Notices from the ISMS

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CONTENTS

INTERNATIONAL SOCIETY FOR MATHEMATICAL SCIENCES · CREATIO VERITAS UTILITAS · SCIENTIAE MATHEMATICAE JAPONICAE · FOUNDED 1948

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Promoting Mathematical Sciences

INTERNATIONAL SOCIETY FOR MATHEMATICAL SCIENCES
Scientiae Mathematicae Japonicae, Notices from the ISMS

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THE WORLD LINE OF KANTOROVICH

S. S. Kutateladze

ABSTRACT. This is an overview of the life and scientific legacy of Leonid V. Kantorovich (1912--1986) who stood at the cradle of linear programming, vector lattices, and computational mathematics.

Linear programming as well as mathematical economics belongs to or at least borders the realm of applied mathematics. The epithets "pure" and "applied" for mathematics have many deficiencies provoking endless discussion and controversy. Nevertheless the corresponding brand names persist and proliferate in scientific usage, signifying some definite cultural phenomena. Scientometricians and fellow mathematicians, pondering over this matter, claim usually that the hallmark of the Russian mathematics is the prevalent trend to unity of the applied and pure mathematicians, which is reflected in the common mathematical infrastructure of all major instances of academic life in Russia. Any glimpse of a gap or contradistinction between the pure and applied mathematics usually brings about the smell of collision, emotion, or at least discomfort to every specialist of a Russian provenance. This is in outright contradistinction to the feelings of American mathematicians. The separate existence of the American Mathematical Society and the Society for Industrial and Applied Mathematics is perfectly natural and universally confortable throughout the States.

(1975 awarded the Nobel Prize)

It is rarely taken into account that these special features of the social life of a particular country are linked with the stances and activities of their creative citizens.

Leonid Vital'evich Kantorovich (1912--1986) will always rank among those Russian scholars trespassed the border between the pure and applied mathematics by personal contribution. He is an exception even in this noble company because of his extraordinary traits stemming from a quite rare combination of the generous gifts of a polymath and practical economist. Describing the place of Kantorovich in synthesizing the exact and verbal methods of reasoning, I. M. Gelfand, the last of the mathematical giants of the 20th century, wrote:[1]

> *Only a very few people of the 20th century turned out to be capable of the required synthesis of the two cultures---mathematics and the human sciences. Among them I can name Andrei Kolmogorov who always understood the world as a unified whole. This was also understood, perhaps on a more naive level, with a strong technocratic influence, by John von Neumann. In the field of social sciences, closer to the humanities, this synthesis was effected by Leonid Kantorovich.*

Date: August 16, 2006.

Key words and phrases. Kantorovich, vector lattice, successive approximations, linear programming.

Based on the author's talk on the closing ceremony at the International Conference "Knatorovich Memorial. Mathematics and Economics: Old Problems and New Approaches" held in the Euler International Mathematical Institute at St. Petersburg on January 8-13, 2004

[1] Gelfand I. M. "Leonid Kantorovich and the Synthesis of Two Cultures. " In:Kantorovich L.V. *Selected works. Part Ⅰ*. Gordon and Brech, 1996, pp.7-9.

When I say ***synthesis*** *I don't mean that the two parts of Kantorovich's heritage are two sides of his personality, that he is sometimes a mathematician, sometimes a specialist in the human sciences. Such combinations occur often: they do not concern us here. What I mean is the all-prevailing light of the spirit that appears in all his creative work...*

Some facts of the life of Kantorovich are now in order to present a rough draft of the list of events and achievements as well as the relevant historic background.

Kantorovich was born in the family of a venereologist at St. Petersburg on January 19, 1912 (January 6, according to the old Russian style). He was a prodigy and so his talent revealed itself very early. In 1926, just at the age of 14, he entered St. Petersburg (then Leningrad) State University (SPSU). Soon he started participating in a circle of G. M. Fikhtengolts for students and in a seminar on descriptive function theory. It is natural that the first academic years formed his life-time environment: D. K. Faddeev, I. P. Natanson, S. L. Sobolev, S. G. Mikhlin, and a few others with whom Kantorovich was friendly all his life also participated in Fikhtengolts's circle.

After graduation from SPSU in 1930, Kantorovich started teaching, combining it with intensive scientific research. Already in 1932 he became a full professor at the Leningrad Institute of Civil Engineering and an assistant professor at SPSU.

From 1934 Kantorovich was a full professor at his alma mater. His close connection with SPSU and the Leningrad Division of the Steklov Mathematical Institute of the Academy of Sciences lasted until his transition to Novosibirsk on the staff of the Institute of Mathematics of the Siberian Division of the Academy of Sciences (now, the Sobolev Institute) at the end of the 1950s.

The letter of Academician N. N. Luzin, written on April 29, 1934, was found in the personal archive of Kantorovich not long ago.[2] This letter demonstrates the attitude of Luzin, one of the most eminent and influential mathematicians of that time, to the brilliance of the young prodigy. Luzin remarked:

...you must know my attitude to you. I do not know you as a man completely but I guess a warm and admirable personality.
However, one thing I know for certain: the range of your mental powers which, so far as I accustomed myself to guess people, open up limitless possibilities in science. I will not utter the appropriate word---what for? Talent---this would belittle you. You are entitled to get more...

Kantorovich had written practically all of his major mathematical works in his "Leningrad" period. (1939 when he invented Kantorovich spaces and linear programming)
Moreover, in the 1930s he mostly published articles on pure mathematics whereas the 1940s became his season of computational mathematics in which he was soon acknowledged as an established and acclaimed leader.

