THE n-TH OPERATOR VALUED DIVERGENCES $\Delta_{i,x}^{[n]}(A|B)$

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Abstract. Let A and B be strictly positive linear operators on a Hilbert space $\mathcal{H}$. As a generalization of the relative operator entropy $S(A|B) \equiv A^{\frac{1}{2}}(\log A^{-\frac{1}{2}}BA^{-\frac{1}{2}})A^{\frac{1}{2}}$ and the Tsallis relative operator entropy $T_x(A|B) \equiv A^{\frac{1}{2}}\dfrac{(A^{-\frac{1}{2}}BA^{-\frac{1}{2}})^x - I}{x}A^{\frac{1}{2}}$, we have introduced the n-th relative operator entropy $S^{[n]}(A|B)$ and the n-th Tsallis relative operator entropy $T_x^{[n]}(A|B)$ for $n \in \mathbb{N}$ and $x \in \mathbb{R}$. In this paper, we define the n-th generalized Petz-Bregman divergence $\mathscr{D}_x^{[n]}(A|B) \equiv T_x^{[n]}(A|B) - S^{[n]}(A|B)$ $(x \in R)$ corresponding to the operator valued divergence $\Delta_{1,\alpha}(A|B) \equiv T_\alpha(A|B) - S(A|B)$ $(\alpha \in [0,1])$ which is a generalization of Petz-Bregman divergence $D_{FK}(A|B) \equiv B - A - S(A|B)$. Similatly, by using $\mathscr{D}_x^{[n]}(A|B)$, we introduce the n-th operator valued divergences $\Delta_{2,x}^{[n]}(A|B)$, $\Delta_{3,x}^{[n]}(A|B)$ and $\Delta_{4,x}^{[n]}(A|B)$ corresponding to $\Delta_{2,\alpha}(A|B) \equiv S_\alpha(A|B) - T_\alpha(A|B)$, $\Delta_{3,\alpha}(A|B) \equiv -T_{1-\alpha}(B|A) - S_\alpha(A|B)$ and $\Delta_{4,\alpha}(A|B) \equiv S_1(A|B) + T_{1-\alpha}(B|A)$, respectively, and show their properties and relations among them.