FURUTA TYPE INEQUALITIES RELATED TO ANDO-HIAI INEQUALITY WITH NEGATIVE POWERS

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Abstract. Furuta inequality and Ando-Hiai inequality have been actively investigated since they were established about thirty years ago. Recently, Kian and Seo obtained the Ando-Hiai type inequality with negative powers as follows: For $A, B > 0$, $A \, \natural_{-\alpha} \, B \leq I$ for $\alpha \in [0, 1]$ implies $A^r \, \natural_{-\alpha} \, B^r \leq I$ for $r \geq 1$. Related to this result, Fujii and Nakamoto obtained Furuta type inequality with negative powers. Moreover, they discussed these generalizations. In this paper, we improve their results based on properties of Furuta inequality and Ando-Hiai inequality.

Key words and phrases. Positive operators, operator mean, Furuta inequality, Ando-Hiai inequality and grand Furuta inequality.