On some matrix mean inequalities with Kantorovich constant

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Abstract. Let A and B be positive definite matrices with $0 < m \leq A, B \leq M$ for some scalar $0 < m \leq M$, and σ, τ two arbitrary means between the harmonic and the arithmetic means. Put $h = \frac{M}{m}$. Then for every unital positive linear map Φ,

$$\begin{aligned}
\Phi^2(A\sigma B) &\leq K^2(h)\Phi^2(A\tau B),\\
\Phi^2(A\sigma B) &\leq K^2(h)\left(\Phi(A)\tau\Phi(B)\right)^2,\\
(\Phi(A)\sigma\Phi(B))^2 &\leq K^2(h)\Phi^2(A\tau B),\\
(\Phi(A)\sigma\Phi(B))^2 &\leq K^2(h)(\Phi(A)\tau\Phi(B))^2,
\end{aligned}$$

where $K(h) = \dfrac{(h+1)^2}{4h}$ is the Kantorovich constant.

We also give a new characterization of the trace property and operator monotonicity by the squared Cauchy inequality.