[2] Reshetnyak Yu. G. and Kutateladze S. S. "A. Letter of N. N. Luzin to L.V. Kantorovich, "*Vestnik Ross. Acad . Nauk*, **72**:8 (2002), pp. 740-742

At the end of the 1930s Kantorovich revealed his outstanding gift of an economist. His booklet *Mathematical Methods in the Organization and Planning of Production* is a material evidence of the birth of linear programming. The economic works of Kantorovich were hardly visible at the surface of the scientific information flow in the 1940s. However, the problems of economics prevailed in his creative studies. During the Second World War he completed the first version of his book *The Best Use of Economic Resources* which led to the Nobel prize in economics awarded to him and Tjalling C. Koopmans in 1975.[3]

The priority of the Kantorovich invention was never questioned. George B. Dantzig wrote in his classical book[4] on linear programming:

> *The Russian mathematician L. V. Kantorovich has for a number of years been interested in the application of mathematics to programming problems. He published an extensive monograph in 1939 entitled Mathematical Methods in the Organization and Planning of Production...*
>
> *Kantorovich should be credited with being the first to recognize that certain important broad classes of production problems had well-defined mathematical structures which, he believed, were amenable to practical numerical evaluation and could be numerically solved.*
>
> *In the first part of his work Kantorovich is concerned with what we now call the weighted two-index distribution problems. These were generalized first to include a single linear side condition, then a class of problems with processes having several simultaneous outputs(mathematically the latter is equivalent to a general linear program). He outlined a solution approach based on having on hand an initial feasible solution to the dual. (For the particular problems studied, the latter did not present any difficulty.) Although the dual variables were not called "prices" the general idea is that the assigned values of these "resolving multipliers" for resources in short supply can be increased to a point where it pays to shift to resources that are in surplus. Kantorovich showed on simple examples how to make the shifts to surplus resources. In general, however, how to shift turns out to be a linear program in itself for which no computational method was given. The report contains an outstanding collection of potential applications...*
>
> *If Kantorovich's earlier efforts had been appreciated at the time they were first presented, it is possible that linear programming would be more advanced today. However, his early work in this field remained unknown both in the Soviet Union and elsewhere for nearly two decades while linear programming became a highly developed art.*

In 1957 Kantorovich was invited to join the Siberian Division of the Academy of Sciences to be constructed in the picturesque suburbs of Novosibirsk. He agreed and soon was elected a corresponding member of the Division of Economics of the Academy of Sciences of the USSR. Since then his major publications were devoted to economics, with the exception of the celebrated course of functional analysis[5] - Kantorovich and Akilov" in the students' jargon.

The 1960s became the decade of his recognition. In 1964 he was elected a full member of the Department of Mathematics of the Academy of Sciences of the USSR,and in 1965 he was awarded the Lenin Prize. In these years he vigorously propounded and maintained his views of interplay between mathematics and economics and exerted great efforts to instill the ideas and methods of modern science into the top economic management of the Soviet Union, which was almost in vain.

[3]This award is formally known as the Bank of Sweden Prize in Economic Sciences in Memory of Alfred Nobel.

[4]Dantzig G. B. *Linear Programming and Extensions* . Princeton, Princeton University Press, 1963, pp. 22-23.

[5]Kantorovich L.V. and Akilov G.P *Functional Analysis.* Oxford etc. , Pergamon Press, 1982.

At the beginning of the 1970s Kantorovich left Novosibirsk for Moscow where he was still engaged in economic analysis, not seizing his efforts to influence the everyday economic practice and decision making in the national economy. These years witnessed a considerable mathematical Renaissance of Kantorovich. Although he never resumed proving theorems, his impact on the mathematical life of Russia increased sharply due to the sweeping changes in the Moscow academic life on the eve of Gorbi's "perestroika." Cancer terminated his path in science on April 7, 1986. He was buried at Novodevichy Cemetery in Moscow.

Kantorovich started his scientific research in rather abstract and sophisticated sections of mathematics such as descriptive set theory, approximation theory, and functional analysis. It should be stressed that at the beginning of the 1930s these areas were most topical, prestigious, and difficult. Kantorovich's fundamental contribution to theoretic mathematics, now indisputable and universally acknowledged, consists in his pioneering works in the above-mentioned areas. Note also that in the "mathematical" years of his career he was primarily famous for his research into the approximate methods of analysis, the ancient euphemism for the computational mathematics of today.

The first works of Kantorovich on computational mathematics were published in 1933. He suggested some approaches to approximate solution of the problem of finding a conformal mapping between domains. These methods used the idea of embedding the original domains into some one-parameter family of domains. Expanding in
A parameter, Kantorovich found out new explicit formulas for approximate calculation of conformal mappings between multiply-connected domains.

In 1933 one of Kantorovich's teachers, V. I. Smirnov included these methods in his multivolume treatise *A Course of Higher Mathematics* which belongs now to the world-class deskbooks.

Kantorovich paid much attention to direct variational methods. He suggested an original method for approximate solution of second order elliptic equations which was based on reduction of the initial problem to minimization of a functional over some functions of one variable. This technique is now called reduction to ordinary differential equations.

The variational method was developed in his subsequent works under the influence of other questions. For instance, his collocation method was suggested in an article about calculations for a beam on an elastic surface.

A few promising ideas were proposed by Kantorovich in the theory of mechanical quadratures which formed a basis for some numerical methods of solution of a general integral equation with a singularity.

This period of his research into applied mathematics was crowned with a joint book with V.~I.~Krylov *Methods for Approximate Solution of Partial Differential Equations* whose further expanded editions appeared under the title *Approximate Methods of Higher Analysis*.

Functional analysis occupies a specific place in the scientific legacy of Kantorovich. He is listed among cofathers of the theory of ordered vector spaces together with G. Birkhoff, H. Freudenthal, M. G. Krein, H. Nakano, T. Ogasawara, F.Riesz, et al. At present, the theory of ordered vector spaces constitutes a vast and fruitful area of contemporary mathematics.

Kantorovich contributed much to making functional analysis a natural language of computational mathematics. His article "Functional analysis and applied Mathematics" in *Russian Mathematical Surveys* (1948) made a record in the personal file of Kantorovich as well as in the history of mathematics in Russia. Kantorovich wrote in the introduction to this article:[6]

[6]Kantorovich L.V. *Selected Works. Part II.* Gordon and Breach, 1996, pp.171- 280.

