

A Generalization of the van den Berg - Kesten - Reimer Inequality

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Received May 6, 2021 ; Revised April 27, 2022

ABSTRACT. We introduce the Inclusion-Exclusion polynomials, and the concept of total monotonicity, which we combine to generalize the celebrated van den Berg - Kesten - Reimer inequality (BKR inequality) to real polynomials. The original BKR inequality gives an upper bound for the probability of the ‘disjoint occurrence’ of two events on the space $\{0, 1\}^n$, in terms of the product of the individual probabilities of these events. We begin by focusing on an equivalent form of this inequality expressed through the cardinalities of the cosets, then use this as the base for our generalization to real polynomials. The relation between difference equations and the Inclusion-Exclusion expansions are investigated and clarified.