meta-controlled Boltzmann machine($K_l = 0.0$)

	Amount of investing funds		
	1 billion yen	500 million yen	100 million yen
Expected Return Rate	0.00147	0.00169	0.00138
Risk	0.000050	0.000085	0.000087
Investing funds (yen)	990,775,000	487,778,000	83,679,000
Computing Time (second)	10.92	10.47	10.58

Table 10: Results of 225-stock problem by Meta-controlled Boltzmann machine($K_l = 0.3$)

	Amount of investing funds		
	1 billion yen	500 million yen	100 million yen
Expected Return Rate	0.00260	0.00223	0.00146
Risk	0.000122	0.000103	0.000083
Investing funds (yen)	982,501,000	484,836,000	84,531,000
Computing Time (second)	11.72	11.10	10.29

Table 11: Results of 225-stock problem by Meta-controlled Boltzmann machine($K_l = 0.5$)

	Amount of investing funds		
	1 billion yen	500 million yen	100 million yen
Expected Return Rate	0.00273	0.00251	0.00145
Risk	0.000158	0.000166	0.000083
Investing funds (yen)	983,740,000	484,113,000	82,288,000
Computing Time (second)	11.97	11.19	10.53

Table 12: Results of 225-stock problem by LINGO 8.0

	Amount of investing funds		
	1 billion yen	500 million yen	100 million yen
Expected Return Rate	0.00105	0.00101	0.00108
Risk	0.000045	0.000035	0.000028
Investing funds (yen)	981,540,500	499,977,500	88,388,000
Computing Time (second)	79.0	339.0	238.0

a local optimum [4]. In this numerical example, the solution of LINGO 8.0 is not a global optimum solution but a local optimum solution.

Let us compare computing time. Table 12 shows that LINGO 8.0 has taken 79.0 seconds by the shortest. Moreover, the case where the investing funds is smaller, LINGO 8.0 has taken hundreds seconds to obtain the solution. Therefore, its solution is not efficient. On the other hand, the Meta-controlled Boltzmann machine can obtain the solution in 10 to 12 seconds.

Moreover, as the value of K_l changes, Tables 9 to 11 shows that the various expected return rate and risk are obtained to satisfy the decision maker's aspiration level as same as in a 15-stock problem.

The above result shows that our Meta-controlled Boltzmann machine is an effective and efficient method in order to solve an integer quadratic programming problem.

5 Concluding remarks In a real investment, a stock is dealt according to a block of shares. In the formulation of a conventional portfolio selection model, it is not discussed about a block of shares for each of stocks. However, in a real investment, a decision maker should buy each stock according to a block of shares. Therefore, for example, the case where the amount of funds for investment is small, if the decision maker allocates the amount of funds according to solution of the portfolio selection model, he might not buy a stock only as one block of shares.

In this paper, we proposed the portfolio selection model in consideration of the amount of funds and a block of shares. A proposed model is formulated as an integer quadratic programming model based on Markowitz model. In this model, we expressed the investing ratio using a stock unit price, a block of shares, the number of dealing blocks and the amount of funds which a decision maker can manage.

In general, the integer nonlinear programming problem will be very difficult to solve for all but the smallest cases. Therefore, in this paper, we proposed the efficient approach to obtaining a solution employing a Meta-controlled Boltzmann machine.

The following points should be emphasized from the result of numerical examples:

- Our model which is shown in Formulation 3 can obtain a solution not with an investment ratio but with the number of dealing blocks. So the solution of our model provides more realistic solution or a real problem.

- Comparing results of a 15-stock problem, we can obtain the sufficiently good approximate solution by employing the Meta-controlled Boltzmann machine.

- Comparing results of a 225-stock problem, it is shown that our approximate solution employing a Meta-controlled Boltzmann machine is an effective model in order to solve an integer quadratic programming problem.

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