...there is now a tradition of viewing functional analysis as a purely theoretic discipline far removed from direct applications, a discipline which cannot deal with practical questions.

This article is an attempt to break with this tradition, at least to a certain extend, and to reveal the relationship between functional analysis and questions of applied mathematics, an attempt to show that functional analysis can be useful to mathematicians dealing with practical applications.

Namely, we would try to show that the ideas and methods of functional analysis can be readily used to construct and analyze effective practical algorithms for solving mathematical problems with the same success as they were used for the theoretic studies of the problems.

The mathematical ideas of this article remain classical by now: The method of finite-dimensional approximations, estimation of the inverse operator, and, last but not least, the Newton--Kantorovich method are well known to the majority of the persons recently educated in mathematics.

The general theory by Kantorovich for analysis and solution of functional equations bases on variation of “data” (operators and spaces) and provides not only estimates for the rate of convergence but also proofs of the very fact of convergence.

As an instance of incarnation of the idea of unity of functional analysis and computational mathematics Kantorovich suggested at the end of the 1940s that the Mechanics and Mathematics Department of SPSU began to prepare specialists in the area of computational mathematics for the first time in Russia. He prolonged this line in Novosibirsk State University where he founded the chair of computational mathematics which delivered graduate courses in functional analysis in the years when Kantorovich hold the chair. I had a privilege of specialization in functional analysis which was offered by the chair of computational mathematics in that unforgettable span of time.

It should be emphasized that Kantorovich tied the progress of linear programming as an area of applied mathematics with the general demand for improving the functional-analytical techniques pertinent to optimization: the theory of topological vector spaces, convex analysis, the theory of extremal problems, etc. Several major sections of functional analysis (in particular, nonlinear functional analysis) underwent drastic changes under the impetus of new applications.

The scientific legacy of Kantorovich is immense. His research in the areas of functional analysis, computational mathematics, optimization, and descriptive set theory has had a dramatic impact on the foundation and progress of these disciplines. Kantorovich deserved his status of one of the father founders of the modern economic-mathematical methods. Linear programming, his most popular and celebrated discovery, has changed the image of economics.

Kantorovich authored more than 300 articles. When we discussed with him the first edition of an annotated bibliography of his publications in the early 1980s, he suggested to combine them in the nine sections:

(1) descriptive function theory and set theory;
(2) constructive function theory;
(3) approximate methods of analysis;
(4) functional analysis;
(5) functional analysis and applied mathematics;
(6) linear programming;
(7) hardware and software;
(8) optimal planning and optimal prices;
(9) the economic problems of a planned economy.

The impressive diversity of these areas of research rests upon not only the traits of Kantorovich but also his methodological views. He always emphasized the innate integrity of his scientific research as well as mutual penetration and synthesis of the methods and techniques he used in solving the most diverse theoretic and applied problems of mathematics and economics. I leave a thorough analysis of the methodology of Kantorovich's contribution a challenge to professional scientometricians. It deserves mentioning right away only that the abstract ideas of Kantorovich in the theory of Dedekind complete vector lattices, now called ***Kantorovich spaces or K-spaces***,[7] turn out to be closely connected with the art of linear programming and the approximate methods of analysis.

Kantorovich told me in the fall of 1983 that his main mathematical achievement is the development of *K*-space theory within functional analysis while remarking that his most useful deed is linear programming. *K*-space, a beautiful pearl of his scientific legacy, deserves a special discussion.

Let us look back at the origin of *K*-space. The first work of Kantorovich in the area of ordered vector spaces appeared in 1935 as a short note in *Doklady*.[8] Therein he treated the members of a *K*-space as generalized numbers and propounded the *heuristic transfer principle*. He wrote:

> *In this note, I define a new type of space that I call a semiordered linear space. The introduction of such a space allows us to study linear operations of one abstract class (those with values in these spaces) in the same way as linear functionals.*

It is worth noting that his definition of a semiordered linear space contains the axiom of Dedekind completeness which was denoted by *I*6. Therefore, Kantorovich selected the class of *K*-spaces, now named after him, in his first article on ordered vector spaces. He applied *K*-spaces to widening the scope of the fundamental Hahn--Banach Theorem and stated Theorem 3 which is now known as the Hahn--Banach--Kantorovich Theorem. This theorem claims in fact that the heuristic transfer principle is applicable to the classical Dominated Extension Theorem; i.e., one may abstract the Hahn--Banach Theorem on substituting the elements of an arbitrary *K*-space for reals and replacing linear functionals with operators acting into this space.

The diversity of Kantorovich's contributions combines with methodological integrity. It is no wonder so that Kantorovich tried to apply semiordered spaces to numerical methods in his earliest papers. In a note [9] of 1936 he described the background for his approach as follows:

> *The method of successive approximations is often applied to proving existence of solutions to various classes of functional equations; moreover, the proof of convergence of these approximations leans on the fact that the equation under study may be majorized by another equation of a simple kind. Similar proofs may be encountered in the theory of infinitely many simultaneous linear equations and in the theory of integral and differential equations. Consideration of semiordered spaces and operations between them enables us to easily develop a complete theory of such functional equations in abstract form.*

There is no denying that the classical method of majorants which stems from the works of A. Cauchy acquires its natural and final form within *K*-space theory.

Inspired by some applied problems, Kantorovich propounded the idea of a *lattice-normed space* or *Bk*-space and introduced a special decomposability axiom for the lattice norm of a *Bk*-space. This axiom looked bizarre and was often omitted in the subsequent publications of other authors as definitely immaterial. The principal importance of this axiom was revealed only within Boolean valued analysis in the 1980s. As typical of Kantorovich, the motivation of *Bk*-space, now called *Banach--Kantorovich space,* was deeply rooted in abstractions as well as in applications. The general domination method of Kantorovich was substantially developed by himself and his students and followers and occupies a noble place in the theoretic toolkit of computational mathematics.

