INTUITIONISTIC FUZZY ASSOCIATIVE $\mathcal{I}$-IDEALS OF IS-ALGEBRAS

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Abstract. In this paper, we introduce the concept of intuitionistic fuzzy associative $\mathcal{I}$-ideals of IS-algebras and investigate some related properties.

1.Introduction and Preliminaries

The notion of BCI-algebras was proposed by Iseki in 1966. For the general development of BCI-algebras, the ideal theory plays an important role. In 1993, Jun et al.[1] introduced a new class of algebras related to BCI-algebras and semigroups, called a BCI-semigroup, In 1998, for the convenience of study, Jun et al.[3] renamed the BCI-semigruop as IS-algebra and studied further properties of these algebras. In [6] Roh et al. introduced the concept of associative $\mathcal{I}$-ideals and strong $\mathcal{I}$-ideals in an IS-algebra. Jun et al. [8] established the fuzzification of $\mathcal{I}$-ideals in IS-algebras and E.H.Roh [4] studied the properties of fuzzy associative $\mathcal{I}$-ideals of IS-algebras.

In this paper, we introduce the concept of intuitionistic fuzzy associative $\mathcal{I}$-ideals of IS-algebras and investigate some related properties.

By a BCI-algebra we mean an algebra $(X;*,0)$ of type $(2,0)$ satisfying the following conditions:

(I) $((x*y)*(x*z))*(z*y)=0$

(II) $(x*(x*y))*y=0$

(III) $x*x=0$

(IV) $x*y=0$ and $y*x=0$ imply $x=y$

A partial ordering on X can be defined by $x\leq y$ if and only if $x*y=0$.

A nonempty subset I of a BCI-algebra X is called an ideal of X if

(i) $0\in I$

(ii) $x*y\in I$ and $y\in I$ imply $x\in I$ for all $x,y\in X$.

A fuzzy set μ is a function $\mu : X\to[0,1]$, and the complement of μ, denoted by $\overline{\mu}$, is the fuzzy set in X given by $\overline{\mu}(x)=1-\mu(x)$ for all $x\in X$. We shall write $a\wedge b$ for $min\{a,b\}$ and $a\vee b$ for $max\{a,b\}$, where a and b are any real numbers.

A fuzzy set μ in a BCI-algebra X is called a fuzzy ideal of X if (i) $\mu(0)\geq\mu(x)$, (ii) $\mu(x)\geq\mu(x*y)\wedge\mu(y)$ for all $x,y\in X$.

An intuitionistic fuzzy set (briefly, IFS) A in nonempty set X is an object having the form $A=\{(x,\alpha_A(x),\beta_A(x))\mid x\in X\}$, where the function $\alpha_A : X\to[0,1]$ and $\beta_A : X\to[0,1]$ denote the degree of membership and the degree of nonmembership, respectively, and

$$0\leq\alpha_A(x)+\beta_A(x)\leq 1,\qquad\qquad \forall\in X$$

An intuitionistic fuzzy set $A=\{(x,\alpha_A(x),\beta_A(x))\mid x\in X\}$ in X can be identified to an ordered pair (α_A,β_A) in $I^X\times I^X$. For the sake of simplicity, we shall use the symbol $A=(\alpha_A,\beta_A)$ for the $IFSA=\{(x,\alpha_A(x),\beta_A(x))\mid x\in X\}$.

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An IS-algebra X is a nonempty set X with two binary operations "*" and "·" and constant 0 satisfying the axioms:

(I) $I(X) = (X; *, 0)$ is a BCI-algebra,

(II) $S(X) = (X, \cdot)$ is a semigroup,

(III) The operation "·" is distribute over the operation "∗", that is, $x\cdot(y*z) = (x\cdot y)*(x\cdot z)$ and $(x * y) \cdot z = (x \cdot z) * (y \cdot z)$ for all $x, y, z \in X$.

A nonempty subset A of a semigroup $S(X) = (X, \cdot)$ is said to be stable if $xa \in A$, whenever $x \in S(X)$ and $a \in A$.

A nonempty subset A of an IS-algebra X is called an $\mathcal{I}$-ideal of X if

(i) A is a stable subset of $S(X)$,

(ii) for any $x, y \in I(X), x * y \in A$ and $y \in A$ imply that $x \in A$.

