NORM INEQUALITIES RELATED TO THE MATRIX GEOMETRIC
MEAN OF NEGATIVE POWER

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Abstract. In this paper, we show norm inequalities related to the matrix geometric mean of negative power for positive definite matrices: For positive definite matrices $A$ and $B$,

$$\left\| e^{(1-\beta) \log A + \beta \log B} \right\| \leq \|A \#_\beta B\| \leq \|A^{1-\beta} B^\beta\|$$

for every unitarily invariant norm and $-1 \leq \beta \leq -\frac{1}{2}$, where the $\beta$-quasi geometric mean $A \#_\beta B$ is defined by $A \#_\beta B = A^{\frac{1}{2}} (A^{-\frac{1}{2}} B A^{-\frac{1}{2}})^\beta A^{\frac{1}{2}}$. For our purposes, we show the Ando-Hiai log-majorization of negative power.

Key words and phrases. Ando-Hiai inequality, matrix geometric mean, unitarily invariant norm, positive definite matrix.