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[7] Kantrovich wrote about “my spaces” in his personal memos.

[8] Kantorovich L.V. *Selected Works. Part I*. Gordon and Breach, 1996, pp. 213-216.

[9] Kantorovich L. V. On one class of functional equations. *Dokl. Akad. Nauk SSSR,* **4**:5 (1936), pp. 211-216.

The above-mentioned informal principle was corroborated many times in the works of Kantorovich and his students and followers. Attempts at formalizing the heuristic ideas by Kantorovich started at the initial stages of *K*-space theory and yielded the so-called identity preservation theorems. They assert that if some algebraic proposition with finitely many function variables is satisfied by the assignment of all real values then it remains valid after replacement of reals with members of an arbitrary *K*-space.

Unfortunately, no satisfactory explanation was suggested for the internal mechanism behind the phenomenon of identity preservation. Rather obscure remained the limits on the heuristic transfer principle. The same applies to the general reasons for similarity and parallelism between the reals and their analogs in *K*-space which reveal themselves every now and then. The omnipotence and omnipresence of Kantorovich's transfer principle found its full explanation only within Boolean valued analysis in the 1970s.

Boolean valued analysis (the term was minted by G. Takeuti) is a branch of functional analysis which uses special set-theoretic models with truth-values in an arbitrary Boolean algebra. Since recently this term has been treated in a broader sense of nonstandard analysis, implying the tricks and tools that stem from comparison between the implementations of a mathematical concept or construct in two distinct Boolean valued models.

Note that the invention of Boolean valued analysis was not connected with the theory of vector lattices. The appropriate language and technique had already been available within mathematical logic by 1960. Nevertheless, the main idea was still absent for rapid progress in model theory and its applications. This idea emerged when P. J. Cohen demonstrated in 1960 that the classical continuum hypothesis is undecidable in a rigorous mathematical sense.

It was the Cohen method of forcing whose comprehension led to the invention of Boolean valued models of set theory which is attributed to the efforts by D. Scott, R. Solovay, and P. Vopenka.

The Boolean valued status of the notion of Kantorovich space was first demonstrated by Gordon's Theorem[10] dated from the mid 1970s. This fact can be reformulated as follows: *A universally complete K-space serves as interpretation of the reals in a suitable Boolean valued model.* (Parenthetically speaking, every Archimedean vector lattice admits universal completion.) Furthermore, every theorem of ZFC about real numbers has a full analog for the corresponding *K*-space.[11] Translation of one theorem into the other is fulfilled by some precisely-defined Escher-type procedures: ascent, descent, canonical embedding, etc., i.e., by algorithm, as a matter of fact. Thus, Kantorovich's motto: “The elements of a *K*-space are generalized numbers” acquires a rigorous mathematical formulation within Boolean valued analysis. On the other hand, the heuristic transfer principle finished its auxiliary role of a guiding nature in many studies of the pre-Boolean-valued *K*-space theory and becomes a powerful and precise method of research within Boolean valued analysis.

Further progress of Boolean valued analysis revealed that this translation (transfer or interpretation) making new theorems from available facts is possible not only for *K*-spaces but also for practically all objects related to them such as *Bk*-spaces, various classes of linear and nonlinear operators, operator algebras, etc. A.G. Kusraev proved that the heuristic transfer principle for *Bk*-spaces (to within elementary stipulations) reads formally as follows: [12]*Each Banach--Kantorovich space embeds in a Boolean valued model, becoming a Banach space.* In other words, a *Bk*-space is a Boolean valued interpretation of a Banach space. Moreover, it is the “bizarre” decomposability axiom of Kantorovich that guarantees the possibility of this embedding.

Returning to the background ideas of *K*-space theory in his last mathematical paper, [13]Kantorovich wrote just before his death:

[10]Gordon E. I. "Real numbers in Boolean-valued models of set theory, and K-spaces," Soviet Math. Doklady, **18** (1977), pp. 1481-1484. (This article was communicated by Kantorovich.)

[11]ZFC stands for *Zermelo-Frankel set theory* with choice.

[12]Kusraev A..G.“ On Banach –Kantrovich spaces,” *Siberian Math. J.*, **26**:2 (1985), pp.254-259.

[13]Kantorovich L.V. Functional analysis (basic ideas), *Siberian Math. J.*,**28**:1 (1987), pp.1-8.

> *One aspect of reality was temporarily omitted in the development of the theory of function spaces. Of great importance is the relation of comparison between practical*

objects, alongside algebraic and other relations between them. Simple comparison applicable to every pair of objects is of a depleted character; for instance, we may order all items by weight which is of little avail. That type of ordering is more natural which is defined or distinguished when this is reasonable and which is left indefinite otherwise (partial ordering or semiorder). For instance, two sets of goods must undoubtedly be considered as comparable and one greater than the other if each item of the former set is quantitatively greater than its counterpart in the latter. If some part of the goods of one set is greater and another part is less than the corresponding part of the other then we can avoid prescribing any orderbetween these sets. It is with this in mind that the theory of ordered vector spaces was propounded and, in particular, the theory of the above-defined K-spaces. It found various applications not only in the theoretic problems of analysis but also in construction of some applied methods, for instance the theory of majorants in connection with the study of successive approximations. At the same time the opportunities it offers are not revealed fully yet. The importance for economics is underestimated of this branch of functional analysis. However, the comparison and correspondence relations play an extraordinary role in economics and it was definitely clear even at the cradle of K-spaces that they will find their place in economic analysis and yield luscious fruits.

The theory of K-spaces has another important feature: their elements can be treated as numbers. In particular, we may use elements of such a space, finite- or infinite-dimensional, as a norm in construction of analogs of Banach spaces. This choice of norms for objects is much more accurate. Say, a function is normed not by its maximum on the whole interval but a dozen of numbers, its maxima on parts of this interval.