Note that if A is an $\mathcal{I}$-ideal of an IS-algebra, then $0 \in A$. Thus A is an ideal of $I(X)$.

A fuzzy set in a semigroup $S(X) = (X, \cdot)$ is said to be fuzzy stable if $\mu(x \cdot y) \geq \mu(y)$ for all $x, y \in X$.

A fuzzy set μ in an IS-algebra X is called a fuzzy $\mathcal{I}$-ideal of X if (i) μ is a fuzzy stable set in $S(X)$; (ii) μ is a fuzzy ideal of a BCI-algebra X.

Definition 1.1([6]) A nonempty subset A of an IS-algebra X is called an associative $\mathcal{I}$-ideal (briefly, $A\mathcal{I}$-ideal) of X if (i) A is a stable subset of $S(X)$; (ii) for any $x, y, z \in I(X), (x * y) * z \in A$ and $y * z \in A$ imply that $x \in A$.

Definition 1.2([4]) A fuzzy set μ in an IS-algebra X is called a fuzzy associative $\mathcal{I}$-ideal (briefly, $FA\mathcal{I}$-ideal) of X if (i) μ is a fuzzy stable set in $S(X)$; (ii) $\mu(x) \geq \mu((x*y)*z)\wedge\mu(y*z)$ for all $x, y, z \in X$.

2. Intuitionistic fuzzy associative $\mathcal{I}$-ideals of IS-algebras

Definition 2.1 An $IFSA = (\alpha_A, \beta_A)$ in an IS-algebra X is called an intuitionistic fuzzy $\mathcal{I}$-ideal (briefly, $IF\mathcal{I}$-ideal) of X if

(I) $\alpha_A(x \cdot y) \geq \alpha_A(y)$,

(II) $\beta_A(x \cdot y) \leq \beta_A(y)$,

(III) $\alpha_A(x) \geq \alpha_A(x * y) \wedge \alpha_A(y)$,

(IV) $\beta_A(x) \leq \beta_A(x * y) \vee \beta_A(y)$.

for all $x, y \in X$.

Definition 2.2 An $IFSA = (\alpha_A, \beta_A)$ in an IS-algebra X is called an intuitionistic fuzzy associative $\mathcal{I}$-ideal(briefly, $IFA\mathcal{I}$-ideal) of X if

(I) $\alpha_A(x \cdot y) \geq \alpha_A(y)$,

(II) $\beta_A(x \cdot y) \leq \beta_A(y)$,

(III) $\alpha_A(x) \geq \alpha_A((x * y) * z) \wedge \alpha_A(y * z)$,

(IV) $\beta_A(x) \leq \beta_A((x * y) * z) \vee \beta_A(y * z)$.

for all $x, y, z \in X$.

Example 2.3 Consider an IS-algebra $X = \{0, a, b, c\}$ with the following Cayley tables;

$*$	0	a	b	c
0	0	0	b	b
a	a	0	c	b
b	b	b	0	0
c	c	b	a	0

$\cdot$	0	a	b	c
0	0	0	0	0
a	0	a	0	a
b	0	0	b	b
c	0	a	b	c

Define an $IFSA = (\alpha_A, \beta_A)$ in X as follows:

$\alpha_A(0) = \alpha_A(b) = 0.6$ and $\alpha_A(a) = \alpha_A(c) = 0.2$; $\beta_A(0) = \beta_A(b) = 0$ and $\beta_A(a) = \beta_A(c) = 0.3$. Then $A = (\alpha_A, \beta_A)$ is an $IF\mathcal{I}$-ideal of X.

Example 2.4 Consider an IS-algebra $X = \{0, a, b, c\}$ with Cayley tables as follows:

$*$	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

$\cdot$	0	a	b	c
0	0	0	0	0
a	0	a	b	c
b	0	a	b	c
c	0	0	0	0

Define an $IFSA = (\alpha_A, \beta_A)$ in X as follows:
$\alpha_A(0) = \alpha_A(a) = 1$ and $\alpha_A(b) = \alpha_A(c) = t$; $\beta_A(0) = \beta_A(a) = 0$ and $\beta_A(b) = \beta_A(c) = s$, where $t \in [0, 1], s \in [0, 1]$ and $t + s \leq 1$. Then $A = (\alpha_A, \beta_A)$ is an $IFA\mathcal{I}$-ideal of X.

