JI-DISTRIBUTIVE, DUALLY QUASI-DE MORGAN SEMI-HEYTING AND HEYTING ALGEBRAS

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Dedicated to Professor P.N. Shivakumar
A Great Humanitarian who changed the course of my life

Abstract. The variety $\textbf{DQD}$ of dually quasi-De Morgan semi-Heyting algebras and several of its subvarieties were investigated in the series [26] - [31]. In this paper we define and investigate a new subvariety $\textbf{JID}$ of $\textbf{DQD}$, called “JI-distributive, dually quasi-De Morgan semi-Heyting algebras”, defined by the identity: $x' \lor (y \to z) \approx (x' \lor y) \to (x' \lor z)$, as well as the (closely related) variety $\textbf{DSt}$ of dually Stone semi-Heyting algebras. Firstly, we prove that $\textbf{DSt}$ and $\textbf{JID}$ are discriminator varieties of level 1 and level 2 respectively. Secondly, we give a characterization of subdirectly irreducible algebras of the subvariety $\textbf{JID}_1$ of $\textbf{JID}$ of level 1. As applications, we derive that the variety $\textbf{JID}_1$ is the join of the variety $\textbf{DSt}$ and the variety of De Morgan Boolean semi-Heyting algebras, give a concrete description of the subdirectly irreducible algebras in the subvariety $\textbf{JIDL}_1$ of $\textbf{JID}_1$ defined by the linear identity: $(x \to y) \lor (y \to x) \approx 1$, and deduce that the variety $\textbf{JIDL}_1$ is the join of the variety $\textbf{DStHC}$ generated by the dually Stone Heyting chains and the variety generated by the 4-element De Morgan Boolean Heyting algebra. Furthermore, we present an explicit description of the lattice of subvarieties of $\textbf{JIDL}_1$ and equational bases for all subvarieties of $\textbf{JIDL}_1$. Finally, we prove that the amalgamation property holds for all subvarieties of $\textbf{DStHC}$.

Key words and phrases. JI-distributive, dually quasi-De Morgan semi-Heyting algebra, De Morgan semi-Heyting algebra, De Morgan Heyting algebra, dually Stone semi-Heyting algebra, dually Stone Heyting algebra, discriminator variety, simple algebra, subdirectly irreducible algebra, equational base.