Observe that this excerpt of the Kantorovich article draws attention to the close connection of *K*-spaces with the theory of inequalities and economic topics. It is also worth noting that the ideas of linear programming are immanent to *K*-space theory in the following rigorous sense: The validity of each of the universally accepted formulations of the duality principle with prices in some algebraic structure necessitates that this structure is a *K*-space.

Magically prophetic happens to be the claim of Kantorovich that the elements of a *K*-space are generalized numbers. The heuristic transfer principle of Kantorovich found a brilliant justification in the framework of modern mathematical logic. Guaranteeing a profusion of unbelievable models of the real axis, the spaces of Kantorovich will stay in the treasure-trove of the world science for ever.

Alfred Marshall (1842--1924), the founder of the Cambridge school of neoclassicals, “Marshallians,” wrote in his magnum opus:

The function then of analysis and deduction in economics is not to forge a few long chains of reasoning, but to forge rightly many short chains and single connecting links..[14]

It is obvious that there is no room in economics for long trains of deductive reasoning.[15]

At the same time, there is no gainsay in ascribing the beauty and power of mathematics to the axiomatic method which consists ideally in deriving new bits and bobs of knowledge from however lengthy chains of formal implications.

The conspicuous discrepancy between economists and mathematicians in mentality has hindered their mutual understanding and cooperation. Many partitions, invisible but ubiquitous, were erected in ratiocination, isolating the economic community from its mathematical counterpart and vice versa.

[14]Marshall A. *Principles of Economics.* 8th edition, Macmillian and Co.,Ltd.,1920. Appendix C: The Scope and Method of Economics.

[15] Ibid. Appendix D: Use of Abstract Reasoning in Economics.

This status quo with deep roots in history was always a challenge to Kantorovich, contradicting his views of interaction between mathematics and economics. His path in science is well marked with the signposts conveying the slogan: “Mathematicians and Economists of the World, Unite!” His message has been received as witnessed by the curricula and syllabi of every economics department in a major university throughout the world.

Despite the antediluvian opinion that “the mathematical scientist emperor of mainstream economics is without any clothes,”[16] the gadgets of mathematics and the idea of optimality will come in handy for a practical economist. Calculation will supersede prophesy. Economics as a boon companion of mathematics will avoid merging into any esoteric part of the humanities, or politics, or belles-lettres. The new generations of mathematicians will treat the puzzling problems of economics as an inexhaustible source of inspiration and an attractive arena for applying and refining their formal methods.

The years of Kantorovich's life dim in the past. Yet Kantorovich's world line, lit with his phenomenal personality and seminal ideas, becomes clearer and brighter helping us to chart new roads between mathematics and economics. Most of them will lead to the turnpike of linear programming Kantorovich was the first to traverse...

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[16]Davidson P. "Is 'Mathematical Science ‘ an Oxymoron When Used to Describe Economics?” *J.Post Keynesian Economics,* 25: 4 (2003)

Communications

(1) Conferences for Young Algebraists

*Klaus Denecke

Twenty years ago some mathematicians from universities in Dresden, Potsdam and Rostock (cities in East Germany) started to organize special conferences in the field of general algebra and its applications for young mathematicians.

I was often asked: What is a young mathematician, what is a young algebraist? With the years it became more and more difficult to give an answer. When Dietlinde Lau from Rostock, Reinhard Pöschel from Dresden and myself started to organize these conferences we were young, looked young and nobody asked us this question. A few years later I have met Saunders Mc Lane, he was around eighty, and invited him to take part in one of the conferences for young algebraists. He was a bit surprised and asked: "Do you think that I am young?" "Of course", was my answer.

Our idea twenty years ago was to encourage graduate students of mathematics to do research. As algebraists we were especially interested in young people which are enthusiastic for our field.

Mathematics research is the long-term, open-ended exploration of a set of related mathematics questions whose answers connect to and build upon each other. Problems are open-ended because any solution creates a new problem. For students problems are open-ended because they continually come up with new questions to ask based on their observations. Students use or should learn to use the same general methods which are used by experienced researchers in Mathematics. But there are some additional characteristics of student research which include:

- Students develop questions, approaches and results, that are, only for them, original products.
- Students have to learn to work through cycles of data-gathering, visualization, abstraction, conjecturing, and proof.
- Students have to learn to communicate mathematically: to describe their thinking, to write definitions and conjectures, to use symbols, to justify their conclusions, to read and to write mathematics.
- The young researchers, as a part of the community of mathematicians have to learn how to share and to build on each other's questions, conjectures, and theorems.

One of the basic ideas of the classical university is that of the unity of teaching, education and research.

Mathematics research influences students learning and personalities in several ways:

- Research provides students with an understanding of what it means to do mathematics and of mathematics as a living, growing field.
- Students develop mastery of mathematics topics. We don't learn the basics by studying the basics but by being engaged in rich activities which require them. Research involves repeated application of technical skills.
- Students develop their own mathematical aesthetic as they practice making choices about which aspects of a problem to investigate.
- Students develop confidence as mathematical thinkers and enthusiasm to do more mathematics.
- Doing research is challenging and can be frustrating.
- The habit of looking for counterexamples to claims is a core skill for critical thinkers in all aspects of life.

For many participants in the conferences of young algebraists it is the first time to present their own research results to an international audience of mathematicians. So, we invite as well experienced researchers and good speakers which are able to show the young participants in which way we have to give our presentation. Mentors can help in a number of ways. They can

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*Klaus Denecke is a professor at the University of Potsdam、 Institute of Mathematics, an Editor of SCMJ, and an Editor of Notices from the ISMS.

- make suggestions about next steps,
- ask clarifying questions about the student's mathematical statements,
- help students to learn how to prove their claims,
- provide emotional support such as encouragement, perspective, and advice,
- identify resources.

During the conferences there is always enough time to ask questions, to make critical remarks and to give suggestions.