Proposition 2.5 Every $IFA\mathcal{I}$-ideal is an $IF\mathcal{I}$-ideal.

Proof Let $IFSA = (\alpha_A, \beta_A)$ be an $IFA\mathcal{I}$-ideals of an IS-algebra X and let $x, y \in X$. Then $\alpha_A(x) \geq \alpha_A((x*y)*0) \wedge \alpha_A(y*0) = \alpha_A(x*y) \wedge \alpha_A(y)$ and $\beta_A(x) \leq \beta_A((x*y)*0) \vee \beta_A(y*0) = \beta_A(x*y) \vee \beta_A(y)$. Hence $IFSA = (\alpha_A, \beta_A)$ is an $IF\mathcal{I}$-ideal of X.

The following example shows that the converse of proposition 2.5 may not be true.

Example 2.6 Let X be an IS-algebra in Example 2.3 and let $IFSA = (\alpha_A, \beta_A)$ defined by $\alpha_A(0) = \alpha_A(b) = 0.6$ and $\alpha_A(a) = \alpha_A(c) = 0.2$; $\beta_A(0) = \beta_A(b) = 0$ and $\beta_A(a) = \beta_A(c) = 0$. It's routine to check that $IFSA$ is an $IF\mathcal{I}$-ideal. But $IFSA$ is not an $IFA\mathcal{I}$-ideal of X, since $\alpha_A(a) < \alpha_A((a*b)*c) \wedge \alpha_A(b*c)$.

Proposition 2.7 Let $IFSA = (\alpha_A, \beta_A)$ be an $IF\mathcal{I}$-ideal of an IS-algebra X. If $x \leq y$ in X, then $\alpha_A(x) \geq \alpha_A(y)$ and $\beta_A(x) \leq \beta_A(y)$, that is, α_A is order-reserving and β_A is order-preserving.

Proof Let $x, y \in X$ be such that $x \leq y$. Then $x*y = 0$ and so $\alpha_A(x) \geq \alpha_A(x*y) \wedge \alpha_A(y) = \alpha_A(0) \wedge \alpha_A(y) = \alpha_A(y)$ and $\beta_A(x) \leq \beta_A(x*y) \vee \beta_A(y) = \beta_A(0) \vee \beta_A(y) = \beta_A(y)$.

Proposition 2.8 Let $IFSA$=(α_A, β_A) be an $IF\mathcal{I}$-ideal of an IS-algebra X. Then $IFSA$ is an $IFA\mathcal{I}$-ideal of X if and only if it satisfies $\alpha_A(x) \geq \alpha_A((x*y)*y)$; $\beta_A(x) \leq \beta_A((x*y)*y)$ for all $x, y \in X$.

Proof Let $IFSA$=(α_A, β_A) be an $IFA\mathcal{I}$-ideal of X, $\alpha_A(x) \geq \alpha_A((x*y)*y) \wedge \alpha_A(y*y) = \alpha_A((x*y)*y) \wedge \alpha_A(0) = \alpha_A((x*y)*y)$ and $\beta_A(x) \leq \beta_A((x*y)*y) \vee \beta_A(y*y) = \beta_A((x*y)*y) \vee \beta_A(0) = \beta_A((x*y)*y)$.

Conversely, note that $((x*z)*z)*(y*z) = ((x*z)*(y*z))*z \leq (x*y)*z$ for all $x, y, z \in X$. If follows that $\alpha_A(x) \geq \alpha_A((x*z)*z) \geq \alpha_A(((x*z)*z)*(y*z)) \wedge \alpha_A(y*z) \geq \alpha_A((x*y)*z) \wedge \alpha_A(y*z)$ and $\beta_A(x) \leq \beta_A((x*z)*z) \leq \beta_A(((x*z)*z)*(y*z)) \vee \beta_A(y*z) \leq \beta_A((x*y)*z) \vee \beta_A(y*z)$ for all $x, y, z \in X$. This completes the proof.

Lemma 2.9 An $IFSA$=(α_A, β_A) is an $IFA\mathcal{I}$-ideal of X if and only if the fuzzy sets α_A and $\overline{\beta}_A$ are $FA\mathcal{I}$-ideals of X.

