Characterization of closed balls via metric projections

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Abstract. Consider the following property (P) for a bounded closed convex set C in a Banach space X. (P): For every $x \in X$, a positive-scalar multiple of x gives a nearest point in C to x. Then it is clear that a closed ball with its center at the origin has this property. The converse of this assertion is the subject of this paper, and it is proved that a bounded closed convex set $C \subset X$ with $0 \in \operatorname{Int} C$ possessing property (P) is a closed ball with center 0, provided $\dim X > 1$. The proof is achieved by reducing the general case to that of 2-dimensional spaces.