In the evening sessions we discussed problems of general interest as

- How to prepare a good presentation of my research result? What is a good presentation?
- How to write a mathematical text?
- How to apply for a project and how to find financial sources for a project?

After twenty years, several former participants are now experienced researchers in our field.

Now we are organizing the conferences of young algebraists once a year in Germany, Austria, Czech Republic or Poland.

In 2000 we started a similar activity in Thailand together with former Ph.D. students and cooperators from the University of the Thai Chamber of Commerce in Bangkok, the Khonkaen University, the University of Mahasarakham, the Chiangmai University, the Silpakorn University in Nakhon Pathom and the Naresuan University in Pitsanulok.

In Europe there is another series of conferences on general algebra - the workshops on general algebra (Arbeitstagung Allgemeine Algebra, AAA).

The traditional AAA was founded in 1970 by Rudolf Wille, Technical University Darmstadt. Since then, twice a year researchers meet each other to exchange their views and results in algebra and related topics.

After the German reunification it was intended to merge this conference series with similar activities in East Germany such as the "Arbeitstagung Algebra and Grenzgebiete" (organized by Hans-Jürgen Hoehnke) and in particular, with the "Tagung junger Algebraiker" (Conferences of Young Algebraists/CYA) that was regularly held at Potsdam University (organized by my group of algebraists).

The following list gives some information on the future conferences of young algebraists:

We are looking forward seeing you at the next conferences:

1. CYA (73. AAA), Klagenfurt (Austria), February 1-4, 2007
2. AAA Tampere (Finland), June 7-10, 2007
3. CYA (75. AAA), Darmstadt (Germany), November 1-4, 2007
4. AAA Linz (Austria), May 22-25, 2008
5. CYA (77. AAA), Potsdam, Februar 2009
   (dedicated to K. Denecke)
6. AAA Bern (Switzerland), June 11-14, 2009
7. CYA (79. AAA), Olomouc (Czech Republic), February 2010

The 73rd AAA / 22nd CYA meeting in this series will be held at the Department of Mathematics of Klagenfurt University, Austria, February 1 (evening) - 4 (lunchtime), 2007.

Colleagues which are interested in participating, are cordially invited to fill in the online registration form available from mid of November 2006. They will be informed in a second announcement on further
details like the schedule of lectures in January 2007. The scientific program will start Friday, February 2, at 9 a.m. and will end Sunday, February 4, probably at 1 p.m. On Thursday evening, February 1, there will be a welcome reception.

The following sections are planned:

* Universal Algebra and Lattice Theory
* Classical Algebra
* Applications of Algebra

It is intended to publish the proceedings of the conference as volume 18 in the series "Contributions to General Algebra". As usual all papers for this volume will be refereed. If there are further questions please use e-mail: aaa73@uni-klu.ac.at or write to:

Workshop on General Algebra

Universitaet Klagenfurt, Institut für Mathematik Universitaetsstraße 65, A-9020 Klagenfurt, AUSTRIA

fax: +43-(0)463 2700/3199

Latest information can be found under: http://www.uni-klu.ac.at/aaa73

(2) Announcement of Meetings in Topology

Communicated by Gerhard Preuss*

July 2007

Summer Conference on Topology, and its Applications 2007 Unibversitat Jaume I, Castellón, Spain.
Organizers: J. Font, J.Galindo, S.Hernandez, S.Macario, M.Sanchis, A.Rodenas
See: www.sumtop07.uji.es

August 6-August 12, 2007:

International Conference on Quantum Topology, Hanoi Institute of Mathematics
Hanoi,Vietnam
Description: The conference aims to educate young researchers in Quantum Topology, Discuss new developments in the area, and bring together different points of view of Geometry, Topology, Algebra, and Quantum Field Theory.
Organizers: Do Ngoc Diep and Nguyen Viet Dung (Institute of Mathematics, Hanoi, Vietnam), Stavros Garoufalidis and Thang Le (Georgia Tech).
Scientific Committee: C.Gordon, University of Texas, Austin, V.Jones, University of California, Berkeley, V.Trraev, University of Strasbourg.
See: http://www.math.gatech.edu/~stavros/vietnam.html

(3) Announcement of QTNA 2007 Communicated by Wuyi Yue**

The Second Asia-Pacific Symposium on Queueing Theory and Network Applications (QTNA2007)

August 1-August 4, 2007 International Conference Center, Kobe, Japan

Webpage of QTNA2007 is: http://www.iict.konan-u.ac.jp/QTNA2007/.

(Notices from the ISMS November 2006 P.6)

(4) BIOCOMP2007 Communicated by L.M. Ricciardi ***

stimulated by some friends, and on the grounds of the successful experience of *BIOCOMP2002 and* "*BIOCOMP2005*: Conferences, Prof. L.M. Ricciardi has now been induced to plan another Conference to be held in the same location (Vietri sul Mare, Italy), September 24-28, 2007.

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*Gerhard Preuss is a professor at the Freie Universität Berlin, FB Mathematik, an Editor of SCMJ, and an Editor of Notices from the ISMS.

**Wuyi Yue is a professor of Konan University, an Editor of SCMJ, and an Editor of Notices from the ISMS.

*** L.M.Ricciardi is a professor at Dipartimento di Matematica e Applicazioni, Universita di Napoli Federico II, and an International Advisor of SCMJ.

The title is ***BIOCOMP2007 - Collective Dynamics: Topics on Competition and Cooperation in the Biosciences..***

The title is motivated by the nature of our sponsors and supporting grants, but the main purpose of this Conference is to bring together a limited number of well-known specialists in the fields of applied mathematics, physics and theoretical biology for an in-depth discussion of model building and computational strategies in some selected areas of the life sciences with special emphasis on theoretical neurobiology, molecular motors and quantitative problems in ecology and population dynamics.

This will be implemented through a program of plenary talks, parallel sessions and a poster session.