Proof Let $IFSA$=(α_A, β_A) be an $IFA\mathcal{I}$-ideal of X, clearly α_A is an $FA\mathcal{I}$-ideal of X. For any $x, y, z \in X$, we have $\overline{\beta}_A(x \cdot y) = 1 - \beta_A(x \cdot y) \geq 1 - \beta_A(y) = \beta_A(y)$ and $\overline{\beta}_A(x) = 1-\beta_A((x*y)*z) \vee \beta_A(y*z) = (1-\beta_A((x*y)*z)) \wedge (1-\beta_A(y*z)) = \overline{\beta}_A((x*y)*z) \wedge \overline{\beta}_A(y*z)$. Hence $\overline{\beta}_A(y)$ is an $FA\mathcal{I}$-ideal of X.

Conversely, assume that α_A and $\overline{\beta}_A$ are $FA\mathcal{I}$-ideals of X. For any $x, y, z \in X$, we get(1) $\overline{\beta}_A(x * y) \geq \overline{\beta}_A(y)$ and that $\beta_A(x \cdot y) \leq \beta_A(y)$; (2) $\overline{\beta}_A(x) \geq \overline{\beta}_A((x * y) * z) \wedge \overline{\beta}_A(y * z)$ and that $1 - \beta_A(x) \geq (1 - \beta_A((x * y) * z)) \wedge (1 - \beta_A(y * z)) = 1 - \beta_A((x * y) * z) \vee \beta_A(y * z)$, that is, $\beta_A(x) \leq \beta_A((x * y) * z) \vee \beta_A(y * z)$. Hence $IFSA$=(α_A, β_A) is an $IFA\mathcal{I}$-ideal of X.

Theorem 2.10 Let $A = (\alpha_A, \beta_A)$ be an IFS in an IS-algebra X. Then $A = (\alpha_A, \beta_A)$ is an $IFA\mathcal{I}$-ideal of X if and only if $\Box A = (\alpha_A, \overline{\alpha}_A)$ and $\Diamond A = (\overline{\beta}_A, \beta_A)$ are $IFA\mathcal{I}$-ideals of X.

Proof If $A = (\alpha_A, \beta_A)$ is an $IFA\mathcal{I}$-ideal, then $\alpha_A = \overline{\overline{\alpha}}_A$ and β_A are $FA\mathcal{I}$-ideals of X from Lemma 2.9, hence $\Box A = (\alpha_A, \overline{\alpha}_A)$ and $\Diamond A = (\overline{\beta}_A, \beta_A)$ are $IFA\mathcal{I}$-ideals of X. Conversely, if $\Box A = (\alpha_A, \overline{\alpha}_A)$ and $\Diamond A = (\overline{\beta}_A, \beta_A)$ are $IFA\mathcal{I}$-ideals of X, then α_A and $\overline{\beta}_A$ are $FA\mathcal{I}$-ideals of X, hence $A = (\alpha_A, \beta_A)$ is an $IFA\mathcal{I}$-ideal of X.

For any $t \in [0, 1]$ and a fuzzy set μ in a nonempty set X, the set $U(\mu; t) = \{x \in X \mid \mu(x) \geq t\}$ is called an upper t-level cut of μ and the set $L(\mu; t) = \{x \in X \mid \mu(x) \leq t\}$ is called a lower t-level cut of μ.

Theorem 2.11 Let $A = (\alpha_A, \beta_A)$ be an IFS in an IS-algebra X, then $IFSA$=(α_A, β_A) is an $IFA\mathcal{I}$-ideal if and only if for all $s, t \in [0, 1]$, the nonempty sets $U(\alpha_A; t)$ and $L(\beta_A; s)$ are $A\mathcal{I}$-ideals of X.

