CLASS $p$-$wA(s,t)$ OPERATORS AND RANGE KERNEL ORTHOGONALITY

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Received March 27, 2017; revised June 19, 2017

Abstract. Let $T = U|T|$ be a polar decomposition of a bounded linear operator $T$ on a complex Hilbert space with ker $U = \ker |T|$. $T$ is said to be class $p$-$wA(s,t)$ if $\left(\frac{\|T^*\|^2|T|^2|T^*|^2}{s} + \frac{t}{2}\right)^{\frac{1}{p}} \geq \|T^*\|^2|T|^2$ and $\|T\|^2|T^*|^2 \geq \left(\frac{\|T\|^2|T^*|^2}{s} + \frac{t}{2}\right)^{\frac{1}{p}}$ with $0 < p \leq 1$ and $0 < s, t, s + t \leq 1$. This is a generalization of $p$-hyponormal or class $A$ operators. In this paper we prove following assertions. (i) If $T$ is class $p$-$wA(s,t)$, then $T$ is normaloid and isoloid. (ii) If $T$ is class $p$-$wA(s,t)$ and $\sigma(T) = \{\lambda\}$, then $T = \lambda$. (iii) If $T$ is class $p$-$wA(s,t)$, then $T$ is finite and the range of generalized derivation $\delta_T : B(\mathcal{H}) \ni X \rightarrow TX - XT \in B(\mathcal{H})$ is orthogonal to its kernel. (iv) If $S$ is class $p$-$wA(s,t)$, $T^*$ is an invertible $p$-$wA(t,s)$ operator and $X$ is a Hilbert-Schmidt operator such that $SX = XT$, then $S^*X = XT^*$.

Key words and phrases. class $p$-$wA(s,t)$, normaloid, isoloid, finite, orthogonality.