The interdisciplinary nature of the conference will allow cross-fertilization of recent advances in applied nonlinear mathematics and computational approaches. Several invited lectures on different topics of biomathematical interest will also be given, especially tailored on the needs of graduate students and young researches.

(5) The 7th International Conference on Optimization (ICOTA 7) Communicates by Wuyi Yue

The 7th International Conference on Optimization(ICOTA 7) : Techniques and applications
December 12-December 15, 2007, International Conference Center, Kobe, Japan
ICOTA webpage is: http://www.iict.konan-u.ac.jp/ICOTA7/
(Notices from the ISMS November 2006 p.8)

The ISMS

Result of votes (Bylaws 2007)

The vote of Bylaws 2007 was 14 in favor versus 0 against. Therefore Bylaws 2007 have been approved and the changes (in bold) are approved.

(1) The number of secretaries increases by fourteen (14) people from Bylaws 2006 and becomes **twenty nine** (29).

(2) The number of Council members increases by fourteen (14) people from Bylaws 2006 and becomes **fifty** (50)

The **roles** of the above fourteen (14) secretaries are:

(a) Four (4) secretaries in charge of **publication of Notices**

(b) Three (3) secretaries in charge of **IVMS and Distance Symposium**

(c) Four (4) secretaries in charge of **international co-sponsored meetings**

(d) Three (3) secretaries in charge of **prizes**

Call for candidates for secretaries

In accordance with the above increases, we hereby accept candidates for the secretaries with the deadline of February 20, 2007. The candidacy should be followed by the recommendation of at least two ISMS members notifying the role of the above (a) -(d). The names of the candidates shall be announced in Vol. 65, No. 2 (March 2007) of SCMJ. The term of office for the fourteen secretaries starts on July 1, 2007 and ends June 30, 2009.

Call for Academic and Institutional Members

Discounted subscription price: When organizations become the Academic and Institutioanl Members of the ISMS, they can subscribe our journal Scientiae Mathematicae Japonicae at the yearly price of US$300. At this price, they can add the subscription of the online version upon their request.

Invitation of two associate members: We would like to invite two persons from the organizations to the associate members with no membership fees. The two persons will enjoy almost the same privileges as the individual members do including the discount of the page charge. Although the associate members cannot have their own ID Name and Password to read the online version of SCMJ, they can read the online version of SCMJ at their organization.

To apply for the Academic and Institutional Member of ISMS, please use the following application form.

Application for Academic and Institutional Member of ISMS

Subscription of SCMJ Check one of the two.	☐Print (US$300) ☐Print + Online (US$300)
University (Institution)	
Department	
Postal Address where SCMJ should be sent	
E-mail address	
Person in charge	Name: Signature:
Payment Check one of the two.	☐Bank transfer ☐Credit Card (Visa, Master)
Name of Associate Membership	1.
	2.

Call for regular Members
ISMS Membership Dues from 2007

A new category "life member" has been established and can be applied for from 2005. An eligible member may become a life member by making a one-time payment of dues. A member who has been an ISMS member for ten years or more is eligible for a life member. The amounts of dues are : \70,000 for the domestic members, US$ 600 (€480)for the foreign members, and US$ 500 (€400)for the members in developing countries.

We have reduced the ISMS memership dues since 2001 and copies of the printed journal have not been distributed to the members, free of charge. Instead, we give User Name and Password to each member so that he/she can view or print out the full text of the papers published in SCMJ except papers in the international plaza from our Web site (http://www.jams.or.jp).

The Membership Dues for each category is as follows. Applications for the 3-year members can be made only in 2005 and in every three years.

Membership Dues for 2007

Membership	JAPAN	S-JAPAN	Foreign	S-Foreign	Developing
1-year	A1 ¥7,000	SA1 ¥3,500	F1 US$50 €40	SF1 US$30 €24	D1 US$30 €24
3-year	A3 ¥18,000	SA3 ¥9,000	F3 US$120 €96	SF3 US$60 €48	D3 US$70 €56
Life Member	Life ¥70,000	Life ¥70,000	FL US$600 €480	FL US$600 €480	DL US$500 €400

Category D is for those who reside in the countries of Eastern Europe, CIS or developing countries. Category S is for students and for the aged (older than 70). The figure 1 and 3 means a year and 3 years respectively.

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Payment Instructions

Payment can be made through a post office or a bank, or by credit card. Members may choose the most convenient way of remittance. Please note that we do not accept payment by bank drafts (checks). For more information, please refer to an invoice.

Methods of Overseas Payment:

Payment can be made through (1) a post office, (2) a bank, (3) by credit card, or (4) UNESCO Coupons.

Authors or members may choose the most convenient way of remittance as are shown below. Please note that **we do not accept payment by bank drafts (checks).**

(1) Remittance through a post office to our giro account No. 00930-1-11872 or send International Postal Money Order to our postal address (2) Remittance through a bank to our account No. 94103518 at Shinsaibashi Branch of CITIBANK (3) **Payment by credit cards** (AMEX, VISA, MASTER or NICOS), or (4) Payment by UNESCO Coupons.

Methods of Domestic Payment:

Make remittance
(1) to our Post Office Transfer Account - 00930-3-73982 or
(2) to our account No.1565679 at SUMITOMO BANK, Sakai, Osaka, Japan.

All the correspondences concerning subscriptions, back numbers, individual and institutional memberships, should be addressed to the Publications Department, International Society for Mathematical Sciences.

Membership Application Form (from 2007)

To determine what membership category you are eligible for, read "Join ISMS" on the inside of the back cover.

1. Name: Family Name, First Name, Middle Name (in this order)
2. Home Address
3. Name of Firm or Institution affiliation
4. Postal address to which correspondence should be sent
5. e-mail address
6. Telephone Number, Fax Number
7. Membership Category
8. Panel (Please choose one out of the following 12panels and write the panel number. You could choose one or more.)