Proof Let $IFSA$=(α_A, β_A) be an $IFA\mathcal{I}$-ideal of X and let $x \in S(X)$ and $y \in U(\alpha_A; t)$. Then $\alpha_A(y) \geq t$ and so $\alpha_A(x \cdot y) \geq \alpha_A(y) \geq t$, which implies that $x \cdot y \in U(\alpha_A; t)$. Hence $U(\alpha_A; t)$ is a stable subset of $S(X)$. Let $x, y, z \in I(X)$ be such that $(x * y) * z \in U(\alpha_A; t)$ and $y * z \in U(\alpha_A; t)$. Then $\alpha_A((x * y) * z) \geq t$ and $\alpha_A(y * z) \geq t$. It follows that $\alpha_A(x) \geq \alpha_A((x*y)*z) \wedge \alpha_A(y*z) \geq t$, so that $x \in U(\alpha_A; t)$. Hence $U(\alpha_A; t)$ is an $A\mathcal{I}$-ideal of X. Now let $x \in S(X)$ and $y \in L(\beta_A; s)$. Then $\beta_A(y) \leq s$ and so $\beta_A(x \cdot y) \leq \beta_A(y) \leq s$, which implies that $x \cdot y \in L(\beta_A; s)$. Hence $L(\beta_A; s)$ is a stable subset of $S(X)$. Let $x, y, z \in I(X)$ be such that $(x * y) * z \in L(\beta_A; s)$ and $y * z \in L(\beta_A; s)$. Then $\beta_A((x * y) * z) \leq s$ and $\beta_A(y * z) \leq s$, so that $x \in L(\beta_A; s)$. Hence $L(\beta_A; s)$ is an $A\mathcal{I}$-ideal of X.

Conversely, assume that for each $s, t \in [0, 1]$, the nonempty sets $U(\alpha_A; t)$ and $L(\beta_A; s)$ are $A\mathcal{I}$-ideals of X. If there are $x_0, y_0 \in S(X)$ such that $\alpha_A(x_0 \cdot y_0) < \alpha_A(y_0)$, then taking $t_0 = (\alpha_A(x_0 \cdot y_0) + \alpha_A(y_0))/2$, we have $\alpha_A(x_0 \cdot y_0) < t_0 < \alpha_A(y_0)$. It follows that $y_0 \in U(\alpha_A; t_0)$ and $x_0 \cdot y_0 \notin U(\alpha_A; t_0)$. This is a contradiction. Therefore α_A is a fuzzy stable set in $S(X)$. If there are $x_0, y_0 \in S(X)$ such that $\beta_A(x_0 \cdot y_0) < \beta_A(y_0)$, then taking $s_0 = (\beta_A(x_0 \cdot y_0) + \beta_A(y_0))/2$, we have $\beta_A(x_0 \cdot y_0) > s_0 > \beta_A(y_0)$, it follows that $y_0 \in L(\beta_A; s_0)$ and $x_0 \cdot y_0 \notin L(\beta_A; s_0)$. This is a contradiction, Therefore β_A is a fuzzy stable set in $S(X)$. Suppose that $\alpha_A(x_0) < \alpha_A((x_0 * y_0) * z_0) \wedge \alpha_A(y_0 * z_0)$ for some $x_0, y_0, z_0 \in X$, putting $t_0 = (\alpha_A(x_0) + \alpha_A((x_0 * y_0) * z_0) \wedge \alpha_A(y_0 * z_0))/2$, then $\alpha_A(x_0) < t_0 < \alpha_A((x_0 * y_0) * z_0) \wedge \alpha_A(y_0 * z_0)$. Which shows that $(x_0 * y_0) * z_0, y_0 * z_0 \in U(\alpha_A; t_0)$ and $x_0 \notin U(\alpha_A; t_0)$. This is impossible. Finally, assume that $a, b, c \in X$ such that $\beta_A(a) > \beta_A((a * b) * c) \vee \beta_A(b * c)$. Taking $s_0 = (\beta_A(a) + \beta_A((a * b) * c))/2$, then $\beta_A((a * b) * c) \vee \beta_A(b * c) < s_0 < \beta_A(a)$. Therefore $(a * b) * c$ and $b * c \in L(\beta_A; s_0)$, but $a \notin L(\beta_A; s_0)$, which is a contradiction, this completes the proof.

Let J be a nonempty subset of $[0, 1]$.

