Mixed Schwarz Inequalities
via the Matrix Geometric Mean

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Abstract. In this paper, by using the Cauchy-Schwarz inequality for matrices via
the matrix geometric mean due to J.I. Fujii, we show the following matrix version of a
mixed Schwarz inequality for any square matrices: Let $A$ be an $n$-square matrix. For
any $n$-square matrices $X, Y$

$$|Y^*AX| \leq X^*|A|^{2\alpha}X \oplus U^*Y^*|A|^\alpha YU$$

holds for all $\alpha, \beta \in [0, 1]$ with $\alpha + \beta = 1$, where $U$ is a unitary matrix in a polar
decomposition of $Y^*AX = U|Y^*AX|$. As applications, we show matrix Parseval’s
equation, Lin’s type extensions for a weighted version of a mixed Schwarz inequality,
and a weighted version of the Wielandt inequality for matrices.

Key words and phrases. weighted mixed Schwarz inequality, matrix geometric mean, Lin’s type exten-
sion, Wielandt inequality.