(e-1)Mathematical Logic, Set Theory, Lattice Theory, Ordered Systems.
(e-2)Algebra, Algebraic Geometry, Number Theory, Combinatorics, Cryptology.
(e-3) Topology, Geometry, Imaging.
(e-4) Real Analysis, Functional Analysis, Complex Functions.
(e-5) Differenation Equations, Integral Equations, Functional Equations.
(e-6) Fluid Dynamics, Rheology, Imaging and other Applied Analysis, Control Theory, Numerical Analysis、Simulation.
(e-7) Probability, Statistics, Data Mining, Decision theory. Quality Control.
(e-8) Game, Finance, Operations Research, Mathematical Economics. Ecology
(e-9) Informatics, Computer Sciences.
(e-10) Biomathematics, Neuroinformatics, Genome Sciences, Nanoscience.
(e-11) Mathematical Education, History of Mathematics.
(e-12) Over several fields.(Ex. Fixed Point Theory, Semi-group)

9. Would you like to buy the printed copies of SCMJ, whose prices a year are US$60(6,000yen) for 1-year-members(A1, D1, S-A1, S-D1)and US$55(5,500yen) for 4-year-members(A4, D4, S-A4, S-D4) ? Type YES or NO.
10. If you apply for an aged member (70 years old or over), please type the year of your birth.
11. If you wish to be a student member, please verify.
12. Is your university (institution) an Academic or Institutional Member of the ISMS? Yes or No.
13. If the answer of 12 is Yes, please answer the following. Are you designated associate member by your university (institution)?
14. Date
15. Signature

For Japanese Applicants, please send two application forms, one in English and the other in Japanese.

I wish to enroll as a member of ISMS and will pay to International Society for Mathematical Sciences the annual dues upon presentation of an invoice. Copies of Mathematica Japonica, Scientiae Mathematicae and Scientiae Mathematicae Japonicae received as an ISMS member will be for my personal use and shall not be placed in institutional, university or other libraries or organizations, nor can membership subscriptions be used for library purposes.

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Join ISMS !

ISMS Publications: We published **Mathematica Japonica (M.J.)**, which enjoyed an international reputation, for about sixty years in print and its offshoot **Scientiae Mathematicae (SCM)** both online and in print. In January 2001, the two publications were unified and changed to **Scientiae Mathematicae Japonicae (SCMJ)**, which is the "21st Century New Unified Series of Mathematica Japonica and Scientiae Mathematicae" and published both online and in print. Ahead of this, the online version of SCMJ was first published in September 2000. The number of the annual total pages of the print version has been from 900 to 1,200 pages in six issues since January 1978. The whole number of SCMJ exceeds 240, which is the largest amount in the publications of mathematical sciences in Japan.

The features of SCMJ are:

1) About 90 eminent professors and researchers of not only Japan but also 20 foreign countries join the Editorial Board. The submitted papers are received directly by the editors and are refereed quickly. The accepted papers are published online with no lead time after compiling or proofreading. SCMJ is reviewed by Mathematical Review and Zentralblatt from cover to cover.
2) SCMJ is distributed to many libraries of the world. The papers in SCMJ are introduced to the relevant research groups for the positive exchanges between researchers.
3) The original papers and surveys of distinguished mathematical scientist appear in every issue of SCMJ. The section called "International Plaza" of SCMJ has very interesting expository papers written by the eminent mathematical scientist of the world. Presentations of recent research frontier including award lectures by the winners of the ISMS Prize or Shimizu Prize are made.
4) **ISMS Annual Meeting:** Many researchers of ISMS members and non-members gather and take time to make presentations and discussions in their research groups every year.
5) The ISMS holds inter-regional videoconferences called **International Videoconference of Mathematical Sciences (IVMS)** via internet. There is no need for the participants to travel abroad.

Privileges to ISMS Members: (1) Free access (**including printing out**) to the online version of SCMJ, (2)Discounted price for the printed version of SCMJ (See **Table 1**), (3) Discounted page charges (See **Table 2**).
Privileges to Institutional Members: (1) Two associate members can be registered, free of charge, from an institution. (2) The discounted page charges (Table 2) are applied to the associate members.

Table 1: Subscription Price (from 2007)

	Individual 1-year mem.	Individual 3-year mem.	Institutional member	List Price
Print / year	¥6,000 US$60, €48	¥5,500 * US$55, €44	¥33,000 US$300, €240	¥45,000 US$400, €320
Online/year	Free	Free	—	—
Online+Print / year	¥6,000 US$60, €48	¥5,500 * US$55, €44	¥33,000 US$300, €240	¥45,000 US$400, €320

Postal charge is US$2 (€1.6) per issue. *In case three-year members make the payment at a time in advance, the price for 3 years is \15,000 (US$150, €120). The authors can buy a copy of the print version at a price of ¥1,200 (US$12) per issue including postage.

Table 2: Page Charge per printed page

	Individual/Associate Member	Non Member
Paper : P	¥3,850 (US$35, €28)	¥4,450 (US$43、€35)
TeX: T	¥2,200 (US$18, €14)	¥2,800 (US$26, €21)
ISMS style: Js	¥1,100 (US$8, €7)	¥1,700 (US$16, €14)

The above page charges include 20 offprints.

Table 3: Membership Dues for this year

Categories	Domestic	Overseas	Developing countries
1-year member (1A)	A1: ¥7,000	F1: US$50 , €40	D1: US$30, €24
3-year member (3A)	A3: ¥18,000	F3: US$120, €96	D3: US$70, €56
1-year students or aged (1S)	SA1: ¥3,500	SF1: US$30, €24	SD1: US$20, €16
3-year students or aged (3S)	SA3: ¥9,000	SF3: US$70, €56	SD3: US$50, €40
Life member* (L)	AL: ¥70,000	FL: US$600, €480	DL: US$500, €400

*The members who have been the ISMS members for more than 10 years are eligible for this category.
The categories 1S and 3S are for students or persons over 70 years old.