Theorem 2.12 Let $\{I_t \mid t \in J\}$ be a collection of $A\mathcal{I}$-ideals of IS-algebra X such that
(i) $X = \bigcup_{t\in J} I_t$,
(ii) $s > t$ if and only if $I_s \subset I_t$ for all $s, t \in J$. Then an $IFSA$=(α_A, β_A) in X defined by

$$\alpha_A(x) = sup\{t \in J \mid x \in I_t\}, \beta_A(x) = inf\{t \in J \mid x \in I_t\}$$

for all $x \in X$ is an $IFA\mathcal{I}$-ideal of X.

Proof According to Theorem 2.11, it is sufficient to show that $U(\alpha_A; t)$ and $L(\beta_A; s)$ are $A\mathcal{I}$-ideals of X for every $t \in [0, \alpha_A(0)]$ and $s \in [\beta_A(0), 1]$. In order to prove that $U(\alpha_A; t)$ is an $A\mathcal{I}$-ideal of X, we divide the proof into the following two cases:
(i) $t = sup\{q \in J \mid q < t\}$
(ii) $t \neq sup\{q \in J \mid q < t\}$
For the case (i) imply that

$$x \in U(\alpha_A; t) \Leftrightarrow x \in I_q, q < t \Leftrightarrow x \in \bigcap_{q<t} I_q$$

and that $U(\alpha_A; t) = \bigcap_{q<t} I_q$, which is an $A\mathcal{I}$-ideal of X. For the case (ii), we claim that $U(\alpha_A; t) = \bigcup_{q\geq t} I_q$. If $x \in \bigcup_{q\geq t} I_q$, then $x \in I_q$ for some $q \geq t$, It follows that $\alpha_A(x) \geq q \geq t$, so that $x \in U(\alpha_A; t)$. This shows that $\bigcup_{q\geq t} I_q \subseteq U(\alpha_A; t)$. Now assume that $x \notin \bigcup_{q\geq t} I_q$, Then $x \notin I_q$ for all $q \geq t$, Since $t \neq \{q \in J \mid q < t\}$, there exists $\varepsilon > 0$ such that $(t - \varepsilon, t) \bigcap J = \varnothing$. Hence $x \notin I_q$ for all $q > t - \varepsilon$, which means that $x \in I_q$, then $q \leq t - \varepsilon$. Thus $\alpha_A(x) \leq t - \varepsilon < t$, and so $x \notin U(\alpha_A; t)$. Therefore $U(\alpha_A; t) \subseteq \bigcup_{q\geq t} I_q$, and thus $U(\alpha_A; t) = \bigcup_{q\geq t} I_q$, which is an $A\mathcal{I}$-ideal of X. Next we prove that $L(\beta_A; s)$ is $A\mathcal{I}$-ideal of X. We consider the following two cases:
(iii) $s = inf\{r \in J \mid s < r\}$ (iv) $s \neq inf\{r \in J \mid s < r\}$
For the case (iii), we have

$$x \in L(\beta_A; s) \Leftrightarrow x \in I_r, \forall s < r \Leftrightarrow x \in \bigcap_{s<r} I_r$$

and hence $L(\beta_A; s) = \bigcap_{s<r} I_r$ which is an $A\mathcal{I}$-ideal of X. For the case (iv), there exists $\varepsilon > 0$ such that $(s, s + \varepsilon) \bigcap J = \varnothing$. We will show that $L(\beta_A; s) = \bigcup_{s\geq r} I_r$. If $x \in \bigcup_{s\geq r} I_r$, then $x \in I_r$ for some $r \leq s$. It follows that $\beta_A(x) \leq r \leq s$, so that $x \in L(\beta_A; s)$. Hence $\bigcup_{s\geq r} I_r \subseteq L(\beta_A; s)$. Now if $x \notin \bigcup_{s\geq r} I_r$, then $x \notin I_r$ for all $r \leq s$, which implies that $x \in I_r$ for all $r < s + \varepsilon$, that is, if $x \in I_r$, then $r \geq s + \varepsilon$. Thus $\beta_A(x) \geq s + \varepsilon > s$, that is, $x \notin L(\beta_A; s)$. Therefore $L(\beta_A; s) \subseteq \bigcup_{s\geq r} I_r$ and consequently $L(\beta_A; s) = \bigcup_{s\geq r} I_r$ which is an $A\mathcal{I}$-ideal of X. This completes the proof.

3. On homomorphism of IS-algebras

Definition 3.1 A mapping $f : X \to Y$ of IS-algebras is called a homomorphism if
(i) $f(x * y) = f(x) * f(y)$ for all $x, y \in I(X)$
(ii) $f(x \cdot y) = f(x) \cdot f(y)$ for all $x, y \in S(X)$
For any $IFSA$=(α_A, β_A) in Y. We define a new $IFSAf$=(α_A^f, β_A^f) in X by

$$\alpha_A^f(x) = \alpha_A(f(x)), \beta_A^f(x) = \beta_A(f(x)), \qquad \forall x \in X$$

Theorem 3.2 Let $f : X \to Y$ be a homomorphism of IS-algebras. If $IFSA$=(α_A, β_A) is an $IFA\mathcal{I}$-ideal of Y, then $IFSA^f = (\alpha_A^f, \beta_A^f)$ in X is an $IFA\mathcal{I}$-ideal of X.

Proof Suppose $IFSA$=(α_A, β_A) is an $IFA\mathcal{I}$-ideal of Y, then $\alpha_A^f(x \cdot y) = \alpha_A(f(x \cdot y)) = \alpha_A(f(x) \cdot f(y)) \geq \alpha_A(f(y)) = \alpha_A^f(y)$ and $\beta_A^f(x \cdot y) = \beta_A(f(x \cdot y)) = \beta_A(f(x) \cdot f(y)) \leq \beta_A(f(y)) = \beta_A^f(y)$. Now let $x, y, z \in X$, then $\alpha_A^f(x) = \alpha_A(f(x)) \geq \alpha_A((f(x) * f(y)) * f(z)) \wedge \alpha_A(f(y) * f(z)) = \alpha_A(f((x * y) * z)) \wedge \alpha_A(f(y * z)) = \alpha_A^f((x * y) * z) \wedge \alpha_A^f(y * z)$ and $\beta_A^f(x) = \beta_A(f(x)) \leq \beta_A((f(x) * f(y)) * f(z)) \vee \beta_A(f(y) * f(z)) = \beta_A(f((x * y) * z)) \vee \beta_A(f(y * z)) = \beta_A^f((x * y) * z) \vee \beta_A^f(y * z)$. Hence $IFSA^f = (\alpha_A^f, \beta_A^f)$ is an $IFA\mathcal{I}$-ideal of X.

If we strengthen the condition of f, then we can construct the converse of Theorem 3.2 as follows:

Theorem 3.3 Let $f : X \to Y$ be an epimorphism of IS-algebras and let $IFSA = (\alpha_A, \beta_A)$ be an IFS in Y. If $IFSA^f = (\alpha_A^f, \beta_A^f)$ is an $IFA\mathcal{I}$-ideal of X, then $IFSA = (\alpha_A, \beta_A)$ is an $IFA\mathcal{I}$-ideal of Y.

Proof For any $x, y \in Y$, there exist $a, b \in X$ such that $f(a) = x$ and $f(b) = y$. Then $\alpha_A(x \cdot y) = \alpha_A(f(a) \cdot f(b)) = \alpha_A(f(a \cdot b)) = \alpha_A^f(a \cdot b) \geq \alpha_A^f(b) = \alpha_A(f(b)) = \alpha_A(y)$. Now let $x, y, z \in Y$, then $f(a) = x, f(b) = y$ and $f(c) = z$ for some $a, b, c \in X$. It follows that $\alpha_A(x) = \alpha_A(f(a)) = \alpha_A^f(a) \geq \alpha_A^f((a * b) * c) \wedge \alpha_A^f(b * c) = \alpha_A(f((a * b) * c)) \wedge \alpha_A(f(b * c)) = \alpha_A((f(a) * f(b)) * f(c)) \wedge \alpha_A(f(b) * f(c)) = \alpha_A((x * y) * z) \wedge \alpha_A(y * z)$ and $\beta_A(x) = \beta_A(f(a)) = \beta_A^f(a) \leq \beta_A^f((a * b) * c) \vee \beta_A^f(b * c) = \beta_A(f((a * b) * c)) \vee \beta_A(f(b * c)) = \beta_A((f(a) * f(b)) * f(c)) \vee \beta_A(f(b) * f(c)) = \beta_A((x * y) * z) \vee \beta_A(y * z)$. This completes the proof.

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